# 7.3 The Tangent Ratio

**Student Text Pages**
352–365

**Suggested Timing**
160 min

**Tools**
- grid paper
- protractor
- ruler

**Technology Tools**
- computer
- *The Geometer's Sketchpad*®
- graphing calculator
- Cabri® Jr.

**Related Resources**
- G–1 Grid Paper
- G–4 Protractor
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 7–10 Section 7.3 Practice Master
- A–7 Thinking General Scoring Rubric
- A–22 Report Checklist

**TI-Navigator™**

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

## Teaching Suggestions

- The purpose of this section is to introduce the tangent ratio and connect it to the slope of a line segment. Students discover that both of these are constant for a given slope angle, independent of the size of the ramp or triangle from which it is measured. Students then learn how to apply the tangent ratio to solve various problems, and how to use a scientific or graphing calculator to perform the tangent function and its inverse.
- The tangent ratio is introduced before the sine and cosine ratios, because it relates to students' prior knowledge of slope.

### Investigate

- The **Investigate** can be carried out in three ways—using pencil and paper, *The Geometer's Sketchpad*® (GSP), or Cabri® Jr. on a graphing calculator. The recommended method is GSP. Use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity. (40–80 min)
- The purpose of the activity is for students to learn that the tangent ratio of a right triangle is defined as the ratio of the opposite side to the adjacent side as related to one of the acute angles. Students discover that this ratio does not change as the size of the triangle changes, holding the angles constant. They also find that the tangent changes when the measure of the angle changes.

### Examples

- Discuss the **Examples**. (30–40 min)
- **Example 1** provides an opportunity for students to identify the opposite and adjacent sides of triangles in various orientations. Parts b) and c) illustrate how these sides become reversed depending on which acute angle is considered. Students should note that the tangent ratio can be expressed as a fraction or as a decimal, the choice of which may depend on the application.
- **Example 2** shows how the tangent function of a scientific or graphing calculator works. Students need to explore their own calculator to determine which sequence of keys to use and how to adjust the mode. Refer to the Technology Tip in the margin for additional support.
- **Example 3** introduces the inverse tangent function, which performs the opposite operation of the tangent function. This is useful to find an angle from known information about the lengths of the opposite and adjacent sides. Once one of the two unknown acute angles is found, the tangent ratio can be applied a second time or geometric reasoning can be applied to find the measure of the third angle. These are presented as Method 1 and Method 2, respectively.
- **Example 4** shows how the tangent ratio can be applied to find a missing opposite side length when the adjacent side and the angle are known.

- **Example 5** presents a problem that requires a number of steps to solve. The first step involves finding the adjacent side, given the opposite side and the value of the angle. This is slightly more complicated than finding the opposite side, because both sides of the equation are multiplied by $d$ and then divided by tan 65° to isolate the variable. Once the adjacent side is found, the Pythagorean theorem is applied to solve for the hypotenuse. The graphing calculator screen shows how the final calculation can be performed in one step.

## Communicate Your Understanding

- Review the vocabulary in this section (slope angle, tangent of an angle) before discussing the **Communicate Your Understanding** questions as a class or in small groups. (5 min)
- Use these questions to assess student understanding of what the tangent ratio is, how to identify the opposite and adjacent sides of a triangle, and how to apply the tangent ratio to find unknown side lengths and angle measures.
- Use **BLM 7–10 Section 7.3 Practice Master** for remediation or extra practice.

### Investigate Answers (pages 252–256)

**Method 1**

**1.** Diagrams may vary.

**2. a)** Answers may vary. Note: Slope values for the triangles with a 10° slope angle should be about 0.176, triangles with a 25° slope angle should be about 0.466, and triangles with a 60° slope angle should be about 1.732.

**b)** The slopes of the ramps for similar right triangles are equal.

**3. a)** Diagrams may vary. The slopes of the ramps for similar right triangles with the same slope angle are equal.

**b)** The tangent ratio of the slope angle is equal to the slope.

**4.** Answers may vary. For example: The tangent of the other acute angle can be found by dividing the length of the opposite side by the length of the adjacent side.

**5. a)** The tangent ratio of an angle is the length of the opposite side divided by the length of the adjacent side in a triangle with a right angle. This is the same as the rise (vertical change) divided by the run (horizontal change).

**b)** The tangent of the angle remains the same.

**Method 2**

**3. b)** Answers may vary.

**c)** The ratio represents the slope of the ramp.

**4.** Answers may vary.

**5. a)** Answers may vary. For example: If the vertex to the left of the right angle in the triangle is clicked and dragged, the length of the rise and the length of the run both change, but the slope and the slope angle do not change. If the vertex above the right angle is clicked and dragged, the length of the rise, the slope, and the slope angle all change, but the length of the run does not change. If the vertex that is the right angle is clicked and dragged, the length of the rise and the length of the run both change, but the slope and the slope angle do not change.

**b)** Yes, the triangle remains a right triangle. Answers may vary.

**6. a)** Answers may vary. For example: The slope angle, the rise, and the slope change, but the run does not change.

**b)** Answers may vary. For example: The rise and the run change, but the slope and the slope angle do not change.

**7.** The slope of the ramp does not change if the angle remains the same. Answers may vary.

**8.** The tangent of each slope angle is equal to the slope of the ramp for each slope angle.

**9.** Answers may vary. For example: From the **Measure** menu, choose **Calculate**. A calculator appears. Select the run measure. Click on ÷. Select the rise measure. Click **OK**. A ratio appears. This ratio represents the tangent of the other acute angle in the triangle.

**10. a)** The tangent ratio of an angle is the ratio of the side opposite an angle to the side adjacent to the angle. This ratio is the same as the slope.

**b)** The tangent of an angle remains the same when you change the size of the triangle but keep the angle the same.

**Method 3**

**3. b)** This ratio represents the slope of the ramp.

**4.** Answers may vary.

**5. a)** Answers may vary. For example: If the vertex to the left of the right angle in the triangle is clicked and dragged, the length of the rise and the length of the run both change, but the slope and the slope angle do not change. If the vertex above the right angle is clicked and dragged, the length of the rise, the slope, and the slope angle all change, but the length of the run does not change. If the vertex that is the right angle is clicked and dragged, the length of the rise and the length of the run both change, but the slope and the slope angle do not change.

**b)** Yes, the triangle remains a right triangle. Answers may vary.

**6. a)** Answers may vary. For example: The slope angle, the rise, and the slope change, but the run does not change.

**b)** Answers may vary. For example: The rise and the run change, but the slope and the slope angle do not change.

**7.** The slope of the ramp does not change if the angle remains the same. Answers may vary.

**8.** The tangent of each slope angle is equal to the slope of the ramp for each slope angle.

**9.** Answers may vary. For example: Choose **Calculate** from the **F5** menu. Move the cursor to the run measurement, and press ENTER. Move the cursor to the rise measurement. Press ENTER. Then, press ÷. Move the calculation to a convenient location, and press ENTER. This ratio represents the tangent of the other acute angle.

**10. a)** The tangent ratio of an angle is the ratio of the side opposite an angle to the side adjacent to the angle. This ratio is the same as the slope.

**b)** The tangent of an angle remains the same when you change the size of the triangle but keep the angle the same.

## Common Errors

- Some students may mix up the opposite and adjacent sides in a triangle.

$R_x$ Have students draw an arrow from the angle across the triangle to the opposite side, until they are comfortable with identifying the opposite side mentally.

- Calculator outputs do not make sense.

$R_x$ Ensure that the calculator is set to degree mode.

- Some students may misinterpret implied multiplication (e.g., tan 30° read as "tan times 30°").

$R_x$ Reinforce the idea that tangent is a ratio that is determined by the angle formed by the adjacent and hypotenuse sides, and that the multiplication of two terms is not involved.

## Accommodations

**Gifted and Enrichment**—Challenge students to investigate the properties of the tangent function using a graphing calculator.

**Perceptual**—Provide students with diagrams for the questions that do not have diagrams.

**Motor**—Let students work with a partner when completing the Investigate in this section.

**ESL**—Allow students to use their dictionaries and translators to understand the new words in this section.

**Communicate Your Understanding Responses (page 361)**

**C1.** Answers may vary. For example: The tangent of the slope angle is equal to the slope of the ramp.

**C2. a)** Answers may vary. For example: The opposite side is across from a specified acute angle in a right triangle. Once the opposite side has been determined, then the adjacent side is the other side that is not the hypotenuse.

**b)** Depending on which acute angle has been specified, the opposite and adjacent sides can be switched.

**C3.** Answers may vary. For example:

**a)** The tan function finds the tangent ratio for a given angle, and the $\tan^{-1}$ function finds the angle for a given tangent ratio.

**b)** You would use the tan function to find the tangent ratio for a given angle and the $\tan^{-1}$ function to find the angle for a given tangent ratio.

**c)** The functions are inverses of each other.

## Practise

- Ensure that calculators are set to degree mode.
- In **questions 1**, **2**, and **8**, students need to take care when identifying the location of the acute angle and the corresponding opposite and adjacent sides.
- For **question 8**, the computational steps are reduced if the angle is chosen such that the variable representing the unknown side appears in the numerator.
- In **question 12**, students need to apply symmetry and geometric reasoning to derive a right triangle and deduce information about it.
- In **question 13**, connections are made to the (average) speed-distance-time relationship.
- For **questions 15** and **16**, encourage students to begin by drawing a diagram and labelling given information.
- For **questions 17** and **18**, have students draw a diagram showing the two triangles separated. This may make it easier to visualize the problem and identify a starting point.
- **Questions 19** and **23** establish some grounding for further study of the tangent function (in MCR3U) and limits (in MCV4U). Use **A–7 Thinking General Scoring Rubric** when assessing students for **question 19**.
- In **question 20**, drawing diagrams with very short opposite (for tan 0°) and adjacent (for tan 90°) sides may help students to understand the results. Connections should be made to slope and what students understand about the slopes of horizontal and vertical line segments. GSP is a useful tool for exploring these situations as well.
- **Questions 24** and **25** make connections to physics.

## Literacy Connections

Draw attention to the marginal item on page 352, the definition of "slope angle." The definition of "tangent of an angle" is on page 353.

Discuss the Literacy Connections on page 353. Stress that "tan" always has an angle associated with it; it is never just tan (unless you are on a beach). Discuss other Greek letters used for angles. Following theta, the most commonly used letters are alpha $\alpha$ and beta $\beta$. As an extension, have students find other Greek letters used in mathematics and what they symbolize.

Discuss the Literacy Connections on page 357. Again, this relates to the tangent and the fact that it is the tangent of an angle, not just a tangent on its own. As an extension, discuss the idea of a tangent to something, as in the tangent line, and the differences between the tangent of an angle and the tangent as a line.

## Student Success

Have students perform an Internet search and write a report on the history of the tangent ratio. Use **A–22 Report Checklist** to support this activity.

Use a concept attainment strategy to have students develop the tangent ratio.

Refer to the introduction of this Teacher's Resource for more information about how to use a concept attainment strategy.

Discuss the Literacy Connections on page 359, in which the inverse tangent is described.

Add "slope angle," "tangent," and "inverse tangent" to the Word Wall.

## Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

| Process Expectations | Selected Questions |
|---|---|
| Problem Solving | 11, 12, 27, 28 |
| Reasoning and Proving | 19, 20, 23 |
| Reflecting | 12, 22, 26 |
| Selecting Tools and Computational Strategies | 11 |
| Connecting | 9–15, 18, 21, 22–26 |
| Representing | 14, 15, 18–20, 23 |
| Communicating | 12, 19, 20, 22, 23 |

## Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.