7.4 The Sine and Cosine Ratios

Student Text Pages
366–377

Suggested Timing
80–160 min

Tools
- grid paper
- protractor
- ruler

Technology Tools
- computer
- *The Geometer's Sketchpad*®
- graphing calculator
- Cabri® Jr.

Related Resources
- G–1 Grid Paper
- G–4 Protractor
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 7–1 Great North American Trigonometry Race Map
- BLM 7–11 Overlapping Triangles
- BLM 7–12 Section 7.4 Practice Master
- BLM 7–13 Section 7.4 Achievement Check Rubric
- A–7 Thinking General Scoring Rubric
- A–9 Communication General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the files for this section.

Teaching Suggestions

- The purpose of this section is to introduce the sine and cosine ratios and to show how they can be used to solve for unknown side lengths and angle measures in right triangles.
- Discuss the photo, GPS devices, and what role trigonometry plays in navigation. (5 min)

Investigate

- In this activity students discover the other two primary trigonometric ratios, sine and cosine. Because of the angle properties of similar triangles, the sine and cosine ratios remain constant for a given angle. The following mnemonic may be helpful in remembering the definitions of the three primary trigonometric ratios: SOH CAH TOA
  SOH **S**ine is **O**pposite over **H**ypotenuse
  CAH **C**osine is **A**djacent over **H**ypotenuse
  TOA **T**angent is **O**pposite over **A**djacent
  Refer students to the Literacy Connections on page 368.
- The **Investigate** can be done with either grid paper, *The Geometer's Sketchpad*® (GSP), or Cabri® Jr. on a graphing calculator. The Technology Tip in the margin of page 367 describes the steps to produce several overlapping similar triangles using dynamic geometry software. Use **BLM 7–11 Overlapping Triangles**, or **T–4 *The Geometer's Sketchpad*® 3**, or **T–5 *The Geometer's Sketchpad*® 4** to support this activity.
- Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the ready-made GSP file that can be used for demonstration or investigation purposes. (15–20 min)

Examples

- Discuss the **Examples** as a class. (35–40 min)
- **Example 1** illustrates how to find the three primary trigonometric ratios, given three sides of a right triangle. These can be written as either fractions or decimals. The choice of which to use usually depends on the application of the result, if any. The Making Connections at the bottom of page 368 explains why one might want to express these values correct to three or four decimal places before rounding the final answer to a problem.
- **Example 2** provides an opportunity to explore the sine and cosine functions of a scientific or graphing calculator.
- **Example 3** provides a nautical context to illustrate how the sine and cosine ratios can be used to find an unknown angle. For part a), the diagram takes a top view of the situation. Students should take note of the convention for writing and reading bearings, as outlined in the solution and the Literacy Connections marginal item. Part b) can be best understood by viewing the situation from the side.
- **Example 4** illustrates the process of solving a right triangle. Ensure students realize that there often is more than one valid sequence of steps to solving a triangle. Discuss with the class other approaches that could have been applied to solving the triangle. Students should adopt the convention of labelling vertices using capital letters and labelling sides in lower case letters that correspond to the vertex opposite to them. These concepts are explained in the Literacy Connections marginal items.

Communicate Your Understanding

- Review the vocabulary in this section (primary trigonometric ratios) before discussing the **Communicate Your Understanding** questions as a class or in small groups. (5 min)
- Use these questions to assess students' readiness to solve right triangles. Students should be able to decide which trigonometric ratio or ratios can be applied for a given set of measures, and be able to outline a sequence of steps that can be taken to solve for all side lengths and angle measures in a right triangle.
- Use **BLM 7–12 Section 7.4 Practice Master** for remediation or extra practice.

Investigate Answers (pages 366–367)

1. a) Diagrams may vary.

b)–d) Answers may vary.

2. a) Diagrams may vary.

b) The corresponding angles in each of the overlapping triangles are equal to the corresponding angles in the first triangle.

c) Answers may vary.

d) The $\frac{\text{opposite}}{\text{hypotenuse}}$ ratios for each triangle are equal.

3. a) Answers may vary.

b) The $\frac{\text{adjacent}}{\text{hypotenuse}}$ ratios for each triangle are equal.

4. a) Diagrams may vary.

b)–d) Answers may vary.

5. Answers may vary. For example: The three primary trigonometric ratios are sine, cosine, and tangent. They are often abbreviated as sin, cos, and tan. The three primary trigonometric ratios for an angle A are defined as follows:

$\sin A = \frac{\text{oppposite}}{\text{hypotenuse}}$, $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$, and $\tan A = \frac{\text{opposite}}{\text{adjacent}}$.

Communicate Your Understanding Responses (page 372)

C1. Answers may vary. For example: The primary trigonometric ratios depend only on a given angle and not the size of a right triangle because all right triangles with a given acute angle will be similar triangles and the ratios of corresponding sides will be equal.

C2. Answers will vary.

C3. a) Answers may vary. For example: Use the fact that in a right triangle the two acute angles are complementary to solve for ∠R. Find side PQ using the tangent ratio of ∠P and side PR using the sine ratio of ∠P.

b) Answers may vary. For example: Use the fact that in a right triangle the two acute angles are complementary to solve for ∠R. Find side PQ using the tangent ratio of ∠R, and side PR using the Pythagorean theorem.

Practise

- Remind students to check that their calculators are set to degree mode.
- In **question 5**, a connection is made to geometric properties of right triangles.
- For **questions 12** through **14**, students must decide which trigonometric ratio to apply in each case. Use the SOH CAH TOA mnemonic to identify which trigonometric ratio relates two pieces of given information.
- For **question 16**, remind students to work in common units of length.
- For **questions 17** through **19**, **22**, and **23**, encourage students to begin by drawing a diagram and labelling given information.
- In **question 26**, a connection is drawn to the speed-distance-time relationship.

Common Errors

- Some students may mix up the opposite and adjacent sides in a triangle.

R_x Have students draw an arrow from the angle across the triangle to the opposite side, until they are comfortable with identifying the opposite side mentally.

- Calculator outputs do not make sense.

R_x Ensure that the calculator is set to degree mode.

- Some students may apply the incorrect trigonometric ratio for a given problem.

R_x Have students clearly identify the angle in question and the opposite, adjacent, and hypotenuse sides of the triangle. Select a trigonometric ratio that relates two pieces of given information to an unknown piece. Use the SOH CAH TOA mnemonic (or another strategy) to help remember how the trigonometric ratios are defined.

- **Question 27** provides an opportunity to discuss how time and effort can be reduced by using the most efficient tool to solve a problem.
- **Question 28** provides an opportunity to assess students' thinking and communication skills. Encourage students to draw a diagram and pick one of the angles to focus on. Find the sine and cosine of the angle and then switch angles and do the same. Repeat for several triangles. Use **A–7 Thinking General Scoring Rubric** or **A–9 Communication General Scoring Rubric** when assessing students for this question.
- **Question 29** is an opportunity to assess students' reasoning and communication skills. Encourage students to support their explanations with diagrams. Use **A–9 Communication General Scoring Rubric** when assessing students for this question.
- Use **BLM 7–1 Great North American Trigonometry Race Map** to support **question 30**.
- For **questions 32** and **33**, encourage students to break the diagram up into two parts.
- In **questions 34** and **35**, students begin to explore properties of related angles. This becomes important in future studies when students learn about trigonometric ratios of angles greater than 90° (in grade 11) and identities (in grade 12).
- **Questions 36** and **37** provide further basis for future work in trigonometric identities. The ability to reason abstractly is vital preparation for future mathematics, science, and engineering students.
- The type of problem (speed triangle) in **question 38** is studied extensively in grade 12 Calculus and Vectors and in grade 12 Physics.

Achievement Check Sample Solution, question 31, page 376

Provide students with **BLM 7–13 Section 7.4 Achievement Check Rubric** to help them understand what is expected.

31. a) The ladder is 2 m, or 200 cm, from the base of the wall.

$$\tan 75° = \frac{h}{200}$$
$$200(\tan 75°) = h$$
$$746 \doteq h$$

The ladder reaches about 746 cm up the wall.

b) $\cos 75° = \frac{200}{x}$

$$x = \frac{200}{\cos 75°}$$
$$x \doteq 773$$

The ladder is about 773 cm long.

c) Find the new height, H, on the wall and the new distance, d, from the base of the wall.

$$\sin 55° = \frac{H}{773} \qquad \cos 55° = \frac{d}{773}$$
$$773(\sin 55°) = H \qquad 773(\cos 55°) = d$$
$$633 \doteq H \qquad 443 \doteq d$$

The top of the ladder has slipped 746 – 633, or 113 cm down the wall.

The base of the ladder has slipped 443 – 200, or 243 cm away from the wall.

So, the end of the ladder on the ground slips 130 cm more than the end against the wall.

Accommodations

Gifted and Enrichment—Challenge students to learn more about Global Positioning System (GPS) devices.

Perceptual—Encourage students to label the sides of a right triangle systematically (in the order hypotenuse, opposite, adjacent) for each triangle, and to use a colour-coding system for the sides.

Language—Let students work with a partner to review the steps required to use a calculator to determine the sine and cosine values for different angles, and to determine the angles for different sine and cosine values.

Memory—Provide students with visual and/or verbal clues for remembering the three primary trigonometric ratios.

Student Success

Use a concept attainment strategy to have students develop the sine and cosine ratios.

Use a graffiti exercise to have students each add a line to the solution of a problem.

Refer to the introduction of this Teacher's Resource for more information about how to use concept attainment and graffiti strategies.

Literacy Connections

Draw attention to the Literacy Connections on page 367, which gives the short forms for sine and cosine. Discuss the fact that each short form has three letters, so there is consistency in the ratios and each one is the sine or cosine of an angle just as the tangent was the tangent of an angle. Ensure that students know that sin θ is pronounced "sign theta" not "sin theta."

There is also a marginal item on page 367 that identifies the primary trigonometric ratios.

The Literacy Connections on page 368 gives the mnemonic SOH CAH TOA as a good way for students to remember the ratio relationships. It will be useful for all questions involving the primary trigonometric ratios. If time permits, read students the article *The Trigonometric Review*. Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the article.

Another Literacy Connections appears on page 369 in which the term "bearing" is explained. Ask students involved in Cadets if they have used this in their cadet group. Alternatively, ask students if they have seen or heard this word used in films or on television shows. There are many applications using the term "bearing."

Two more Literacy Connections are on page 370. One discusses the idea of solving a triangle; the other explains the convention of using capital letters to denote the angles and lower case letters to represent the lengths of the sides opposite.

Add "sine," "cosine," "primary trigonometric ratios," "SOH CAH TOA," and "bearing" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	26, 28, 30, 31, 38, 40, 41
Reasoning and Proving	5, 16, 28–31, 35–37
Reflecting	26, 28, 37
Selecting Tools and Computational Strategies	12–16, 25, 27, 28, 30–33, 38
Connecting	15, 16, 19, 21, 24, 26, 27, 30, 31, 38
Representing	5, 13, 14, 17, 18, 22, 23, 28–31
Communicating	5, 16, 26, 28, 29, 31, 34

Ongoing Assessment

- Use Achievement Check question 31 to monitor student success. See Achievement Check Answers and **BLM 7–13 Section 7.4 Achievement Check Rubric**.
- Chapter Problem question 30 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.