# 3

Pythagorean Relationship

Many board games and puzzles include squares and triangles in their design. Checkers, chess, and SCRABBLE® have game boards made of squares. The game called Playing Leader, includes both squares and triangles in its game board design.

Squares and triangles are also important geometric figures in construction, art, and mathematics. There are many connections between squares and triangles in mathematics. In previous math courses, you have discovered some connections between these two shapes.

What You Will Learn

❏ to find the squares and square roots of whole numbers

❏ to estimate square roots of whole numbers

❏ to determine whether a triangle is a right triangle

❏ to apply the Pythagorean relationship to find missing dimensions of triangles and to solve problems
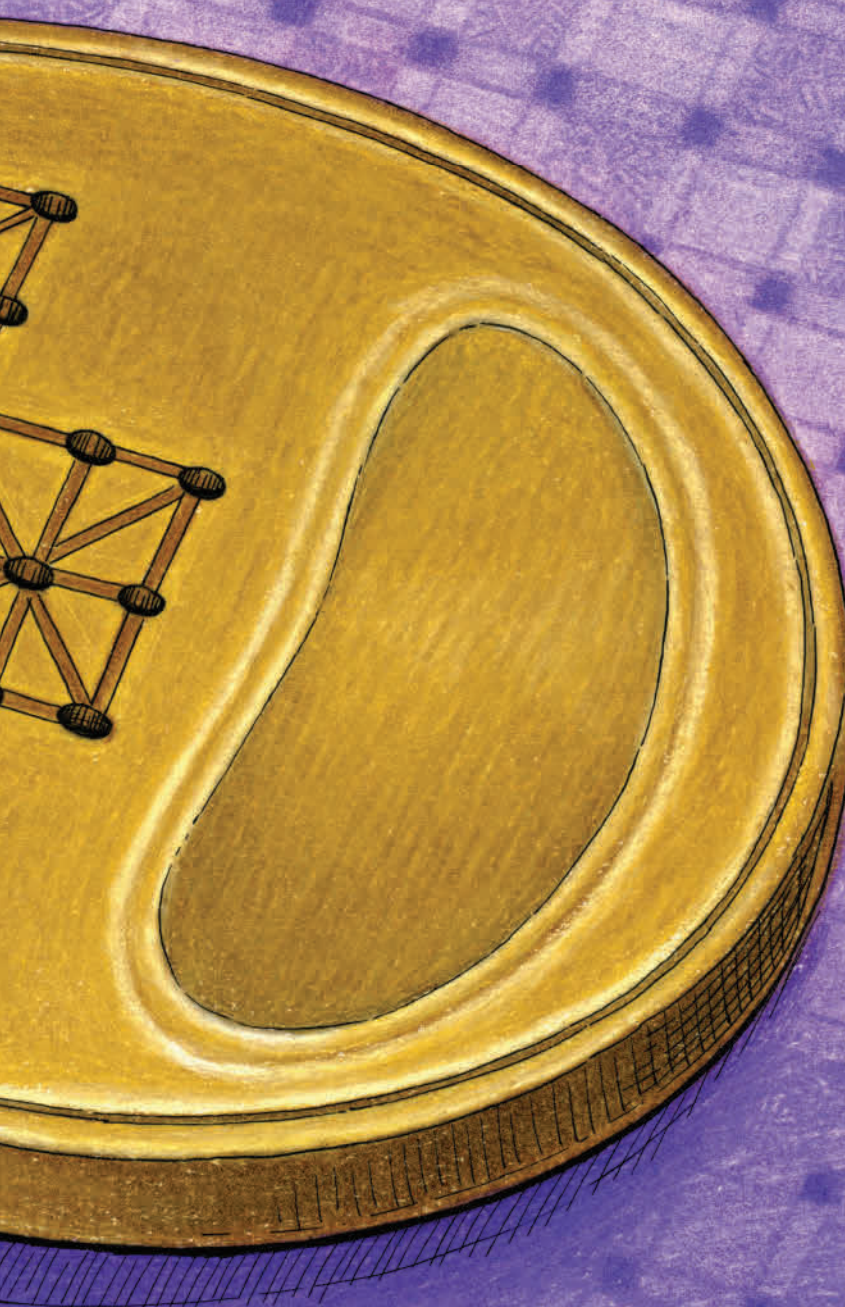
Key Words

- prime factorization
- perfect square
- square root
- hypotenuse
- Pythagorean relationship

Literacy Link

You can use a Verbal Visual Chart (V VC) to help you learn and remember new terms.

Copy the blank V VC into your math journal or notebook and use it for the term *square*.

- Write the term in the first box.
- Draw a diagram in the second box.
- Define the term in the third box. The glossary on pages 517–521 may help you.
- In the fourth box, explain how you will remember the term and what it means. Consider using an example, a characteristic, a memory device, or a visual.

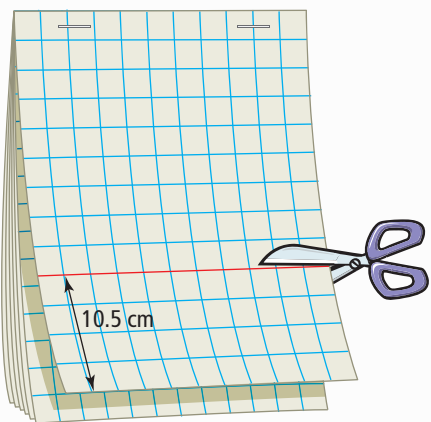| Term	Diagram
Definition	**How I Will Remember It**

Making the Foldable

Materials
- eight sheets of grid paper
- stapler
- ruler
- scissors

Step 1

Staple eight sheets of grid paper together along the top edge.

Step 2

Make a line 10.5 cm up from the bottom of the top page. Cut across the entire width of the page at this mark.
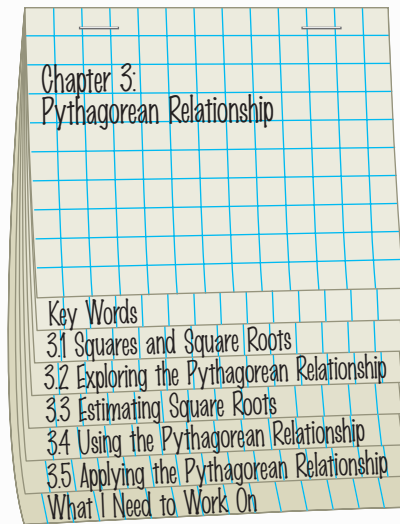

Step 3

Make a line 9 cm up from the bottom of the second page. Cut across the entire page at this mark.

Step 4

In a similar manner, cut off 7.5 cm from the third page, 6 cm from the fourth page, 4.5 cm from the fifth page, 3 cm from the sixth page, and 1.5 cm from the seventh page.

Step 5

Label the tabs as shown.

Using the Foldable

As you work through Chapter 3, define each Key Word on the first tab of your Foldable. Use a visual or example to help you remember each term.

Make notes about examples and Key Ideas under the appropriate tab.

On the last tab, make notes using the heading What I Need to Work On. Check off each item as you deal with it.

Use the back of the Foldable to list your ideas for the Wrap It Up!

MATH LINK

Game Design

Playing Leader is a peg board game. The board design for Playing Leader includes squares and triangles. In this game, two players compete against each other. One player controls 13 green pegs and the opponent controls the leader peg, which is a different colour. All of the pegs start at the top of the board and are moved one hole at a time along the lines on the board. The 13 green pegs may be moved left, right, or down. The leader peg may be moved left, right, up, or down. The green pegs try to surround the leader peg so that it cannot move to another position. The leader peg tries to capture all of the green pegs or advance to the bottom of the board. The leader peg captures a green peg by jumping over it to an empty space. Captured pegs are removed from the board.

Use a copy of the Playing Leader board to help answer the following questions.
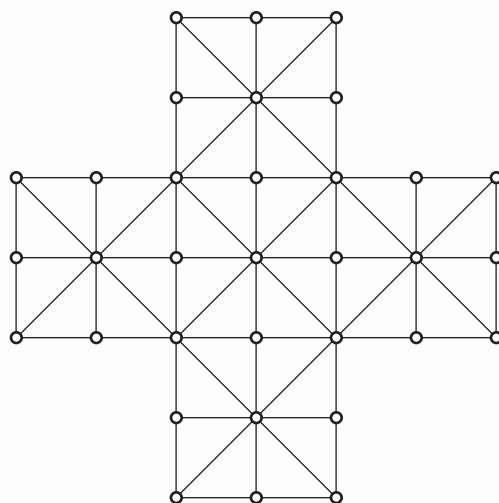
1. Show how to place the 13 green pegs and one leader peg in such a way that the leader peg cannot move.

2. Moving one space at a time, what is the maximum number of moves you can make with one peg in a straight line vertically?

3. How many squares can you count on the board that do not overlap?

4. What words could you use to describe the triangles on the board, for example, *isosceles, equilateral, scalene, acute, right, obtuse*? Draw and label the triangle(s) to show why you used the words you did.

5. The horizontal or vertical distance between two peg holes is 5 cm. Determine the area of the game board in two different ways.

Compare your answers with those of a classmate.

At the end of the chapter, you will design a new board game that includes triangles and squares in the design.

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Squares and Square Roots

The Pythagoreans were members of an academy of study that existed 2500 years ago. They created square numbers by arranging pebbles in equal numbers of rows and columns. Nine pebbles could be arranged in three rows and three columns. Nine is a square number because $3 \times 3 = 9$. The picture shows the first four square numbers that the Pythagoreans found: 1, 4, 9, and 16. How can you determine the next square number?

Literacy ❸ **Link**

A *square number* is the product of the same two numbers. $3 \times 3 = 9$, so 9 is a square number.

A square number is also known as a *perfect square*. A number that is not a perfect square is called a *non-perfect square*.

Did You Know?

Pythagoras (about 580–500 B.C.E.) was the leader of a group of academics called the Pythagoreans. They believed that patterns in whole numbers could help explain the universe.

Explore the Math

Materials
• square tiles

How can you identify a perfect square?

1. Use square tiles to make five rectangles with the dimensions shown.

What is the area of each rectangle?

Length (cm)	Width (cm)
5	3
8	2
9	1
4	3
9	4

2. Try to rearrange the tiles in each rectangle to make a square.

  a) Which rectangles can you make into squares?

  b) What is the side length of each square?

  c) How is the area of each square related to its side length?

3. a) Choose three perfect squares and three non-perfect squares.

  b) Express each number as a product of prime factors.

  c) For each number, how many times does each prime factor appear? Compare your results with a partner's results.

4. a) What do all of the perfect squares have in common?

  b) What do all of the non-perfect squares have in common?

Reflect on Your Findings

5. a) How can square tiles help you to determine if a number is a perfect square?

  b) How can prime factors help you to determine if a number is a perfect square?

Example 1: Identify Perfect Squares

prime factorization

- a number written as the product of its prime factors
- the prime factorization of 6 is 2×3

perfect square

- a number that is the product of the same two factors
- has only an even number of prime factors
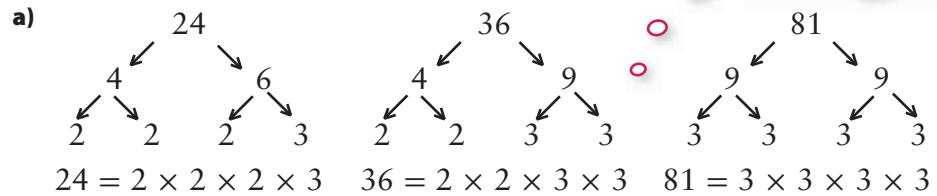- $5 \times 5 = 25$, so 25 is a perfect square

5

5

a) Determine the **prime factorization** of the following numbers: 24, 36, 81.

b) Which of the numbers is a **perfect square**? Explain.

c) For each number that is a perfect square, draw the square and label its side length.

Different factor trees are possible to arrive at the same prime factorization.

Solution

a)

24
4   6
2   2   2   3
$24 = 2 \times 2 \times 2 \times 3$

36
4   9
2   2   3   3
$36 = 2 \times 2 \times 3 \times 3$

81
9   9
3   3   3   3
$81 = 3 \times 3 \times 3 \times 3$

b) To be a perfect square, each prime factor in the prime factorization must occur an even number of times. 36 and 81 are perfect squares because each prime factor occurs an even number of times.

$36 = 2 \times 2 \times 3 \times 3$ — two factors of 2, two factors of 3
$81 = 3 \times 3 \times 3 \times 3$ — four factors of 3

24 is not a perfect square because at least one of the prime factors occurs an odd number of times.

$24 = 2 \times 2 \times 2 \times 3$ — three factors of 2, one factor of 3

c) To determine the side length of the squares, look at the product of prime factors for the area.

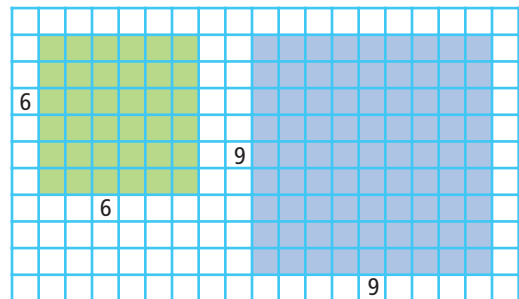$36 = 2 \times 2 \times 3 \times 3$ $81 = 3 \times 3 \times 3 \times 3$

Rearrange the prime factors into two equal groups.

$36 = (2 \times 3) \times (2 \times 3)$
$36 = 6 \times 6$

$81 = (3 \times 3) \times (3 \times 3)$
$81 = 9 \times 9$

6

9

6

9

Show You Know

Write the prime factorization of each number. Which number is not a perfect square? Explain how you know.

a) 45   **b)** 100

Example 2: Determine the Square of a Number

Determine the area of a square picture with a side length of 13 cm.

Solution
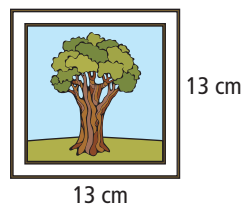
$A = s^2$
$A = 13^2$
$A = 13 \times 13$
$A = 169$
The area is 169 cm².

area of a square = side length $\times$ side length
$A = s \times s$
$A = s^2$

13 cm

13 cm

Show You Know
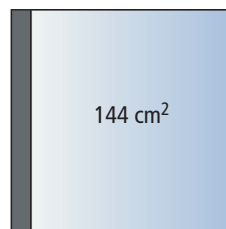
Determine the area of a square with a side length of 16 mm.

Literacy Link

You can write a repeated multiplication like 13×13 as a square: $13 \times 13 = 13^2$. 13^2 is read as thirteen squared.

Example 3: Determine the Square Root of a Perfect Square

Edgar knows that the square case for his computer game has an area of 144 cm². What is the side length of the case?

144 cm²

Solution

Method 1: Use Inspection
To find the side length, determine what positive number when multiplied by itself equals 144.
$12 \times 12 = 144$
The **square root** of 144 is 12, or $\sqrt{144} = 12$.
The side length is 12 cm.

M E

square root

- a number that when multiplied by itself equals a given value
- 6 is the square root of 36 because $6 \times 6 = 36$

Method 2: Use Guess and Check
Find the positive value for the blank boxes.
$\blacksquare \times \blacksquare = 144$
$10 \times 10 = 100$ Too low
$13 \times 13 = 169$ Too high
$12 \times 12 = 144$ Correct!
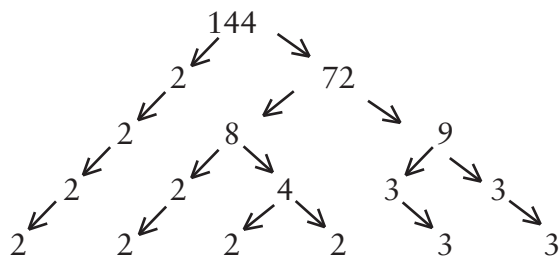$12 = \sqrt{144}$
The side length is 12 cm.

Literacy Link

Reading Square Roots

The symbol for square root is $\sqrt{}$. Read $\sqrt{9}$ as the square root of 9, square root 9, or root 9.

Method 3: Use Prime Factorization

144

2   72

2   8   9

2   2   4   3   3

2   2   2   2   3   3

The prime factorization of 144 is $2 \times 2 \times 2 \times 2 \times 3 \times 3$.

Rearrange the prime factors into two equal groups.

$144 = (2 \times 2 \times 3) \times (2 \times 2 \times 3)$

$144 = 12 \times 12$

$\sqrt{144} = 12$

The side length is 12 cm.

Show You Know

Determine the side length of a square with an area of 196 cm².

Key Ideas

- The square of a number is the number multiplied by itself.
  $5 \times 5 = 25$, or $5^2 = 25$
- The square of a whole number is a perfect square. $2^2 = 4$
  So, 4 is a perfect square.
- The square of a number can be thought of as the area of a square.
  $4^2 = 16$
  The area is 16 cm².
- The square root of a number can be thought of as the side length of a square.
  $\sqrt{16} = 4$
  The side length is 4 cm.
- The square root of a value is a number that when multiplied by itself equals the value.
  $6 \times 6 = 36$, so $\sqrt{36} = 6$
- In the prime factorization of a perfect square, there is an even number of each prime factor.
  $36 = 2 \times 2 \times 3 \times 3$ two factors of 2, two factors of 3

$A = 16$ cm²   4 cm
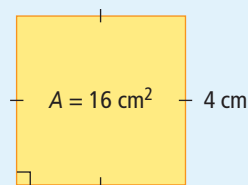
Communicate the Ideas

1. Explain how to square the number 7.

2. How would you use prime factorization to determine the square root of 225? Compare your answer with a classmate's.

3. The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Use words and/or diagrams to explain how you know which factor is the square root of 36.

4. Explain how squaring a number is the reverse of finding the square root of a number. Include an example with your explanation.

Check Your Understanding

Practise

For help with #5 to #8, refer to Example 1 on page 82.

5. a) Determine the prime factorization of 4.
   b) Is 4 a perfect square? Explain.
   c) Draw the square and label its side length.

6. A rectangle has an area of 64 m².
   a) Determine the prime factorization of 64.
   b) Is 64 a perfect square? Explain.
   c) Draw a square with that area and label its side length.

7. Write the prime factorization of each number. Identify the perfect squares.
   a) 42   **b)** 169   **c)** 256

8. Determine the prime factorization of each number. Which numbers are perfect squares?
   a) 144   **b)** 60   **c)** 40

For help with #9 to #12, refer to Example 2 on page 83.

9. What is the area of a square with each side length?
   a) 10   **b)** 16

10. Determine the area of a square with each side length.
   a) 20   **b)** 17

11. What is the square of each number?
   a) 9   **b)** 11

12. Determine the square of each number.
   a) 3   **b)** 18

For help with #13 to #16, refer to Example 3 on pages 83–84.

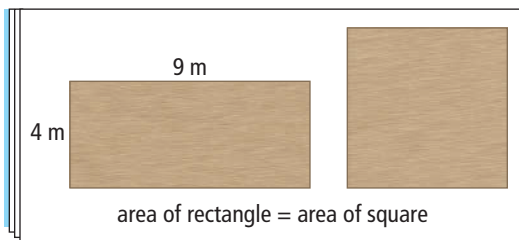**13.** What is the side length of the square shown?

49 mm²
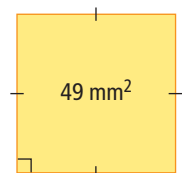
14. Determine the side length of a square with an area of 900 cm².

15. Evaluate.

   a) $\sqrt{49}$ **b)** $\sqrt{64}$ **c)** $\sqrt{625}$

16. Determine the value.

   a) $\sqrt{9}$ **b)** $\sqrt{25}$ **c)** $\sqrt{1600}$

Apply

17. A fridge magnet has an area of 54 mm². Is 54 a perfect square? Use prime factorization to find the answer.

**18.** A floor mat for gymnastics is a square with a side length of 14 m. What is the area of the floor mat in square metres?

19. The gym teacher told the students to run twice around the perimeter of the school field. The area of the square field is 28 900 m². What distance did the students run?

20. Adam's uncle has instructions for building a shed. One page of the instructions, shown below, is not very clear.

9 m

4 m

area of rectangle = area of square

   a) What is the area of the rectangle?
   b) What is the side length of the square?

21. Kate is going to put a patio in her backyard. The patio stones she is using each have an area of 1 m². She has created the rectangular design shown.

4 m

14 m
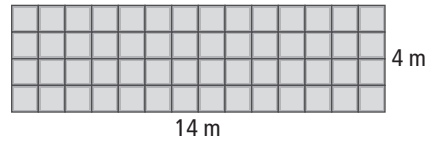
   a) What is the area of the patio?
   b) What are the dimensions of another rectangular patio she could build with the same area?
   c) Kate decides to make a patio with the same area but she wants it to be a square with whole number side lengths. Is this possible? Explain your reasoning.

22. The world's largest city square is Tiananmen Square in Beijing, China. It has an area of 396 900 m².

   a) What are the dimensions of the square?
   b) If the square had dimensions of 629 m by 629 m, what would be the area?
   c) If the square had an area less than 394 000 m² and greater than 386 000 m², what are all of the possible whole number dimensions that it could have?

23. A helicopter landing pad has a square shape. The area is 400 m². Use prime factorization to find the side length of the pad.
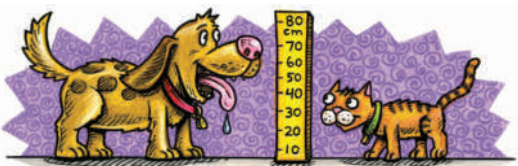
24. The first three triangular numbers are

1   3   6
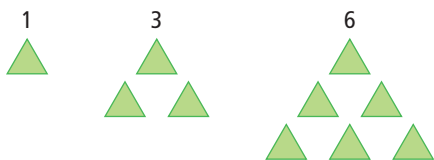
a) What are the next three triangular numbers?

b) Add together any two consecutive triangular numbers. What do you notice about the sums?

25. A square digital photo on the computer has an area of 144 cm².

a) What is the side length of the photo?

b) The photo is enlarged so that the side length is now 36 cm. What is the area of the enlarged photo?

c) How many times as large as the original area is the enlarged area?

Imagine your dog is 80 cm tall and your cat is 40 cm tall. How many times as tall as your cat is your dog? What operation did you perform?

d) How many times as large as the original side length is the enlarged side length?

e) Use what you know about the square root of a perfect square to identify the relationship between the numbers in parts c) and d).

26. a) Determine which of the following numbers are perfect squares: 10, 100, 1000, 10 000, 100 000.

b) State the square root of each perfect square.

c) Choose one of the numbers that is not a perfect square. Explain how you know that it is not a perfect square.

d) Describe a quick method for determining mentally if the numbers are perfect squares.

e) Use your method in part d) to decide if 1 000 000 000 is a perfect square. **M E**

27. a) Determine the square root of each number: 6400, 640 000, 64 000 000.

b) Describe a quick method for determining mentally the square root of each number in part a).

c) Explain why this method does not work for evaluating $\sqrt{640}$.

d) Use your method in part b) to evaluate $\sqrt{640\,000\,000\,000}$. Explain how you determined the answer. **M E**

MATH LINK

Chess is played on a square board. The board is made up of 32 white squares and 32 dark squares.

You decide to make your own chessboard. You are going to cut the board out of the 42 cm x 50 cm piece of wood shown.

42 cm

50 cm

Each square on the board will have whole number side lengths. The chess pieces fit on squares that are no smaller than 9 cm². What are all of the possible dimensions that your board could have?

Exploring the Pythagorean Relationship

Right triangles are found in art, construction, and many other objects. The sail for this sailboat is a right triangle. What makes this shape so special? You will explore some important properties of right triangles in this lesson.

Explore the Math

Materials

- centimetre grid paper
- scissors
- transparent tape
- protractor

What is a relationship that applies to right triangles?

1. From a piece of centimetre grid paper, cut out three squares with the following dimensions:

6 cm × 6 cm   8 cm × 8 cm   10 cm × 10 cm

2. Arrange the squares to form Triangle 1 as shown. Tape the squares onto a sheet of paper. Label Triangle 1.

> The length of side *a* is 6 cm, side *b* is 8 cm, and side *c* is 10 cm.

Triangle 1

a

b

c
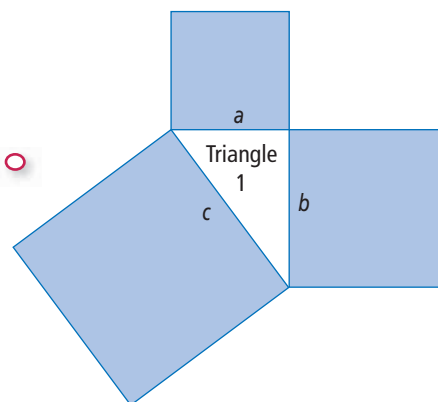
3. Copy the table below into your notebook.

	Side	Side Length (cm)	Angle Opposite the Side (°)	Area of Square (cm²)	Right Triangle? (yes/no)
Triangle 1	a	6	37		
	b	8			
	c	10			
Triangle 2	a	5			
	b	7			
	c	10			
Triangle 3	a	5		25	
	b			144	
	c			169	

4. Measure the angle opposite each side of Triangle 1 with a protractor.

angle opposite side b

angle opposite side c

a

Triangle 1

c

b

angle opposite side a
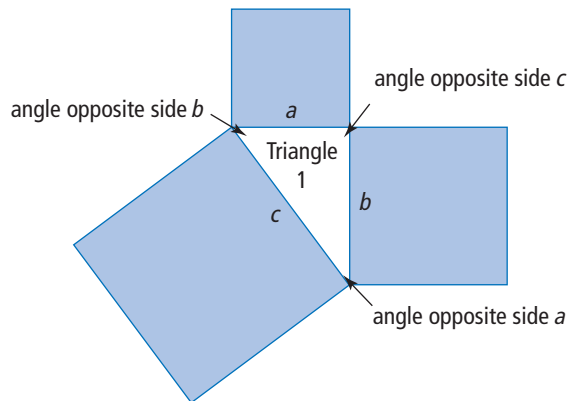
5. In your table, record the angle measures to the nearest degree.

6. Complete the rest of the table for Triangle 1.

7. Repeat the above steps for Triangles 2 and 3 in the table.

Reflect on Your Findings

8. a) Which triangles are right triangles? How do you know?

   b) For each right triangle, write an addition statement showing the relationship between the areas of the three squares.

   c) For each right triangle, describe in words the relationship between the side lengths of the triangle.
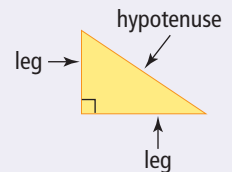
Example 1: Describe Relationships in Right Triangles

a) What is the area of each square?

b) Which side is the hypotenuse of the triangle?

c) Write an addition statement showing the relationship between the areas of the three squares.

d) Describe, using words and symbols, the relationship between the side lengths of the triangle.
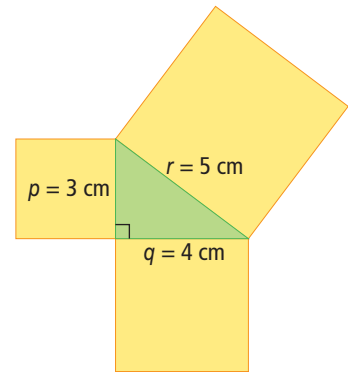
*r* = 5 cm
p = 3 cm
q = 4 cm

Solution

a) $p = 3$ cm $q = 4$ cm $r = 5$ cm
$A = 3^2$ $A = 4^2$ $A = 5^2$
$A = 9$ $A = 16$ $A = 25$
The area is 9 cm². The area is 16 cm². The area is 25 cm².

b) Side r is the hypotenuse.

This relationship is called the **Pythagorean relationship.**

c) $9 + 16 = 25$

d) The sum of the areas of the squares attached to legs p and q equals the area of the square attached to hypotenuse r.

For a right triangle with legs p and q and hypotenuse r, $p^2 + q^2 = r^2$.

Pythagorean relationship

- the relationship between the lengths of the sides of a right triangle
- The sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse.

a^2
b^2
c^2

$a^2 + b^2 = c^2$
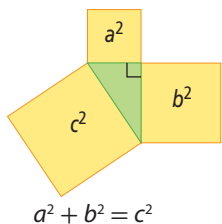
Show You Know

The sides of a right triangle are 9 cm, 12 cm, and 15 cm.

a) Sketch a picture of the triangle. Draw a square on each side of the triangle.

b) What is the area of each square?

c) Write an addition statement using the areas of the three squares.

Example 2: Identify a Right Triangle

A triangle has side lengths of 5 cm, 7 cm, and 9 cm.

a) What are the areas of the three squares that can be drawn on the sides of the triangle?

**b)** Is the triangle a right triangle? Explain your answer.

Literacy Link

The symbol ≠ means "is not equal to."

Solution

a) $5 \times 5 = 25$ $7 \times 7 = 49$ $9 \times 9 = 81$
The area is 25 cm².   The area is 49 cm².   The area is 81 cm².

b) Calculate the sum of the areas of the two smaller squares.
$25 + 49 = 74$
The sum of the areas is 74 cm². The sum does not equal the area of the large square. 74 cm² ≠ 81 cm²
The triangle is not a right triangle.

Show You Know

A triangle has side lengths of 12 cm, 16 cm, and 20 cm.

a) What are the areas of the three squares that can be drawn on the sides of the triangle?

b) Is the triangle a right triangle? Explain.

Key Ideas

- In a right triangle, the sum of the areas of the squares attached to the legs equals the area of the square attached to the hypotenuse.
- The Pythagorean relationship states that in a right triangle with sides s, t, and v, where side v is the hypotenuse, $v^2 = s^2 + t^2$.

$$v^2 = s^2 + t^2$$
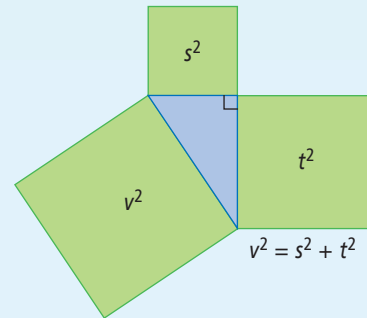
Communicate the Ideas

1. Describe, using words and symbols, the relationship among the areas of the three squares shown.

2. A triangle has side lengths of 7 cm, 11 cm, and 15 cm. Explain how you can determine whether or not it is a right triangle.

3. For the triangle shown, Kendra wrote the Pythagorean relationship as $r^2 = p^2 + q^2$. Is she correct? Explain.
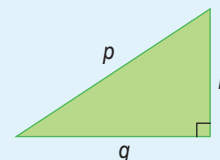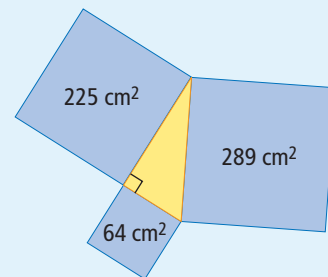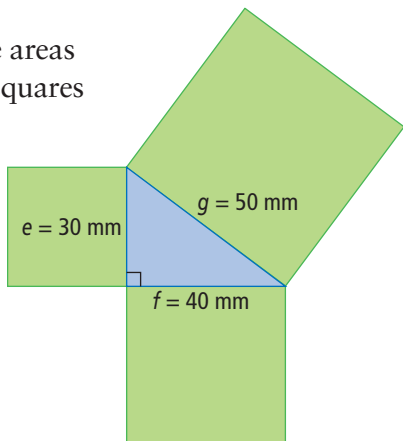
Practise

For help with #4 to #7, refer to Example 1 on page 90.

4. What are the areas of the three squares shown?

$g = 50$ mm

$e = 30$ mm

$f = 40$ mm

5. A right triangle has side lengths of 40 mm, 75 mm, and 85 mm.

a) Sketch the triangle. Draw a square on each side of the triangle.

b) What are the areas of the three squares?

c) Write an addition statement with the areas of the three squares.

6. a) Write an addition statement using the areas of these three squares.
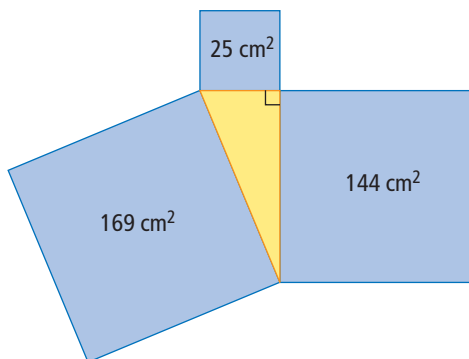
25 cm²

144 cm²

169 cm²

b) What is the side length of each square?

c) Describe, using words and symbols, the relationship between the side lengths of each square.

7. The sides of a right triangle measure 9 cm, 12 cm, and 15 cm.

a) What is the area of each square attached to the three sides of the right triangle?

b) Write an addition statement showing the relationship between the areas of the three squares.

c) Describe, using words and symbols, the relationship between the side lengths of each square.

For help with #8 to #11, refer to Example 2 on pages 90–91.

8. Is the triangle shown a right triangle? Explain your reasoning.

40 cm²

50 cm²

?

20 cm²
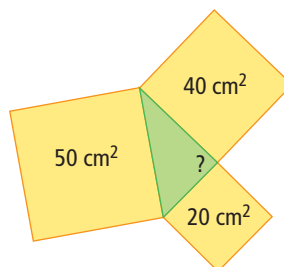
9. a) Calculate the areas of the three squares.

4 cm

3 cm

?

2 cm
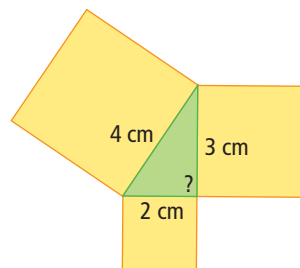
b) Is this triangle a right triangle? Explain.
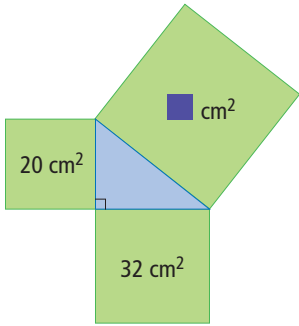
10. A triangle has side lengths of 120 mm, 160 mm, and 200 mm. Is the triangle a right triangle? Explain your reasoning.

11. The side lengths of a triangle are 5 cm, 6 cm, and 8 cm. Determine whether the triangle is a right triangle. Explain.
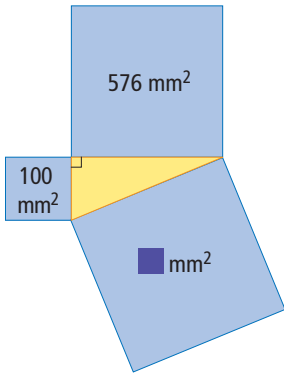
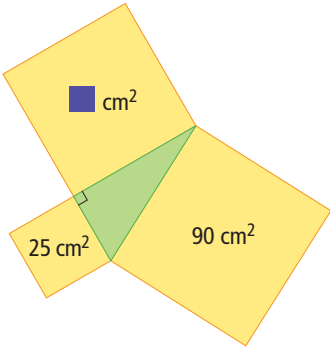**12.** Use the Pythagorean relationship to find the unknown area of each square.

a)

■ cm²

20 cm²

32 cm²

b)

576 mm²

100 mm²

■ mm²
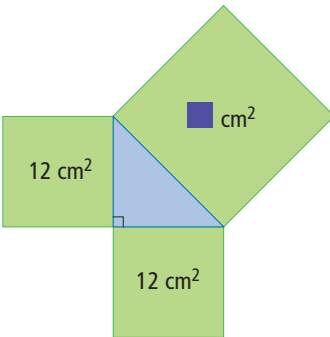
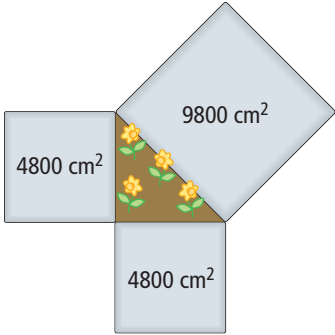**c)**

■ cm²

25 cm²

90 cm²

d)

■ cm²

12 cm²

12 cm²

13. A small triangular flower bed has a square stepping stone at each of its sides. Is the flower bed in the shape of a right triangle? Explain your reasoning.

9800 cm²

4800 cm²

4800 cm²

14. Show whether each triangle in the table is a right triangle.

Triangle	Side Lengths (cm)
A	9, 12, 15
B	7, 8, 11
C	7, 24, 25
D	16, 30, 34
E	10, 11, 14

15. Construction workers have begun to dig a hole for a swimming pool. They want to check that the angle they have dug is 90°. They measure the diagonal as shown to be 9.5 m. Is the angle 90°? Explain your reasoning.

8 m

?

6 m
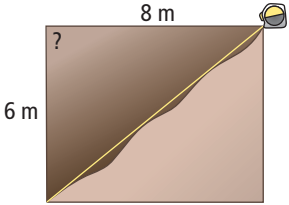
16. Baldeep is building a wooden box for storing coloured pencils. The box will have rectangular sides that are 12 cm wide and 20 cm long. Show how Baldeep can be sure the sides are rectangular, without using a protractor.

**17.** What is the area of the square that can be drawn on side c of each triangle?

a)

21 cm, 28 cm, c

b)

c, 5 mm, 12 mm
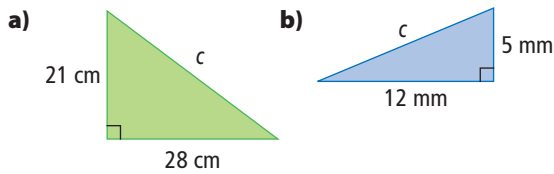
Extend

18. The diagram is made of two right triangles and five squares.

12 m², Y, X, 24 m², 4 m²
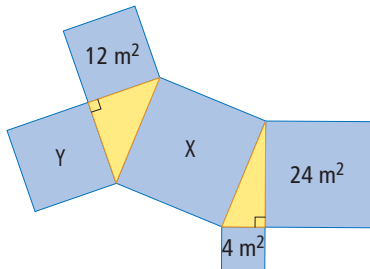
a) What is the area of square X?

b) What is the area of square Y?

19. A right triangle has a square attached to each side. Two of the squares have areas of 10 cm² and 15 cm². What are possible areas for the third square? Draw a sketch for each solution.

20. A right triangle has sides of 3 cm, 4 cm, and 5 cm. Attached to each side is a semi-circle instead of a square. Describe the relationship between the areas of the semi-circles.
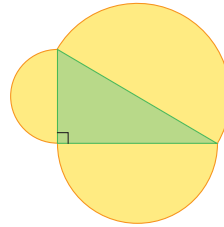
> **Literacy Link**
>
> area of a circle = $\pi \times r^2$

21. An example of a Pythagorean triple is 3, 4, 5.
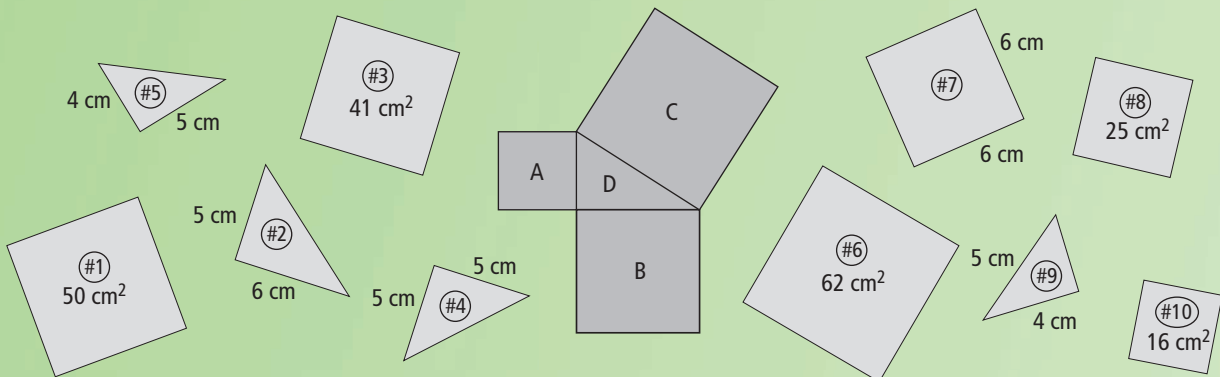
> **Did You Know?**
>
> A Pythagorean triple consists of three whole numbers that form the sides of a right triangle. For example, 3, 4, 5 make a Pythagorean triple because $3^2 + 4^2 = 5^2$.

a) Multiply each number by 2. Show whether the resulting three numbers form a Pythagorean triple.

b) Multiply each number in the triple 3, 4, 5 by a natural number other than 2. Show whether the results form a Pythagorean triple.

c) Is there any natural number that does not make a Pythagorean triple when 3, 4, 5 are multiplied by it? Explain.
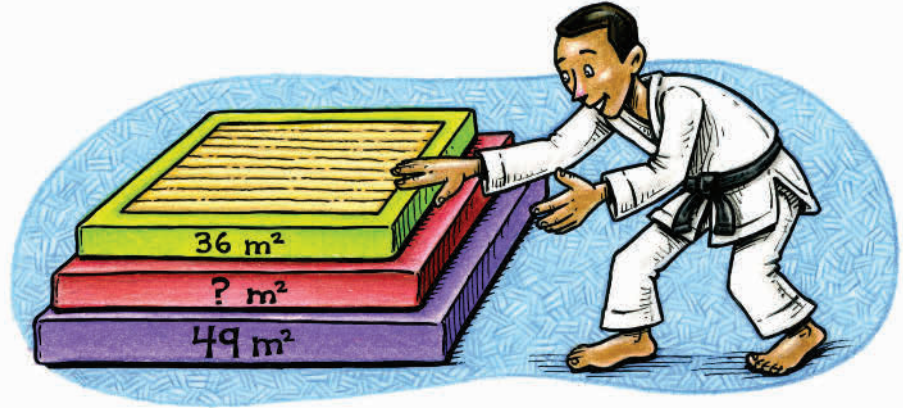
MATH LINK

Identify the right triangle and three squares that complete this Pythagorean puzzle.

4 cm, #5, 5 cm

#3, 41 cm²

6 cm, #7, 6 cm

#8, 25 cm²

A, D, C, B

5 cm, #2, 6 cm

#1, 50 cm²

5 cm, #4, 5 cm

#6, 62 cm²

5 cm, #9, 4 cm

#10, 16 cm²

Estimating Square Roots

The picture shows three tatami mats that are used in judo. Can you think of a way to estimate the side length of the middle mat?

Explore the Math

How do you estimate a square root?

1. What is a reasonable estimate for the area of the middle mat in the picture?

2. What are the side lengths of the smallest and largest mats? Explain how you calculated these dimensions.

3. The number line below shows square roots of perfect squares. Copy the number line into your notebook. Complete the boxes.

$\sqrt{1}$ $\sqrt{4}$ $\sqrt{9}$ $\sqrt{\blacksquare}$ $\sqrt{25}$ $\sqrt{36}$ $\sqrt{\blacksquare}$ $\sqrt{64}$ $\sqrt{\blacksquare}$

0 1 2 $\blacksquare$ 4 5 6 $\blacksquare$ 8 9
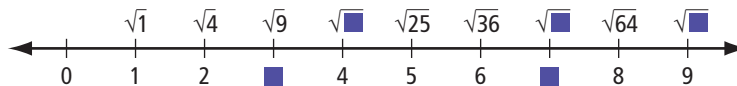
4. Use the number line to estimate the side length for the middle mat. Give your answer to one decimal place.

Reflect on Your Findings

5. a) Compare your estimate of the side length of the middle mat with a classmate's.

   b) Using a calculator, determine the square root of your estimate in #1. Give your answer to the nearest tenth. Compare this approximation to your estimate for the side length.

   c) Explain how you can use perfect squares to estimate a square root.

## Example 1: Estimate the Square Root of a Number

Felicity wants to know if a wading pool will fit in a small space in her yard. She must estimate the side length of the square wading pool, which has an area of 7 m².
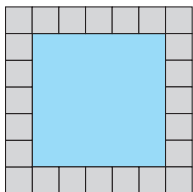
a) What is a reasonable estimate for the side length of the pool? Use perfect squares to estimate. Give your answer to one decimal place.

b) Use a calculator to approximate the side length of the pool, to the nearest tenth of a metre. Compare your estimate in part a) with the calculator's approximate answer.

Solution

a) The side length of the pool is the square root of 7.

The perfect squares on either side of 7 are 4 and 9.

Since 7 is closer to 9, the square root of 7 is closer to the square root of 9.
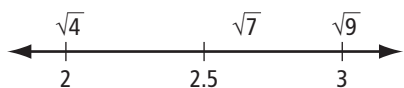
$\sqrt{9} = 3$
$\sqrt{7}$ will be a bit less than 3.
A reasonable estimate is 2.7 m.

b) Approximate the square root of 7.

$\boxed{C}\ \boxed{7}\ \boxed{\sqrt{}}$ 2.645751311

The answer to the nearest tenth of a metre is 2.6 m.
This answer is very close to the estimate of 2.7 m.

> **Strategies**
> **Estimate and Check**

> This value is an approximation. The decimal portion of the exact answer continues forever. The calculator can display only ten digits.
> The square of the approximation shows that it is not an exact answer:
> $2.645751311^2 = 6.999999999658218721$
> ≈ 7

Show You Know

For each of the following, use perfect squares to estimate the square root to one decimal place. Check your answer with a calculator.

a) $\sqrt{18}$ **b)** $\sqrt{23}$ **c)** $\sqrt{35}$

Example 2: Identify a Number With a Square Root Between Two Numbers

a) What is a whole number that has a square root between 6 and 7?

b) How many whole numbers can you find that have a square root between 6 and 7? Show your work.
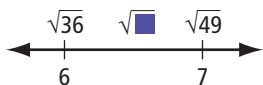
Solution

a) Determine the square of 6.

$6^2 = 36$

Determine the square of 7.

$7^2 = 49$

Draw a number line.

Find a value for ■ on the number line.
Choose any whole number between 36 and 49.
One possible whole number is 40.
$\sqrt{40}$ will have a value between 6 and 7.

Check:

C 40 √ 6.32455532

6.32455532 is between 6 and 7.
40 is a possible answer.

> **Strategies**
> **Estimate and Check**

b) The possible answers are all of the whole numbers larger than 36 and smaller than 49:

37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48

There are 12 whole numbers that have square roots between 6 and 7.

> **Show You Know**
>
> **a)** Identify a whole number with a square root between 8 and 9.
>
> **b)** How many whole numbers can you find that have a square root between 8 and 9? Show your work.
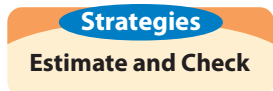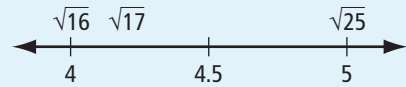
Key Ideas

- To estimate the square root of a whole number that is not a perfect square,
  - locate the perfect squares on either side of the number
  - calculate the square roots of these two perfect squares
  - estimate based on the position between the two perfect squares

For example, estimate the square root of 17:
$\sqrt{17} \approx 4.1$

- To identify a whole number that has a square root between two given numbers,
  - determine the perfect squares of the two consecutive whole numbers
  - choose a whole number between the two perfect squares

For example, identify a whole number that has a square root between 5 and 6:

$5^2 = 25$ $6^2 = 36$

$\sqrt{30}$ will have a value between 5 and 6.

- When using a calculator to find the square root of a natural number that is not a perfect square, the value shown on the calculator is only an approximation.

$\boxed{C}\ \boxed{8}\ \boxed{\sqrt{}}$ 2.828427125

Communicate the Ideas

1. Explain how to estimate $\sqrt{28}$ to one decimal place without using a calculator. Compare your answer with a classmate's.

2. Find a whole number that has a square root between 3 and 4. Explain how you found it.

3. Jason is doing his math homework. He has to find the square root of 10. He presses $\boxed{\sqrt{}}$ **10** on his calculator and the screen displays 3.16227766. However, when 3.16227766 is multiplied by itself, the answer is not 10. Explain.
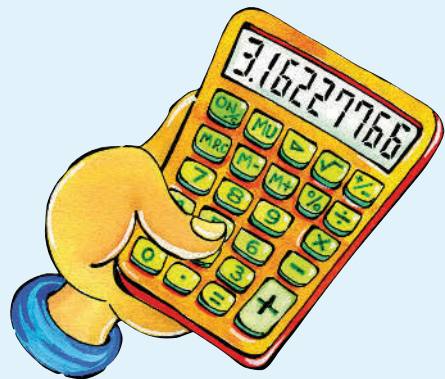
Check Your Understanding

Practise

4. Estimate the square root of each number, to one decimal place. Check with a calculator.

   a) 72   **b)** 103   **c)** 55

5. Estimate each value, to one decimal place. Check your answer with a calculator.

   a) $\sqrt{14}$ **b)** $\sqrt{86}$ **c)** $\sqrt{136}$

6. What is an example of a whole number that has a square root between 9 and 10?

7. Identify a whole number with a square root between 11 and 12.

8. Identify all possible whole numbers with a square root larger than 2 and smaller than 3.

9. What are all possible whole numbers that have a square root between 4 and 5?

Apply

10. Kai uses an entire can of paint on a square backdrop for the school play. The label on the can states that one can covers 27 m² of wall surface. Estimate the backdrop's side length, to one decimal place.

PAINT

covers 27 m² of wall surface

11. The square has an area of 20 cm².

20 cm²
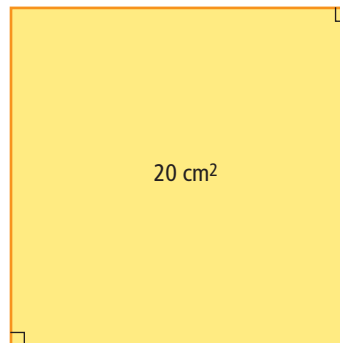
   a) Use perfect squares to estimate the side length to one decimal place.

   b) Check your answer using a ruler to measure the side of the square. Measure to the nearest tenth of a centimetre.

12. While shopping online, Ji Hun finds a square rug with an area of 11 m². He needs to know if it will fit in his 4 m × 5 m bedroom.

   a) Estimate the side length of the rug, to one decimal place.

   b) Check your estimate with a calculator.

   c) Will the rug fit? Explain.

13. Stella is planning an outdoor wedding. She would like a square dance floor with an area of 115 m².

   a) Determine the side length of the dance floor, to the nearest tenth of a metre.

   b) Stella finds out that the dance floor will be made up of floorboards that each measure 1 m². What are the two side lengths the dance floor can have that are closest to what she wants?

   c) What are the two square areas for the dance floor that Stella can choose from?

   d) Which area will Stella choose? Explain.

14. Alex is thinking of a number.

The number has a square root between 7 and 8, and it is a multiple of 12.
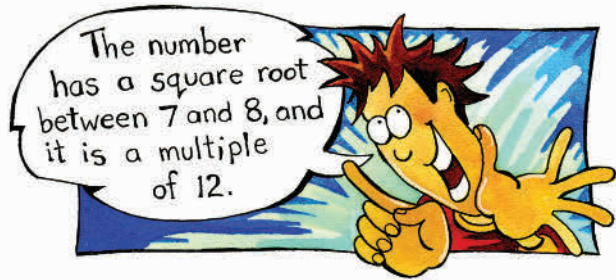
a) What number could he be thinking of?

b) Is there more than one answer? Explain.

15. Order the following numbers from least to greatest: $7, \sqrt{46}, 5.8, \sqrt{27}, 6.3$.

16. A fitness centre will install a square hot tub in a 6 m × 6 m room. They want the tub to fill no more than 75% of the room's area.

a) What is the maximum area of the hot tub?

b) What dimensions, to a tenth of a metre, will the fitness centre order from the manufacturer? Explain.

17. Carmel wants to mount an 18 cm × 18 cm square picture on a square board that is four times the area of the picture.

a) What is the area of the picture?

b) What is the area of the board?

c) What are the dimensions of the board?

Extend

18. a) Evaluate $\sqrt{9}$.

b) Estimate the square root of your answer in part a), to one decimal place.

c) Use a calculator to check your estimate. Express your answer to the nearest hundredth.

d) How close is your estimate in part b) to your calculation in part c)?

19. Estimate $\sqrt{160\ 100}$. Explain how you determined your estimate.

20. What is the smallest natural number value for n if the solution for $\sqrt{56n}$ is also a natural number?

21. Determine two numbers that have a square root between 326 and 327, are divisible by 100, and are a multiple of 6.

MATH LINK

You have created a mini peg board game called Mind Buster. The square game board has a base area of 134 cm². You go to the store to get a box for storing the game. You find five boxes with the base dimensions shown.
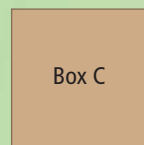
a) Identify which boxes can store the game board. Explain.
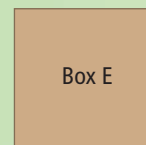
b) Which box would you choose? Why?

Box A 11.3 cm
11.3 cm

Box B 11.3 c m
11.9 cm

Box C 11.7 cm
11.4 cm

Box D 11.7 cm
11.6 cm
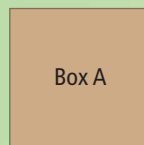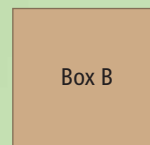
Box E 11.9 cm
11.9 cm

3.4

Using the Pythagorean Relationship

27 m

A baseball diamond is a square. How could you determine the distance from second base to home plate? How many different strategies can you develop?

Explore the Math

### How do you determine the missing side length of a right triangle?

1. On centimetre grid paper, draw a right triangle.

2. Describe two methods for finding the length of the hypotenuse of a right triangle.

Reflect on Your Findings

3. a) Describe a situation in which one method would be better to use than another.

   b) Work with a partner to determine the distance from second base to home plate on a baseball diamond. Share your solution with another pair of classmates.

Materials
- centimetre grid paper
- ruler

Example 1: Determine the Length of the Hypotenuse of a Right Triangle

Determine the length of hypotenuse c. Express your answer to the nearest tenth of a metre.
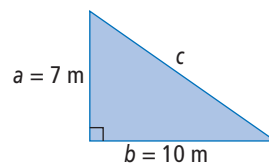
Solution

Use the Pythagorean relationship, $c^2 = a^2 + b^2$, where the length of the hypotenuse is c, and the lengths of the legs are a and b.

$c^2 = 7^2 + 10^2$
$c^2 = 49 + 100$
$c^2 = 149$
$c = \sqrt{149}$
$c \approx 12.2$
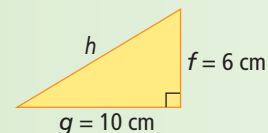
The length of the hypotenuse is approximately 12.2 m.

Show You Know

Determine the length of the hypotenuse for the right triangle, to the nearest centimetre.

Example 2: Determine the Length of a Leg of a Right Triangle

What is the length of leg e of the right triangle?
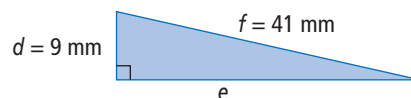
Solution

Use the Pythagorean relationship, $d^2 + e^2 = f^2$, where the length of the hypotenuse is f, and the lengths of the legs are d and e.

$9^2 + e^2 = 41^2$
$81 + e^2 = 1681$
$81 + e^2 - 81 = 1681 - 81$
$e^2 = 1600$
$e = \sqrt{1600}$
$e = 40$

Why do you subtract 81?

The length of the leg is 40 mm.

Show You Know

Determine the length of leg s of the right triangle.
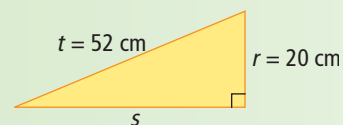
Key Ideas

- The Pythagorean relationship can be used to determine the length of the hypotenuse of a right triangle when the lengths of the two legs are known.
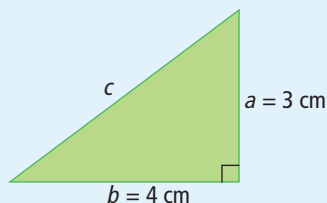
$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2$$
$$c^2 = 9 + 16$$
$$c^2 = 25$$
$$c = \sqrt{25}$$
$$c = 5$$

The length of hypotenuse c is 5 cm.

- The Pythagorean relationship can be used to determine the leg length of a right triangle when the lengths of the hypotenuse and the other leg are known.

$$p^2 + q^2 = r^2$$
$$p^2 + 12^2 = 15^2$$
$$p^2 + 144 = 225$$
$$p^2 + 144 - 144 = 225 - 144$$
$$p^2 = 81$$
$$p = \sqrt{81}$$
$$p = 9$$
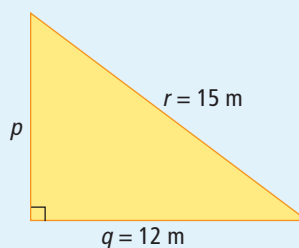
The length of leg p is 9 m.

Communicate the Ideas

1. Jack must determine the missing side length of a triangle. He decides to draw it and then measure it, as shown. Do you agree with the method that Jack is using? Explain.

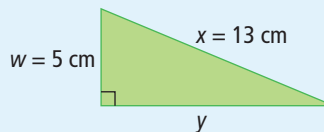**2.** Kira calculated the missing side length of the right triangle.

$$y^2 = 5^2 + 13^2$$
$$y^2 = 25 + 169$$
$$y^2 = 194$$
$$y \approx 13.9$$

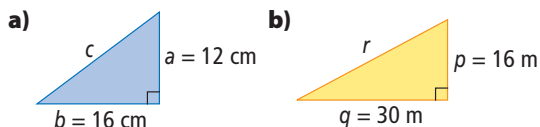The length of side y is approximately 13.9 cm.

Is Kira correct? If she is correct, explain how you know. If she is incorrect, explain the correct method.
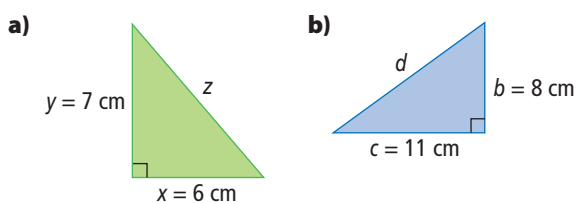
Check Your Understanding

Practise

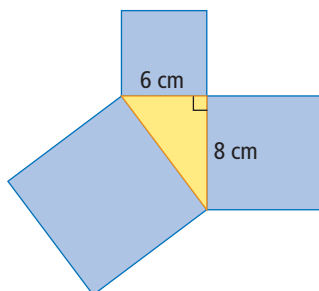*For help with #3 and #4, refer to Example 1 on page 102.*

3. Determine the length of each hypotenuse.

a)

c, $a = 12$ cm, $b = 16$ cm

b)

r, $p = 16$ m, $q = 30$ m

4. What is the length of each hypotenuse? Give your answer to the nearest tenth of a centimetre.

a)

z, $y = 7$ cm, $x = 6$ cm

b)

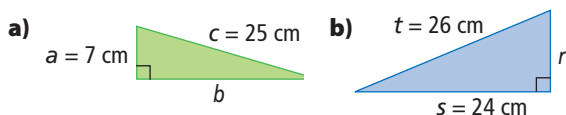d, $b = 8$ cm, $c = 11$ cm

5. a) What is the area of each square attached to the legs of the right triangle?

6 cm
8 cm

b) What is the area of the square attached to the hypotenuse?

c) What is the length of the hypotenuse?

For help with #6 and #7, refer to Example 2 on page 102.

6. Determine the length of the leg for each right triangle.

a)

$c = 25$ cm, $a = 7$ cm, b

b)

$t = 26$ cm, r, $s = 24$ cm

7. What is the missing length of the leg for each triangle? Give your answer to the nearest tenth of a millimetre.

a)

h, $i = 9$ mm, $g = 5$ mm
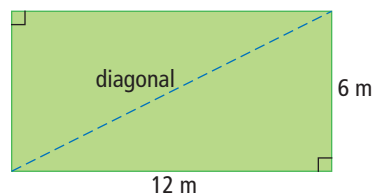
b)

$q = 11$ mm, p, $r = 15$ mm
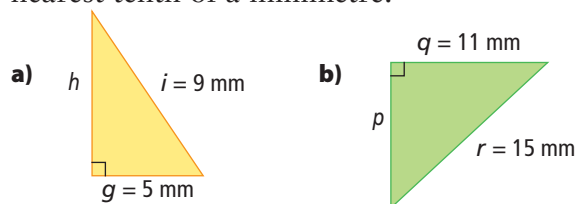
Apply

8. The side view of a ramp at a grocery store is in the shape of a right triangle. Determine the length of the ramp, to the nearest centimetre.

50 cm
200 cm

9. Tina wants to construct a path along the diagonal of her yard. What length will the path be? Express your answer to the nearest tenth of a metre.

diagonal
6 m
12 m

10. What is the minimum distance the player at third base has to throw the ball to get the runner out at first base? Express your answer to the nearest tenth of a metre.


second base
first base
third base
27 m
home plate
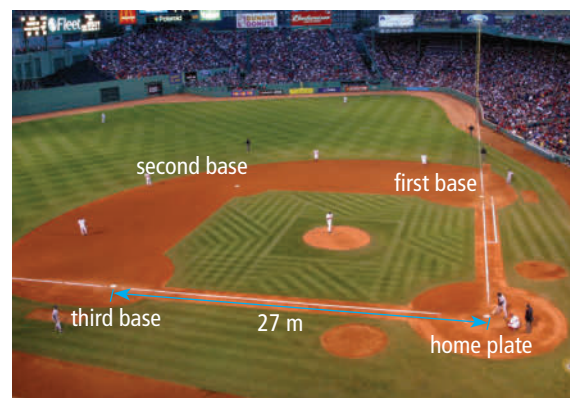
11. The right triangle below has a square attached to its hypotenuse. What is the perimeter of the triangle? Give your answer to the nearest tenth of a centimetre.


914 cm²

17 cm
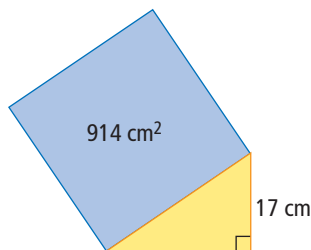
12. The hypotenuse of the triangle cuts the circle in half. What is the diameter of the circle? Express your answer to the nearest tenth of a centimetre.

■ cm

7 cm

5 cm
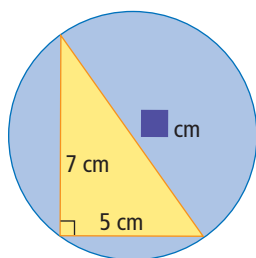
13. Determine the length of the base of the large triangle. Express your answer to the nearest tenth of a millimetre.
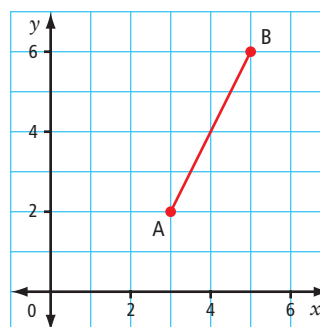

10 mm   10 mm

8 mm
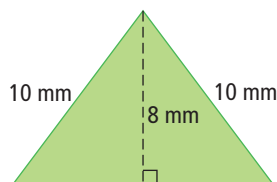
14. What are the lengths of b and c? Write your answer to the nearest tenth of a metre where appropriate.
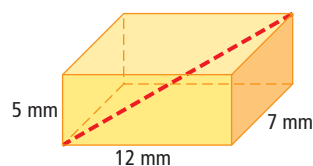

c

b   5 m

3 m   3 m
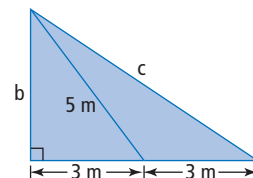
15. The coordinate grid shown was drawn on centimetre grid paper. What is the length of line segment AB? Express your answer to the nearest tenth of a centimetre.


16. What is the length of the red diagonal in the box? Express your answer to the nearest tenth of a millimetre.

5 mm

12 mm   7 mm

MATH LINK

For each of the following questions, express your answer to the nearest tenth of a centimetre.

a) What is the distance between A and B? Explain.
b) If you have to follow the lines on the game board, what is the shortest distance between C and D?
c) If you do not have to follow the lines on the game board, what is the shortest distance between C and D? Justify your answer.
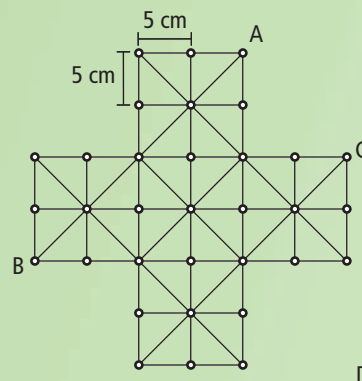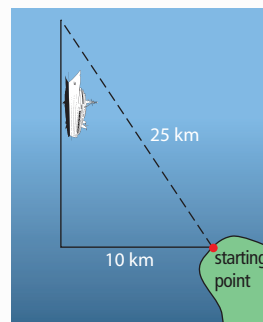

5 cm

5 cm

A

C

B

D

Applying the Pythagorean Relationship

A ship leaves the Pacific coast of British Columbia and travels west for 10 km. Then, it turns and travels north. When the ship is 25 km from its starting point, how could you use the Pythagorean relationship to determine the distance the ship travelled north?
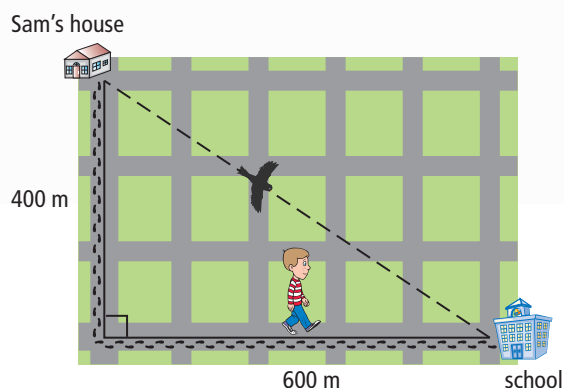

25 km

10 km   starting point

Explore the Math

How can you determine a distance using the Pythagorean relationship?

The diagram shows Sam's trip to school.

1. a) Work with a partner to determine how far his house is from the school.

   b) Share your answer with your classmates. Is there more than one possible answer? Explain.

Sam's house

400 m

600 m   school

2. a) What do you think the expression "as the crow flies" means?

   b) How much farther does Sam travel than the crow? Show your method.

Reflect on Your Findings

**3.** Why is the path that the crow takes from Sam's house to the school difficult to measure directly?

Example 1: Determine Distances With Right Triangles

a) Anthony and Shalima are canoeing on a lake in Saskatchewan. There are two boat ramps on the lake. How far is it by canoe between the boat ramps?

b) How much farther is it for someone to travel by road from ramp A to ramp B than to canoe between the two ramps?

boat ramp A

1500 m

800 m
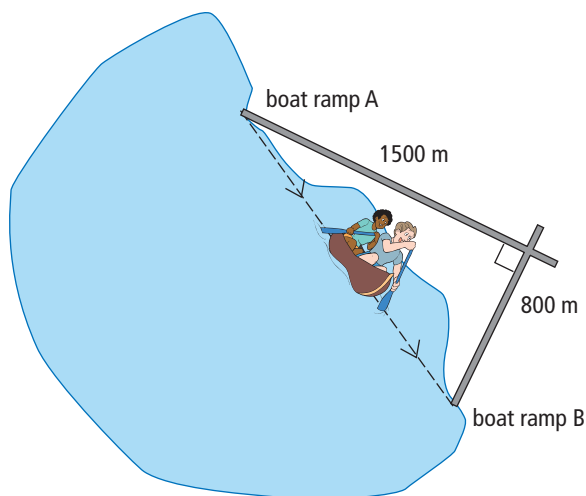
boat ramp B

Solution

a) The two roads leading from the boat ramps make the legs of a right triangle. The distance by canoe is the hypotenuse.

Let d represent the distance by canoe.

Use the Pythagorean relationship.
$$d^2 = 1500^2 + 800^2$$
$$d^2 = 2\,250\,000 + 640\,000$$
$$d^2 = 2\,890\,000$$
$$d = \sqrt{2\,890\,000}$$
$$d = 1700$$
The distance by canoe is 1700 m.

Strategies

Solve an Equation

Strategies

What other method could you use to solve this problem?

b) Determine the total distance by road between the boat ramps.
$$1500 + 800 = 2300$$
The total distance by road is 2300 m.

Determine the difference between the two distances.
$$2300 - 1700 = 600$$
It is 600 m farther to travel by road than by canoe between the boat ramps.
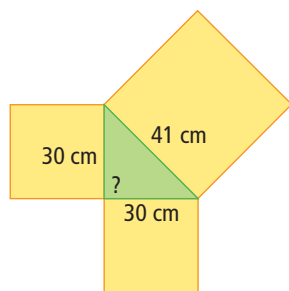
Show You Know

Refer to the opening paragraph and picture on page 106. A ship leaves the Pacific coast of British Columbia and travels west for 10 km. Then, it turns and travels north. If the boat is 25 km from its starting point, what distance did it travel north? Give your answer to the nearest tenth of a kilometre.

Example 2: Verify a Right Angle Triangle

Danelle is trying to install a corner shelf in her bedroom. Since the shelf does not fit properly, she thinks the two walls in her bedroom do not meet at a right angle. She measures a length of 30 cm along the base of each wall away from the corner. Then, she measures the hypotenuse to be 41 cm. Do the walls meet at a right angle? Explain.

Solution

Use the Pythagorean relationship to determine whether the triangle is a right triangle.

Determine whether the sum of the areas of the two smaller squares equals the area of the large square.

Left Side:
$30^2 + 30^2 = 900 + 900$
$\qquad\quad = 1800$
The sum of the areas of the two smaller squares is 1800 cm².

Right Side:
$41^2 = 1681$
The area of the large square is 1681 cm².

$$1800 \text{ cm}^2 \neq 1681 \text{ cm}^2$$

The triangle is not a right triangle. The walls do not meet at a right angle.

Show You Know

A construction company is digging a rectangular foundation with a width of 17 m and a length of 20 m. To check that a corner is a right angle, a worker measures the diagonal length, which is 26.25 m. Is the corner a right angle? Explain.

Key Ideas

- The Pythagorean relationship can be used to determine distances that might be difficult or impossible to measure.
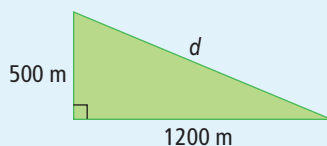
$d^2 = 500^2 + 1200^2$
$d^2 = 250\,000 + 1\,440\,000$
$d^2 = 1\,690\,000$
$d = \sqrt{1\,690\,000}$
$d = 1300$

The hypotenuse is 1300 m.

- The Pythagorean relationship can be used to show if a triangle is a right triangle.

Left Side:
$6^2 + 8^2 = 36 + 64$
$= 100$
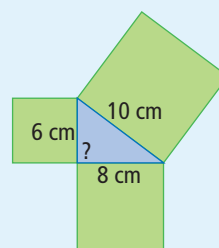The sum of the areas of the two smaller squares is 100 cm².

Right Side:
$10^2 = 100$
The area of the large square is 100 cm².

$$100 \text{ cm}^2 = 100 \text{ cm}^2$$

The triangle is a right triangle.

Communicate the Ideas

1. Use an example from real life to explain how you can apply the Pythagorean relationship to calculate distance.

2. Ilana used the following method to determine whether the diagram shows a right triangle.

Left Side:
The large square is 61 cm.
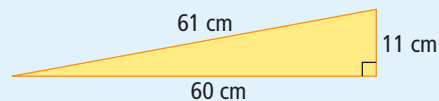
Right Side:
11 + 60 = 71
The two smaller squares are 71 cm.

61 cm ≠ 71 cm
The triangle is not a right triangle.

Is Ilana's method correct? If it is correct, explain how you know. If it is incorrect, explain the method llana should use.

Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on page 107.

3. Walter walks across a rectangular field in a diagonal line. Maria walks around two sides of the field. They meet at the opposite corner.

120 m

300 m
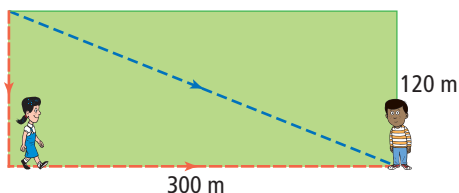
a) How far did Maria walk?

b) How far did Walter walk? Express your answer to the nearest metre.

c) Who walked farther? By how much?

4. Find the height of the pole where the guy wire is attached, to the nearest tenth of a metre.
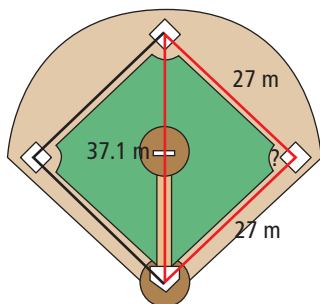
guy wire

10 m

2 m
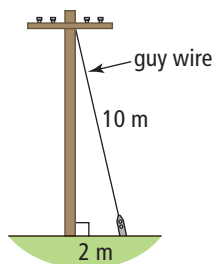
For help with #5 and #6, refer to Example 2 on page 108

5. Martin measured a rectangle and wrote:
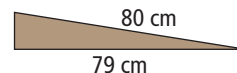
Width: 9 cm   Length: 22 cm   Diagonal: 23.8 cm

Could these measurements form a rectangle? Justify your answer.

6. You are asked to check the design plans for a baseball diamond. Is the triangle a right triangle? Explain.
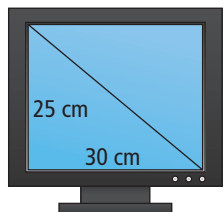
27 m

37.1 m

?

27 m

Apply

7. What is the height of the wheelchair ramp? Give your answer to the nearest tenth of a centimetre.

80 cm

79 cm

8. Shahriar knows that the size of a computer monitor is based on the length of the diagonal of the screen. He thinks that the diagonal is not as large as the ad says. Is he correct? Explain.

GREAT DEAL

42-cm monitor on sale!

25 cm

30 cm

9. A checkerboard is made of 64 small squares that each have a dimension of 3 cm × 3 cm. The 64 small squares are arranged in eight rows of eight.

a) What is the length of the diagonal of a small square? Give your answer to the nearest tenth of a centimetre.

b) What is the total length of the diagonal of the board? Give your answer to the nearest centimetre.

10. A gymnast requires a distance of 16 m for her tumbling routine. If the gymnast is competing on a 12 m × 12 m square mat, does she have enough room to do her routine safely? Explain your answer.
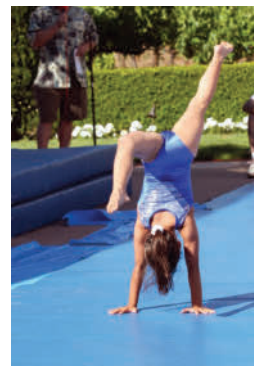
**11.** Johan has a 300-cm ladder that he leans up against a wall. The safety sticker on the side of the ladder shows that the bottom must be placed between 70 cm and 110 cm away from the wall. What are the minimum distance and maximum distance up the wall that the ladder can reach? Give your answers to the nearest tenth of a centimetre.

Extend

12. Sarah has a vegetable garden in the shape of a right triangle. She wants to put fencing all around it to keep the rabbits away.

  a) What total length of fencing does she need? Give your answer to the nearest hundredth of a metre.

  b) If fencing costs $2/m, what will be the total cost of the fencing?
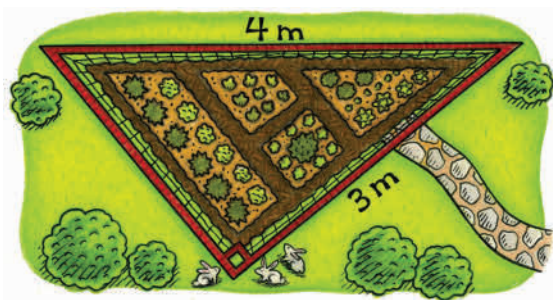
13. A cruise ship travels from Port Cassett north at a speed of 34 km/h for 2.5 h. Then it turns 90° and travels west at 30 km/h for 7.3 h. When it reaches Green Sea Island, how far is the ship from Port Cassett? Express your answer to the nearest kilometre.
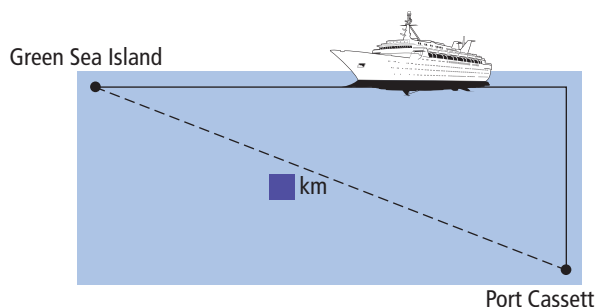

14. The red square has a perimeter of 40 mm and the green square has an area of 4 mm^2. What is the shortest distance between A and B? Give your answer to the nearest tenth of a millimetre.

MATH LINK

The diagram shows the rough plans for a board game designed for a toy manufacturer. The board is composed of a square and four identical right triangles. Complete the plans by answering the following questions. Give your answers to the nearest tenth of a centimetre where appropriate.

a) If the central square has an area of 225 cm^2, what is the perimeter of the game board? Show how you know.

b) The game will be packaged in a box with a square base. Determine the minimum diagonal length of the base of the box.

Chapter Review

3
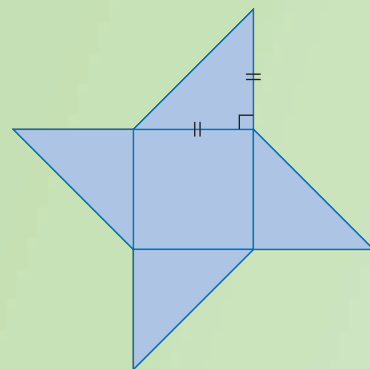
Key Words

For #1 to #5, write in your notebook the terms from the list that complete the sentences below.

*hypotenuse   perfect square*
prime factorization   Pythagorean relationship
square root

1. The ▆▆▆ ▆▆▆ of 36 is 6.

2. The number 25 is a ▆▆▆ ▆▆▆ because it is the product of the same two factors, $5 \times 5 = 25$.

3. In a right triangle, the longest side is known as the ▆▆▆.

4. If the sides of a right triangle are a, b, and c, and c is the longest side, the equation $c^2 = a^2 + b^2$ is known as the ▆▆▆ ▆▆▆.

5. The ▆▆▆ ▆▆▆ of 18 is $2 \times 3 \times 3$.

3.1 Squares and Square Roots, pages 80–87

6. Determine the square of each number.
   a) 6   b) 11   c) 25

7. Determine each square root.
   a) $\sqrt{49}$
   b) $\sqrt{256}$
   c) $\sqrt{100\,000\,000}$

8. Lisa needs at least 17 m² of fabric to make curtains. Is this square piece of fabric large enough? Show your work.
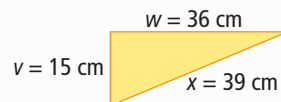
   4 m
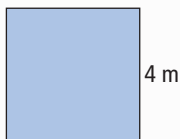
3.2 Exploring the Pythagorean Relationship, pages 88–94

9. A triangle has squares on each of its sides.
   a) Is the triangle a right triangle? Explain.
   b) What is the length of each of the three sides?

   16 cm²   36 cm²   ?   16 cm²

10. Is the triangle a right triangle? Explain.

    $w = 36$ cm $v = 15$ cm $x = 39$ cm
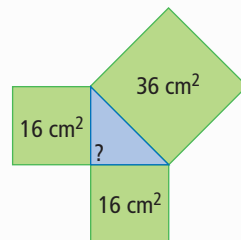
11. The table shows the side lengths of four triangles. Which triangles are right angled?

Triangle	Side x	Side y	Side z
A	9	12	15
B	5	6	7
C	12	35	37
D	30 000	40 000	50 000

3.3 Estimating Square Roots, pages 95–100

12. Cliffmount School is creating invitations for its 50th anniversary celebration. There are three possible designs.

    Cliffmount School
    You are invited to our 50th anniversary celebration!
    25 cm²

    Cliffmount School
    You are invited to our 50th anniversary celebration!
    ▆ cm²

    Cliffmount School
   You are invited to our 50th anniversary celebration!
    36 cm²

   a) What is a possible whole number area for the middle invitation?

b) What is the side length of the smallest one? the largest one?

c) What is an estimate for the side length of the middle invitation? Express your answer to one decimal place.

d) With a calculator, use the area in part a) to check the side length in part c). Give your answer to the nearest tenth of a centimetre.

**13.** Use the number line to answer the following questions.

$$\sqrt{1} \quad \sqrt{4} \quad \sqrt{9} \quad \sqrt{16}$$
$$1 \quad\ \ 2 \quad\ \ 3 \quad\ \ 4$$
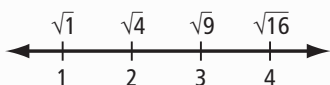
a) What is an estimate for $\sqrt{10}$? Give your answer to one decimal place.

b) Is $\sqrt{6}$ closer to 2 or 3? Explain.

c) A calculator shows that the approximate square root of a certain whole number is 3.61. What is a reasonable value for this whole number? Explain.

3.4 Using the Pythagorean Relationship, pages 101–105

14. Find the missing side length of each triangle.

a)

$c = 12$ m d

$b = 5$ m

b)

$w = 15$ cm

$t = 9$ cm

v
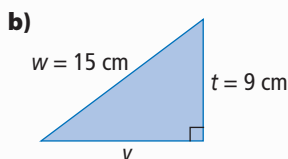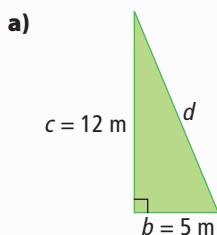
15. The coordinate grid shown was drawn on centimetre grid paper. Answer the following questions to the nearest tenth of a centimetre where appropriate.
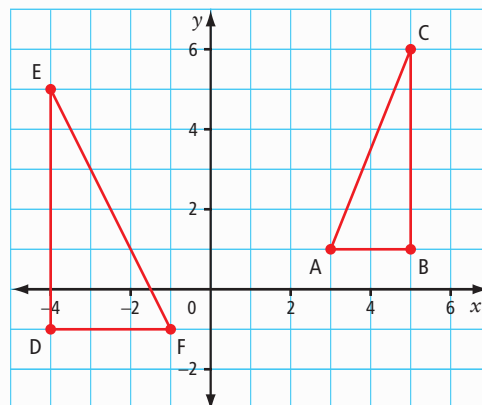

a) What is the length of the hypotenuse in △ABC? in △DEF?

b) What is the perimeter of △DEF?

3.5 Applying the Pythagorean Relationship, pages 106–111

16. A 4-m ladder is being used for a production of *Romeo and Juliet*. The bottom of the ladder will be placed 1 m from the base of Juliet's house. Will the ladder reach the window? Show your work.
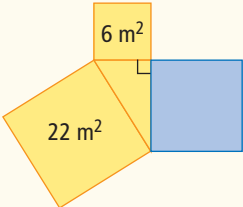

3.9 m
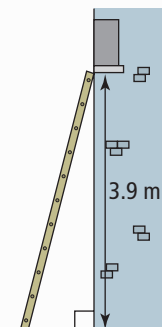
17. Yosef wants to buy a hutch. It must fit in the 90° corner of his dining room. Yosef measures as shown. What should his measurement be? Give your answer to the nearest tenth of a centimetre.
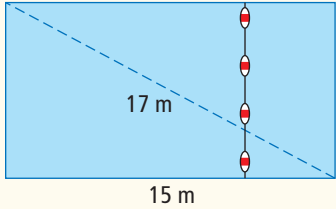

70 cm

70 cm
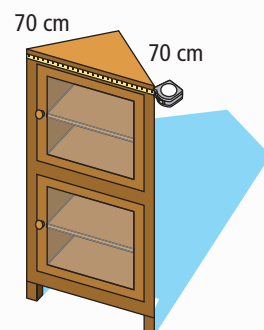
For #1 to #5, choose the best answer.

1. Which number is a perfect square?
   A 10   B 20
   C 50   D 100

2. What is the side length of the square in the diagram?

   81 mm²

   A 6 mm   B 9 mm
   C 12 mm   D 18 mm

3. A square has a side length of 7 cm. What is the area of the square?
   A 14 cm²   B 21 cm²
   C 28 cm²   D 49 cm²

4. A right triangle has squares on each of its sides. What is the area of the blue square?

   6 m²
   22 m²

   A 4 m²   B 14 m²
   C 16 m²   D 28 m²

5. The value of $\sqrt{51}$ is closest to which whole number?
   A 7   B 8
   C 49   D 51

Complete the statements in #6 and #7.

6. For a right triangle with sides a, b, and c, the Pythagorean relationship is $c^2 = a^2 + b^2$. The variable that represents the length of the hypotenuse is ■.

7. A square has an area of 53 cm². When you calculate the side length of the square, to the nearest tenth, the answer is ■.
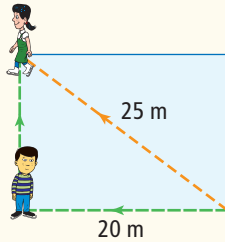
Short Answer

8. The legs of a right triangle measure 3 cm and 7 cm.
   a) Use a calculator to determine the approximate length of the hypotenuse, to the nearest tenth of a centimetre.
   b) Explain why the length is an approximation both before and after you round the answer.

9. The rectangular pool at Wild Water World has a length that measures 15 m and a diagonal that measures 17 m. A float line divides the shallow end and deep end. What is the length of the float line?
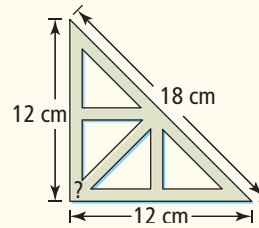
   17 m
   15 m

10. a) Identify a whole number that has its square root between 7 and 8.
    b) How many whole numbers have a square root between 7 and 8? Identify these whole numbers.

11. Use the Pythagorean relationship to determine whether a triangle with sides of 14 mm, 48 mm, and 50 mm is a right triangle. Show your work.

12. Josie skated diagonally across a rectangular ice rink. Han is skating along two sides of the rink and has just reached the first corner. How much farther does he have to skate to meet up with Josie?
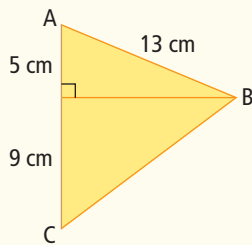
25 m

20 m

Extended Response

13. Determine the perimeter of △ABC.

A
13 cm
5 cm
B
9 cm
C

14. A carpenter's square is a tool in the shape of a right triangle. Joe thinks there may be something wrong with the one he bought. Determine whether the carpenter's square shown is a right triangle. Explain your reasoning.

12 cm

18 cm

?

12 cm

15. The prime factorization of 15 876 is $2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7$.

a) How can you use prime factorization to determine that 15 876 is a perfect square?

b) Use a calculator to check that 15 876 is a perfect square. Show your work.

c) Explain how you can calculate $\sqrt{15\,876}$ using its prime factors.

WRAP IT UP!

Create a game of your own. Include squares and right triangles in the game board. Write rules for your game.

The design of your board or the way you play your game needs to cover the following concepts:

- calculating the square of a number
- calculating the square root of a perfect square
- estimating the square root of a non-perfect square
- using the Pythagorean relationship to determine if a triangle is a right triangle
- determining the missing side length of a right triangle

Show how you have covered the concepts.

Math Games

It's Prime Time

1. Play It's Prime Time! with a partner. These are the rules:
- Each player rolls one die to decide who will play first. If there is a tie, roll again.
- For each turn, roll one die twice. The result of the first roll gives the first digit of a two-digit number. The result of the second roll gives the second digit of the number. For example, if you roll a 5 and then a 2, your number is 52.
- Determine the prime factorization of the two-digit number. For example, the prime factorization of 52 is $2 \times 2 \times 13$.
- Determine the sum of the factors in the prime factorization. This sum is your score for the turn. For instance, for a roll of 52, your score is 17 points.
- Score zero points for the turn if the two-digit number is a prime number. For example, if you roll 41, which is a prime number, your score is zero.
- Score ten bonus points for the turn if the two-digit number is a perfect square. For instance, if you roll 16, which is a perfect square, your score is 18 points.
- The first player to reach 100 points wins.

> $2 + 2 + 13 = 17$

> The prime factorization of 16 is $2 \times 2 \times 2 \times 2$.
> $2 + 2 + 2 + 2 = 8$
> Adding the 10 bonus points gives a score of 18 points.

**2.** Play a different version of the game by modifying the rules as follows:
- For each turn, roll both dice together.
- Record the results in either order to make the two-digit number. For example, if you roll a 2 and a 4, you can choose either 24 or 42 as your two-digit number.
- Award points and decide the winner in the same way as before.

Building a Staircase

Carpenters are well known for their skill in mental math. By studying the blueprint of a house plan, they are able to build many parts of the house such as walls, floors, and stairs.
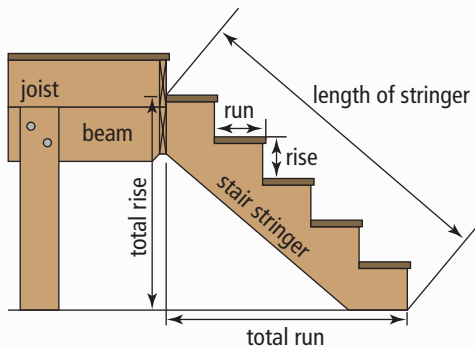
You be the carpenter! Your task is to calculate the length of a stringer for a staircase. The staircase has five steps. Each step has a rise of 18 cm and a run of 24 cm.

1. What is the total rise of the staircase?

2. What is the total run of the staircase?

3. Using the Pythagorean relationship, calculate the length of a piece of wood needed to make the stringer.

4. A retirement residence wants you to build a number of step stools for its residents. Each step stool should be two steps high. Each step should have a rise of 13 cm and a run of 26 cm.
   a) Design and label a step stool.
   b) To the nearest centimetre, calculate the length of stringer needed for the stool.