8.1 The Sine Law

Student Text Pages
396–404

Suggested Timing
80 min

Tools
- ruler
- protractor

Technology Tools
- computer
- *The Geometer's Sketchpad*®

Related Resources
- G–4 Protractor
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 8–3 The Sine Law and *The Geometer's Sketchpad*®
- BLM 8–4 Section 8.1 Practice Master
- A–7 Thinking General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

- Discuss the photo of the Bermuda Triangle and its mysteries. Ask students how big they think this region is. How could they estimate its area or perimeter? (5 min)
- In Chapter 7, students learned how to solve right triangles using trigonometric ratios. Explain that trigonometry can also be used to solve oblique triangles. Students will learn to solve problems involving acute triangles in grade 10 and those involving obtuse triangles in grade 11.

Investigate

- This **Investigate** can be performed with or without technology. If needed, use **BLM 8–3 The Sine Law and *The Geometer's Sketchpad*®** with **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity. If using *The Geometer's Sketchpad*® (GSP), students can explore different triangles by simply clicking and dragging one or more vertices of the triangle. Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the ready-made GSP file that can be used for demonstration or investigation purposes. (10–15 min)
- Whether using GSP or pencil and paper, have students work alone or in small groups, but then compare results for many different triangles to generalize the results (step 3).
- The purpose of the activity is for students to discover that the ratio of a side to the sine of its corresponding angle is a constant for any given acute triangle.
- After students have discovered the sine law, go through the algebraic development of the sine law, as outlined on page 397. Students should be aware that the sine law can be written with either the sides or the sines of the angles in the numerators, according to convenience. This is illustrated on page 399, between **Examples 1** and **2**. The Literacy Connections on that page explains why this property holds true. Generally, it is preferred to use the form in which the unknown appears in the numerator. (10 min)
- Students who finish the **Investigate** early could explore question 19 on page 404 (Is there a corresponding relationship involving cosines or tangents?).

Examples

- Discuss the **Examples** with the class. (25–30 min)
- **Example 1** illustrates how to find an unknown side using the sine law. Students need to recall basic geometric concepts (i.e., the sum of three interior angles in a triangle is 180°) and how to clear fractions when solving equations.
- **Example 2** illustrates how the sine law can be used to solve for an unknown angle. Students need to recall how to use the inverse trigonometric functions of their calculators.
- Graphing calculator output is shown. Slight variations may be necessary for different calculators, particularly scientific ones. Ensure that calculators are set to degree mode.
- **Example 3** applies the sine law twice to solve a contextual perimeter problem. Students must make connections to basic geometry and perimeter concepts. It should be noted that when using the sine law,

once two pairs of sides and angles are known, either pair can be used to find the final unknown side (see Method 1 and Method 2 on page 400). Answers may vary slightly depending on the number of decimal places carried when rounding at various steps in solving a problem. Generally, a final answer should not be expressed with any greater accuracy (i.e., number of significant digits) than any of the initial measures given in the problem. It may be a good idea, however, to carry one or two additional digits through the intermediate calculation steps before rounding at the end.

Communicate Your Understanding

- Review the vocabulary in this section (sine law) before discussing the **Communicate Your Understanding** questions. (5 min)
- It is important for students to understand that only two parts of the sine law are used at any given time. They should also recognize when the sine law can be used (i.e., when you know at least two sides and a non-contained angle, or two angles and a side), and that they can choose whether to express the sides in the numerators or the denominators.
- Use **BLM 8–4 Section 8.1 Practice Master** for remediation or extra practice.

Investigate Answers (page 396)

1. a)–c) Answers will vary.

2. a) Answers will vary.

b) Answers may vary. For example: $\frac{a}{b} = \frac{\sin A}{\sin B}$, $\frac{a}{c} = \frac{\sin A}{\sin C}$, and $\frac{b}{c} = \frac{\sin B}{\sin C}$.

3. a) Answers will vary.

b) The three ratios are equivalent. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

4. Yes. The results are the same for two different triangles. Answers will vary.

5. In an acute triangle the ratios of each of the sides to the opposite angles are equivalent. The relationship is expressed as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Communicate Your Understanding Responses (page 401)

C1. a) The sine law appears to have three parts because there are three ratios that are equal.

b) No. Use the two ratios where you are given three of the four measurements required. Solve the proportion for the unknown value.

C2. For $\triangle ABC$, solve for $\angle C$ using the fact that the sum of the interior angles in a triangle is 180°, then solve for sides a and b using the sine law. For $\triangle PQR$, solve for $\angle P$ using the sine law, then solve for $\angle R$ using the sum of the interior angles in a triangle is 180°, then solve for r using the sine law. $\triangle JKL$ cannot be solved using the sine law because the given angle is not opposite one of the given sides. $\triangle TUV$ cannot be solved using the sine law because no angles are given.

C3. To use the sine law, you need two sides and an angle opposite one of the two sides, or two angles and any side.

C4. It is easier to use $\frac{a}{\sin A} = \frac{b}{\sin B}$ if you are solving for an unknown side, a or b.

It is easier to use $\frac{\sin A}{a} = \frac{\sin B}{b}$ if you are solving for an unknown angle, $\angle A$ or $\angle B$.

Answers will vary.

Common Errors

- Calculator outputs may not make sense.

R_x Have students check that calculators are in degree mode and adjust if necessary.

- Some students may substitute improperly into the sine law.

R_x Have students practise identifying angles and their corresponding sides by drawing an arrow from the angle across the triangle to identify its corresponding side. Encourage students to label angles using capital letters and their corresponding sides with corresponding lower case letters.

- Students provide incomplete solutions.

R_x Have students read carefully what is asked for in a question. In particular, they should note that to solve a triangle means to find all of its side lengths and all of its interior angle measures.

Accommodations

Gifted and Enrichment—Challenge students to research and learn more about the mathematician Heron.

Visual—Allow students to use visual cues, such as highlighting or colour-coding, when substituting the known quantities for sides and angles into the sine law formulas.

Spatial—Encourage students to create diagrams and label the diagrams with the information given in the questions.

Motor—Provide students with enlarged photocopies of the diagrams in the Practise questions.

Practise

- Encourage students to apply the convention of labelling angles with capital letters and corresponding sides with lower case letters. This will help them to identify which part of the sine law to apply. This is particularly important in **questions 4** and **7**, where students must draw a diagram from given information.
- Students should start to become comfortable with rounding to "about" the right number of digits. For example, if a question gives angle measures to the nearest degree, it is reasonable to round answers to the nearest degree, or perhaps tenth of a degree. This concept should not be evaluated as part of student achievement, but students will benefit from discussing this issue and learning to apply their own common sense.
- Some of the diagrams in the questions can be simplified. Encourage students to copy simplified diagrams into their notes and label given and unknown information.
- For questions in which no diagram is given, such as **question 11**, encourage students to sketch a diagram that represents the problem. Have them label vertices with labels connected to the problem.
- Application of simple geometry (i.e., z pattern, complementary angles) is required to solve **question 15**.
- **Question 16** introduces Heron's formula as an alternative way of finding the area of a triangle given the three side lengths. Connection is made back to the initial problem posed in the section opener.
- **Question 17** is another application of Heron's formula, following the application of the sine law.
- In **question 19**, students discover that there is no corresponding cosine or tangent law that appears like the sine law. Students will learn about the cosine law in the next two sections. There is no tangent law. Some students could explore this during the initial investigation if they finish it early. Use **A–7 Thinking General Scoring Rubric** when assessing students.
- **Question 20** challenges students' ability to reason geometrically.
- **Question 21** challenges students' ability to reason algebraically.

Literacy Connections

Note the definition of "sine law" in the margin on page 397. Point out to students that normally only one equal sign is shown in any given line. In this case, it is a relationship indicating three equivalent ratios. Remind students that they should use only two of the ratios at any given time to solve a problem.

Note the Literacy Connections in the margin on page 399.

Discuss the Did You Know? on page 404. Have students add Heron to their list of mathematicians and their inventions. As an extension, have students search the Internet to find out more about Heron.

Create a Word Wall for this chapter. Add "sine law," "angle of elevation," "angle of depression," and "proportion."

Student Success

Use a timed retell strategy to have pairs of students explain why the sine law works.

Have students complete a journal entry explaining the sine law.

Refer to the introduction of this Teacher's Resource for more information about how to use a timed retell strategy.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	10, 18, 19
Reasoning and Proving	10–15, 19–21
Reflecting	8
Selecting Tools and Computational Strategies	11–13, 15, 19
Connecting	9–13, 15–17
Representing	4, 7, 8, 11, 14, 19–21
Communicating	11, 14, 19, 20

Ongoing Assessment

- Chapter Problem question 13 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.