8.4 Solve Problems Using Trigonometry

Student Text Pages
424–429

Suggested Timing
80 min

Tools
- protractor
- ruler

Technology Tools
- Internet Access

Related Resources
- G–4 Protractor
- BLM 8–10 Section 8.4 Practice Master
- A–5 Problem Solving Checklist
- A–18 My Progress as a Problem Solver
- A–22 Report Checklist

Teaching Suggestions

- The purpose of this section is to consolidate student understanding of the application of acute angle trigonometry and related concepts, such as right angle trigonometry, the Pythagorean theorem, and geometric properties of polygons. The problems in this section are typically more challenging than those in the previous sections, and sometimes involve composite figures and/or multiple triangles in three dimensions.

Examples

- Discuss the **Examples** with the class. (25–30 min)
- **Example 1** requires students to use multiple tools and steps. Encourage students to use various techniques to emphasize different parts of the diagram, as needed. For example,
  – highlight different parts using a colour scheme
  – split different parts of the diagram into separate diagrams
  – draw only part of the diagram, as needed
  Encourage students to focus on the part about which the most information is known.
- Encourage students to select from the multiple tools at their disposal. For example, in the first part of the **Example 1** solution, application of the sine and cosine ratios is more straightforward than using a more cumbersome tool such as the sine law. Encourage students to look for opportunities and clues (in this case, the presence of a right angle triangle) that will allow them to apply time-saving, efficient strategies. Similarly, identification of the equilateral property of $\triangle BCE$ and the isosceles property of $\triangle BDE$ in subsequent steps will save computational effort.
- In **Example 1**, once all required missing information on the truss diagram has been found, remind students to ensure that the question has actually been answered. Add all the lengths in the diagram to give the total length of the beams required.
- A solution to **Example 1** using *The Geometer's Sketchpad*® (GSP) is available. Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file.
- **Example 2** requires the use of three-dimensional trigonometric reasoning. Some students may have trouble visualizing the problem. Try presenting a simplified top view isometric diagram of $\triangle BCD$ for the first part, followed by a simplified side view isometric diagram of $\triangle ABC$ for the second part. Note that while the sine law is initially used to find a common side shared between the two triangles, the tangent ratio is sufficient in the subsequent step to find the unknown height.

Common Errors

- Calculator outputs may not make sense.

R_x Have students check that calculators are in degree mode and adjust if necessary.

- Students are uncertain whether to apply the sine law or the cosine law.

R_x Typically, the cosine law is required if the given information is limited to the three sides of a triangle or two sides and the contained angle. In other cases, the sine law will generally work if you are given three pieces of information including at least one side. A reasonable strategy is to always try the sine law first, and then switch to the cosine law if you recognize that you do not have enough information.

- Students provide incomplete solutions.

R_x Have students read carefully what is asked for in a question. Some questions involve more than one triangle and multiple steps. The final step of solving a problem should be to look back and ask yourself
  - Did I answer the question?
  - Does the answer seem reasonable?

Communicate Your Understanding

- Use the Literacy Connections in the margin of page 426 to help explain why a two-letter system is sometimes necessary for labelling side lengths in complicated diagrams.
- These **Communicate Your Understanding** questions focus on helping students identify which trigonometric tools can and should be used for various given situations. Caution students against automatically turning to the sine law or cosine law before considering simpler tools. There is often more than one valid strategy for solving a problem. (5 min)
- Use **A–5 Problem Solving Checklist** to assist you when assessing students. Alternatively, have students use **A–18 My Progress as a Problem Solver** as a self-assessment tool.
- Use **BLM 8–10 Section 8.4 Practice Master** for remediation or extra practice.

Communicate Your Understanding Responses (page 427)

C1. a) Answers may vary. For example: Yes, this was already proved in Sections 8.1 and 8.2.

b) You could use the primary trigonometric ratios or the Pythagorean theorem.

C2. Answers may vary. For example: If you are given three sides and need to find an angle, or if you have two sides and the contained angle, use the cosine law. Otherwise, use the sine law.

C3. a) Answers may vary. For example: Find the missing angles in the isosceles $\triangle ACD$. Find the length of AC using the cosine ratio in $\triangle ABC$. Find the length of AD using the sine law.

b) Answers will vary.

Practise

- "If you don't know what to do, do what you can." Use this as a motivational technique for beginning a problem when a student doesn't know what to do.
- The focus of **question 1** is on identifying the best trigonometric tool to use.
- In **questions 2** and **3**, diagrams involving two triangles in two dimensions are solved. Encourage students to consider which part of the diagram has more information in order to recognize a starting point. Have them consider any geometric properties that may be used.
- **Question 4** applies the geometric properties of parallel lines. After finding two interior angles in the triangle, students can apply the tangent ratio twice to find two parts that comprise the diameter.
- **Question 5** involves celestial motion, one of the original triggers for the development of trigonometry. Some students may know that the orbits of Mars and Earth are not synchronous. Therefore, the relative positions of the three bodies are in a constant state of flux. A GSP sketch for a solution to this problem is available. Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file.
- For **questions 6** and **8**, encourage students to draw a diagram and label the given information.
- For **question 9**, encourage students to tackle the diagram in parts. Some will observe that symmetry renders the left and right sides of the truss congruent, implying that focus can be limited to just the left side. Students need to recall a number of geometric properties (e.g., isosceles triangles, supplementary and complementary angles, and the Pythagorean theorem) as well as acute and right angle trigonometry relationships.
- **Question 10** involves three triangles in three dimensions. Suggest a sketch of simplified isometric top, side, and front view diagrams as a visual aid.

Accommodations

Gifted and Enrichment—Challenge students to solve the questions in this section and then create alternative solutions for the same questions.

Perceptual—Allow students to work in groups to complete the questions.

Spatial—Encourage students to create three-dimensional models for the questions in this section.

Motor—Let students work with an educational assistant who will scribe their answers.

Student Success

Use a jigsaw strategy to have students become experts in the various trigonometry problems.

Use a think aloud strategy to have students discuss trigonometry problems.

Have students construct a decision tree for trigonometry problems.

Have students conduct an Internet search to research astronomical units.

Refer to the introduction of this Teacher's Resource for more information about how to use jigsaw, think aloud, and decision tree strategies.

- For **question 11**, a polyhedron model may be helpful.
- **Question 13** may interest students who plan to go on to study Calculus and Vectors in grade 12.
- **Question 15** challenges students' ability to reason algebraically and geometrically in three dimensions. A shoe box model may be a useful visual aid.
- For **question 16**, students may need some help recalling how to read bearings, which were introduced in Chapter 7.
- For **question 17**, access to the Internet or Career Resource Library may be useful. Use **A–22 Report Checklist** when assessing students.
- Extension: Because of the curvature of Earth, real navigation cannot use plane (flat surface) trigonometry for long distance travel. In fact, it is possible to fly north, turn 90° right, fly south the same distance, turn 90° right, and fly west the same distance to arrive back at the starting point. Use a globe or Internet map service to find three points on Earth's surface that make this possible. Provide a written explanation supported with diagrams.
- Extension: **a)** Find the perimeter of a regular pentagon, whose distance from its centre to each vertex is 15 cm.
  b) Explain your method.
  c) Can this method be adapted to solve for the perimeter of any regular polygon? If so, explain how. If not, explain why.

Literacy Connections

Draw attention to the Literacy Connections on page 426. Using three letters to name an angle or the two endpoints to name a line can always be done. In this case, it must be done to avoid confusion. Have students rewrite the sine law and cosine law using three letters for each angle and two letters for each side.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	8, 10, 11, 13–15, 17
Reasoning and Proving	5, 7–15
Reflecting	2, 8–10, 12, 14
Selecting Tools and Computational Strategies	1–11,13–16
Connecting	5–16
Representing	5, 6, 8, 15, 16
Communicating	5, 8–10, 12, 14, 17

Ongoing Assessment

- Chapter Problem question 13 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.