8.2 The Cosine Law

Student Text Pages
405–411

Suggested Timing
80 min

Tools
- ruler
- protractor

Technology Tools
- computer
- *The Geometer's Sketchpad*®

Related Resources
- G–2 Placemat
- G–4 Protractor
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 8–5 The Cosine Law and *The Geometer's Sketchpad*®
- BLM 8–6 Section 8.2 Practice Master
- BLM 8–7 Section 8.2 Achievement Check Rubric
- A–22 Report Checklist

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

- Use the photo to generate discussion around a contextual trigonometry problem that can not be solved by using the sine law. Students should see why the sine law will not work: you do not have information on at least one corresponding side-angle pair. Use this conclusion to launch the investigation that follows. (5 min)

Investigate

- This **Investigate** can be performed with or without technology. If needed, use **BLM 8–5 The Cosine Law and *The Geometer's Sketchpad*®** with **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity. Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the ready-made GSP file that can be used for demonstration or investigation purposes. (10–15 min)
- The purpose of the activity is for students to recognize that the cosine law holds true. Students should note that the cosine law simplifies to the Pythagorean theorem when the angle in question equals 90°. The cosine law has a correction factor for non-right triangles, which is the term $-2ab(\cos C)$. This is illustrated nicely using GSP, where the angle can be dragged very close to, and then equal to, 90°.
- An alternative approach to the **Investigate** is to revise step 2 to look at an equilateral and an isosceles triangle before looking at scalene acute triangles. For example,
  a) Draw an equilateral triangle, △ABC, with sides 5 cm. Find a^2, b^2, c^2, and $2ab(\cos C)$. Calculate $a^2 + b^2 - 2ab(\cos C)$.
  b) Draw an isosceles triangle, △ABC, with sides AB = 6 cm, AC = 6 cm, and BC = 4 cm. Find a^2, b^2, c^2, and $2ab(\cos C)$. Calculate $a^2 + b^2 - 2ab(\cos C)$.
  c) Make a hypothesis about these quantities.
  d) Preferably using GSP, draw and measure several scalene acute triangles. Find a^2, b^2, c^2, and $2ab(\cos C)$. Compare the values c^2 and $a^2 + b^2 - 2ab(\cos C)$. State your conclusions.
- After students have discovered the validity of the cosine law, lead them through the algebraic development of the cosine law, as outlined on page 406. Students should be aware that the cosine law can be written in three different ways, each relating one of the three angles to the three sides. The choice of which version of the formula to use will depend on the information given in the problem. (10 min)

Examples

- Discuss the **Examples** with the class. (15–20 min)
- **Example 1** illustrates how to find an unknown side using the cosine law. Students need to carefully apply the correct order of operations after substituting given information. Care must also be taken to take the square root of both sides in the final step to find the unknown side length.
- Use the Literacy Connections margin item to help explain what is meant by a "contained angle."
- **Example 2** illustrates the process for solving a triangle, beginning with

the cosine law. Since the mechanical steps involved in applying the cosine law are more complex than those for the sine law, students should realize that once the cosine law has been applied and a fourth piece of information about a triangle has been found, they can generally turn to simpler tools (e.g., sine law, geometric properties) to find the remaining information.

- Graphing calculator output is shown. Slight variations may be necessary for different calculators, particularly scientific ones. Ensure that calculators are set to degree mode.

Communicate Your Understanding

- Review the vocabulary in this section (cosine law) before discussing the **Communicate Your Understanding** questions. (5 min)
- These questions can be used to diagnose whether students are ready to independently apply the cosine law to solve triangles. The ability to identify a contained angle is important because it informs which version of the cosine law to use (i.e., which variables go where). Being able to relate the cosine law to the Pythagorean theorem may reduce the level of abstractness of the appearance of the formula. This may also help students to remember its form.
- Use **BLM 8–6 Section 8.2 Practice Master** for remediation or extra practice.

Investigate Answers (pages 405–406)

1.a)–c) Answers will vary.

2. a) Answers will vary.

b) Answers may vary. For example: $a^2 + b^2 \neq c^2$. The relationship $a^2 + b^2 = c^2$ when △ABC is a right triangle and ∠C = 90°. This relationship is called the Pythagorean theorem.

c) Answers will vary.

d) Answers may vary. For example: $c^2 = a^2 + b^2 - 2ab(\cos C)$.

e) The relationship holds true for the other acute triangles.

3. a) If ∠C = 90°, then the quantity $2ab(\cos C) = 0$ because cos 90° = 0.

b) The Pythagorean relationship applies when ∠C = 90°.

4. a) For an acute △ABC, each of the following three equations relates the cosine of each of the angles in the acute triangle and the three sides of the triangle:
$a^2 = b^2 + c^2 - 2bc(\cos A)$, $b^2 = a^2 + c^2 - 2ac(\cos B)$, and
$c^2 = a^2 + b^2 - 2ab(\cos C)$.

b) When the measure of one of the angles is 90°, each of the relationships change as follows:
If ∠A = 90°, the relationship is $a^2 = b^2 + c^2$.
If ∠B = 90°, the relationship is $b^2 = a^2 + c^2$.
If ∠C = 90°, the relationship is $c^2 = a^2 + b^2$.

Common Errors

- Calculator outputs may not make sense.

R_x Have students check that calculators are in degree mode and adjust if necessary.

- Some students may have difficulty identifying which version of the cosine law to apply.

R_x Point out that the contained angle indicates the corresponding side that sits alone on the left side of the equation. For example, given ∠A, *b*, and *c*, use $a^2 = b^2 + c^2 - 2bc(\cos A)$ to find *a* first.

- Some students may substitute improperly into the cosine law.

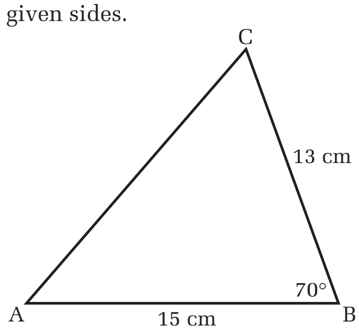
R_x Have students practise identifying angles and their corresponding sides by drawing an arrow from the angle across the triangle to identify its corresponding side. Encourage students to label angles using capital letters and their corresponding sides using the corresponding lower case letters.

- Students provide incomplete solutions.

R_x Have students read carefully what is asked for in a question. In particular, they should note that to solve a triangle means to find all of its side lengths and all of its interior angle measures.

Communicate Your Understanding Responses (page 409)

C1. a) Diagrams may vary. A contained angle is the angle made by, or between, two given sides.



b) It is important that the angle used with the cosine law is the contained angle.

C2. a) Answers may vary. For example: They are similar because the left side of the equation is the square of one side and the right side of the equation involves the sum of the squares of the other two sides. They are different because for the cosine law you still need to subtract a quantity, and the isolated side does not need to be the hypotenuse.

b) Answers may vary. For example: You can apply the Pythagorean theorem only for right triangles. You can apply the cosine law for any type of triangle, including right triangles.

C3. Find *r* using the cosine law, then find either ∠P or ∠Q using the sine law. The final angle can be found using the fact that the sum of the interior angles in a triangle is 180°.

Practise

- One area of focus in this section is identifying and writing the correct form of the cosine law. When two sides and a contained angle are given, that angle's corresponding side appears alone on the left side of the equation. Encourage students to follow the correct order of operations and observe proper form when transcribing solutions.
- The application of acute angle trigonometry is not limited to triangles involving distance. **Question 10**, for example, is a good application of trigonometry involving measures of nautical speed.
- For **question 13**, some students may not realize that the threshold of a runway is its endpoint.
- **Question 14** makes connections between speed, distance, and time.
- **Question 15** challenges students' ability to reason geometrically and algebraically.
- **Question 16** provides an opportunity to make connections between right angle trigonometry and acute angle trigonometry.
- **Questions 17** and **18** extend the cosine law into obtuse angle triangles, which students will study in more depth in grade 11. Use **A–22 Report Checklist** when assessing students for **question 17**.
- **Question 19** provides an opportunity to make connections to circle geometry and measurement. Students may need to recall the formula for the circumference of a circle.

Accommodations

Gifted and Enrichment—Challenge students to learn more about the history and the mathematics involved in the design of totem poles.

Language—Encourage students to work with a partner who will read the questions to them.

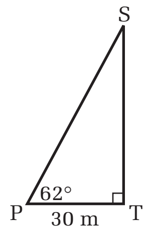
Memory—Allow students to use scientific calculators to calculate the missing sides using the cosine law, and have them practise using the correct keystrokes for the order of operations.

ESL—Let students use their dictionaries to understand the new words in this section.

Achievement Check Sample Solution, question 16, page 411

Provide students with **BLM 8–7 Section 8.2 Achievement Check Rubric** to help them understand what is expected.

16. a) Label △TPS.



Use the tangent ratio.

$\frac{TS}{30} = \tan 62°$

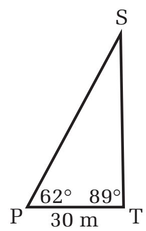
$TS = 30(\tan 62°)$

$TS \doteq 56.4$

The totem pole is approximately 56.4 m tall.

b) Shorter. Point S would move to the left; it would be located lower and to the left, closer to P.

c) Use the new dimensions for △TPS.



$\angle S = 180° - 62° - 89°$
$= 29°$

Use the sine law.

$\frac{TS}{\sin 62°} = \frac{30}{\sin 29°}$

$TS = \sin 62°\left(\frac{30}{\sin 29°}\right)$

$TS \doteq 54.6$

The difference between the two heights is 1.8 m.

Literacy Connections

Note the definition of "cosine law" in the margin on page 406.

Note the Literacy Connections in the margin on page 407. Remind students that often mathematical terms have a basis in English, and that to contain something means to hold it. They can think of the contained angle as being held by the two sides.

Draw students' attention to the Did You Know? on page 410. As an extension, ask students to investigate the origin of the use of "knot."

Add "cosine law," "contained angle," "knot," and "nautical mile" to the Word Wall.

Student Success

Use a placemat activity to have groups of students complete problems using the cosine law. Use **G–2 Placemat** to support this activity.

Given a solution, have students make up a question to fit that solution.

Refer to the introduction of this Teacher's Resource for more information about how to use a placemat strategy.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	12, 16, 17, 19–21
Reasoning and Proving	10, 15–17, 19
Reflecting	6, 12, 16, 19
Selecting Tools and Computational Strategies	16, 17
Connecting	10, 14–16, 19
Representing	3, 5, 6, 10, 12–14, 16, 17, 19
Communicating	10, 12, 16, 17, 19

Ongoing Assessment

- Use Achievement Check question 16 to monitor student success. See Achievement Check Answers and **BLM 8–7 Section 8.2 Achievement Check Rubric**.
- Chapter Problem question 14 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.