Task

Solar Lights

Student Text Pages
436

Suggested Timing
10 min

Tools
- grid paper
- ruler
- protractor
- compasses

Technology Tools
- computer
- *The Geometer's Sketchpad*®

Related Resources
- G–1 Grid Paper
- G–4 Protractor
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 8–16 Task: Solar Lights Rubric

Accommodations

Gifted and Enrichment—Challenge students to use the Internet to learn more about solar lights and the history of light and lighting.

Perceptual—Encourage students to make a diagram or a real-life model of the information given in the Task.

Motor—Let students work in groups to complete the Task.

Memory—Provide students with visual cues and prompts to help them remember the formulas required to complete the Task.

Specific Expectations

Trigonometry

Solving Problems Involving the Trigonometry of Right Triangles

TR2.01 determine, through investigation (e.g., using dynamic geometry software, concrete materials), the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios (e.g., $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$);

TR2.02 determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem;

TR2.03 solve problems involving the measures of sides and angles in right triangles in real-life applications (e.g., in surveying, in navigating, in determining the height of an inaccessible object around the school), using the primary trigonometric ratios and the Pythagorean theorem.

Solving Problems Involving the Trigonometry of Acute Triangles

TR3.01 explore the development of the sine law within acute triangles (e.g., use dynamic geometry software to determine that the ratio of the side lengths equals the ratio of the sines of the opposite angles; follow the algebraic development of the sine law and identify the application of solving systems of equations [student reproduction of the development of the formula is not required]);

TR3.02 explore the development of the cosine law within acute triangles (e.g., use dynamic geometry software to verify the cosine law; follow the algebraic development of the cosine law and identify its relationship to the Pythagorean theorem and the cosine ratio [student reproduction of the development of the formula is not required]);

TR3.03 determine the measures of sides and angles in acute triangles, using the sine law and the cosine law (***Sample problem:*** In triangle ABC, $\angle A = 35°$, $\angle B = 65°$, and AC = 18 cm. Determine BC. Check your result using dynamic geometry software.);

TR3.04 solve problems involving the measures of sides and angles in acute triangles.

Teaching Suggestions

- Students should work independently on the Task.
- Students can complete the Task by using grid paper, ruler, compasses, and protractor, or by using *The Geometer's Sketchpad*®. In either case students should provide a summary of what they are constructing and why.
- If needed, use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity.
- Emphasize that trigonometric methodology is needed to fully justify the solutions.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. When evaluating a response, look for the following:

- Knowledge of the properties of the trigonometric ratios and trigonometry of acute triangles
- Recognition of trigonometric relationships in the problem

- Evidence of communication of mathematical thinking and mathematical vocabulary

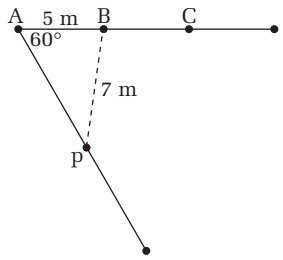
Ongoing Assessment

- Use **BLM 8–16 Task: Solar Lights Rubric** to assess student achievement.

Level 3 Sample Response

a) Light A can light an arc of 7 m, so it will light 7 m along the second path.

b) Let P be the point on the second path that is 7 m from point B. Consider $\triangle ABP$, where $p = 5$, $a = 7$, and $\angle A = 60°$. Side b is the length of the path illuminated by the lights at A and B. Use the sine law to find $\angle P = 38°$. So, $\angle B = 82°$. Apply the sine law again to find $b = 8$. Therefore, the light at A illuminates the first 7 m of the path while the light at B will light an additional 1 m, for a total of 8 m.



c) Consider the perpendicular distance from the light at C to the second path. The light at C will not illuminate the path if this distance is greater than 7 m. This perpendicular distance is given by 10(sin 60°), or 8.66 m. Therefore the light at C does not add to the illumination, which remains 8 m.

Level 3 Notes

Look for the following:

- Clear understanding of trigonometric concepts
- Clear understanding of how to use the sine law
- Planning and thinking in how the context diagram is developed, especially in parts b) and c)
- Mostly accurate calculations in using the sine law
- Correct use of the sine ratio to find a perpendicular distance in part c)
- Well-constructed mathematical arguments
- Use of good form and correct mathematical notation

What Distinguishes Level 2

At this level, look for the following:

- Some understanding of trigonometric concepts
- Some understanding of how to use the sine law
- Some planning and thinking in how the context diagram is developed, especially in parts b) and c)
- Somewhat accurate calculations in using the sine law
- Somewhat correct use of the sine ratio to find a perpendicular distance in part c)
- Some well-constructed mathematical arguments
- Some use of good form and correct mathematical notation

What Distinguishes Level 4

At this level, look for the following:

- Very clear understanding of trigonometric concepts
- Very clear understanding of how to use the sine law
- Detailed planning and thinking in how the context diagram is developed, especially in parts b) and c)
- Accurate calculations in using the sine law
- Detailed use of the sine ratio to find a perpendicular distance in part c)
- Very well constructed mathematical arguments
- Use of very good form and correct mathematical notation

Task — Lighting the Park

Student Text Pages
436

Suggested Timing
25–35 min

Tools
- grid paper
- ruler
- protractor
- compasses

Technology Tools
- computer
- *The Geometer's Sketchpad*®

Related Resources
- G–1 Grid Paper
- G–4 Protractor
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 8–17 Task: Lighting the Park Rubric

Accommodations

Gifted and Enrichment—Challenge students to create an alternative Task with a triangular flower bed and determine the least number of solar lights that will be needed to illuminate the paths around the edges of the flower bed.

Visual—Let students create a real-life model with solar lights to complete the Task.

Perceptual—Encourage students to work together to create a report for this Task.

Language—Allow students to give an oral response when justifying their conclusions for this Task.

Specific Expectations

Trigonometry

Solving Problems Involving the Trigonometry of Right Triangles

TR2.01 determine, through investigation (e.g., using dynamic geometry software, concrete materials), the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios (e.g., $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$);

TR2.02 determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem;

TR2.03 solve problems involving the measures of sides and angles in right triangles in real-life applications (e.g., in surveying, in navigating, in determining the height of an inaccessible object around the school), using the primary trigonometric ratios and the Pythagorean theorem.

Solving Problems Involving the Trigonometry of Acute Triangles

TR3.01 explore the development of the sine law within acute triangles (e.g., use dynamic geometry software to determine that the ratio of the side lengths equals the ratio of the sines of the opposite angles; follow the algebraic development of the sine law and identify the application of solving systems of equations [student reproduction of the development of the formula is not required]);

TR3.02 explore the development of the cosine law within acute triangles (e.g., use dynamic geometry software to verify the cosine law; follow the algebraic development of the cosine law and identify its relationship to the Pythagorean theorem and the cosine ratio [student reproduction of the development of the formula is not required]);

TR3.03 determine the measures of sides and angles in acute triangles, using the sine law and the cosine law (***Sample problem:*** In triangle ABC, $\angle A = 35°$, $\angle B = 65°$, and AC = 18 cm. Determine BC. Check your result using dynamic geometry software.);

TR3.04 solve problems involving the measures of sides and angles in acute triangles.

Teaching Suggestions

- Students can complete the Task by using grid paper, ruler, compasses, and protractor, or by using *The Geometer's Sketchpad*®.
- If needed, use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity.
- Review the sine law and the cosine law.
- It may be helpful for students to work in pairs to plan how to tackle the problem. Afterward, they could compare notes and adjust their answers/strategies.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. When evaluating a response, look for the following:

- Knowledge of the properties of the trigonometric ratios and trigonometry of acute triangles
- Evidence of recognition of how to use trigonometric relationships to solve the problem

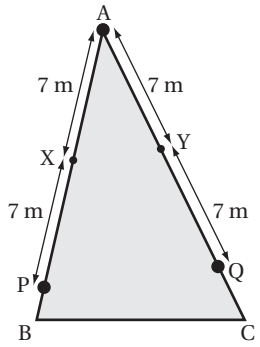
- Evidence of communication of mathematical thinking and mathematical vocabulary

Ongoing Assessment

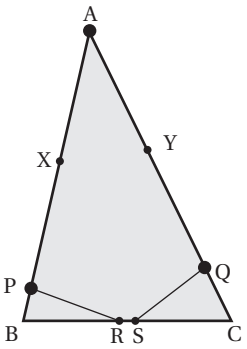
- Use **BLM 8–17 Task: Lighting the Park Rubric** to assess student achievement.

Level 3 Sample Response

If a light is placed at A its range will cover 7 m of AB and AC, or to points X and Y. If lights are placed an additional 7 m from points X and Y, at points P and Q, then all of AB and AC will be illuminated. Check if these lights will illuminate all of BC.



Consider △BPR and △CSQ. Lights placed at P and Q will illuminate all of BC if (BR + CS) is equal to or greater than 11 m.



First find the angles of △ABC. Use the cosine law to find that ∠B = 75.5°. Then, use the sine law to find that ∠C = 65.7°.

Next, apply the sine law in △BPR, where BP = 2 m, PR = 7 m, and ∠PBR = 75.5°.
∠PRB = 16.1° and BR = 7.2 m

Similarly, apply this technique to △CSQ to find CS.
∠CSQ = 23° and CS = 7.7 m

So, (BR + CS) is greater than 11 m. This confirms that the three solar lights at A, P, and Q illuminate the entire path around △ABC.

Note: Some students may notice that one light placed at, say, the midpoint of BC will illuminate the entire path along BC, as well as portions of paths AB and AC. Then, calculations will show that only one more light is necessary to illuminate the entire path around △ABC, for a total of two lights.

Level 3 Notes

Look for the following:

- Clear understanding of trigonometric concepts
- Clear understanding of how to use the sine law and the cosine law
- Planning and thinking in how the context diagram is developed and triangles are solved
- Mostly accurate calculations in using the sine law and the cosine law
- Well-constructed mathematical arguments
- Use of good form and correct mathematical notation

What Distinguishes Level 2

At this level, look for the following:

- Some understanding of trigonometric concepts
- Some understanding of how to use the sine law and the cosine law
- Some planning and thinking in how the context diagram is developed and triangles are solved
- Somewhat accurate calculations in using the sine law and the cosine law
- Some well-constructed mathematical arguments
- Some use of good form and correct mathematical notation

What Distinguishes Level 4

At this level, look for the following:

- Very clear understanding of trigonometric concepts
- Detailed understanding of how to use the sine law and the cosine law
- Detailed planning and thinking in how the context diagram is developed and triangles are solved
- Accurate calculations in using the sine law and the cosine law
- Very well constructed mathematical arguments
- Use of very good form and correct mathematical notation

Task

Trigonometry Using *The Geometer's Sketchpad®*

Student Text Pages
437

Suggested Timing
25–35 min

Technology Tools
- computer
- *The Geometer's Sketchpad®*

Related Resources
- T–4 *The Geometer's Sketchpad®* 3
- T–5 *The Geometer's Sketchpad®* 4
- BLM 8–18 Task: Trigonometry Using *The Geometer's Sketchpad®* Rubric

Accommodations

Gifted and Enrichment—Challenge students to create a presentation to demonstrate the steps required to solve a triangle using GSP.

Motor—Let students work with a partner when using GSP.

Language—Allow students to work with a partner who will read the instructions required to solve a triangle using GSP.

ESL—Encourage students to use their dictionaries or translators to understand the new words in this section.

Specific Expectations

Trigonometry

Solving Problems Involving the Trigonometry of Acute Triangles

TR3.03 determine the measures of sides and angles in acute triangles, using the sine law and the cosine law (***Sample problem:*** In triangle ABC, $\angle A = 35°$, $\angle B = 65°$, and AC = 18 cm. Determine BC. Check your result using dynamic geometry software.);

TR3.04 solve problems involving the measures of sides and angles in acute triangles.

Teaching Suggestions

- This Task requires access to *The Geometer's Sketchpad®* (GSP) software and some facility with its tools and commands. If needed, use **T–4 *The Geometer's Sketchpad®* 3** or **T–5 *The Geometer's Sketchpad®* 4** to support this activity.
- Individualize the Task by giving students different dimensions to start. For example, give students the length of AB as 2.23XX cm, where XX represents the day of the month for their birthday or the last two digits of their student number. This gives the appearance of individuality, but the final results will be very similar (and easy to mark) and it avoids copying.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. When evaluating a response, look for the following:

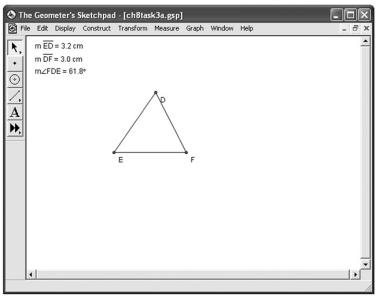
- Knowledge and understanding of trigonometry and GSP tools
- Details of how to make GSP constructions
- Evidence of communication of mathematical thinking and mathematical vocabulary

Ongoing Assessment

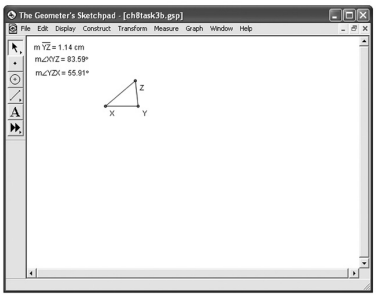
- Use **BLM 8–18 Task: Trigonometry Using *The Geometer's Sketchpad®* Rubric** to assess student achievement.

Level 3 Sample Response

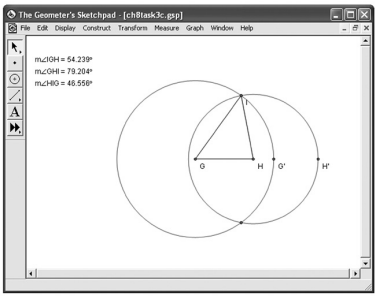
a)



b)



c)



Level 3 Notes

Look for the following:

- Complete triangles with the required measurements indicated
- Some explanation of the method used in each case
- At least three points listed for part d)
- Use of good form and correct mathematical notation

What Distinguishes Level 2

At this level, look for the following:

- Complete triangles where measurements may be approximate, since the student has used only basic techniques to create sides and angles of the required sizes
- Complete solution for only one of the first three parts
- Answers accurate only to the standard two decimal places
- Some use of good form and correct mathematical notation

What Distinguishes Level 4

At this level, look for the following:

- Complete triangles with the required measurements indicated
- Accurate calculations specified to four decimal places; may comment that accuracy is only warranted to the number of decimal places of the least accurate element in the given information
- Use of very good form and correct mathematical notation
- Detailed instructions on how to complete the constructions
- An extensive list of ideas for part d)