

## CURRICULUM CORRELATION

Strand/Outcome	Chapter/Section	Pages
<b>Strand: Number</b>		
<b>General Outcome</b> <i>Develop number sense.</i>	Chapters 2–4 Chapter 6 Chapter 8	pp. 42–155 pp. 194–241 pp. 282–323
<b>Specific Outcomes</b>		
<b>1.</b> Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]	Chapter 3: 3.1 Wrap It Up! Math Games Challenge in Real Life: Building a Staircase Chapters 1–4 Review	pp. 80–87 p. 115 p. 116 p. 117  pp. 156–158
<b>2.</b> Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T]	Chapter 3: 3.3 Wrap It Up! Chapters 1–4 Review	pp. 95–100 p. 115 pp. 156–158
<b>3.</b> Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V]	Chapter 4: 4.1–4.4 Wrap It Up! Math Games Challenge in Real Life: The Buying and Selling Game Chapters 1–4 Review Task: Test the Efficiency of a Ramp	pp. 122–149 p. 153 p. 154 p. 155  pp. 156–158 p. 159
<b>4.</b> Demonstrate an understanding of ratio and rate. [C, CN, V]	Chapter 2: 2.1–2.2 Wrap It Up! Math Games Challenge in Real Life: Life of a Bush Pilot Chapters 1–4 Review Task: Test the Efficiency of a Ramp Task: Put Out a Forest Fire	pp. 46–62 p. 73 p. 74 p. 75  pp. 156–158 p. 159 p. 475
<b>5.</b> Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]	Chapter 2: 2.1–2.3 Wrap It Up! Challenge in Real Life: Life of a Bush Pilot Chapters 1–4 Review Challenge in Real Life: Treasure Hunt Task: Put Out a Forest Fire	pp. 46–69 p. 73 p. 75  pp. 156–158 p. 441 p. 475
<b>6.</b> Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]	Chapter 6: 6.1–6.6 Wrap It Up! Math Games Challenge in Real Life: Rock, Paper, Scissors Task: Fraction Cubes Challenge in Real Life: Treasure Hunt Chapters 5–8 Review	pp. 198–235 p. 239 p. 240 p. 241  p. 327 p. 441 pp. 324–326
<b>7.</b> Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]	Chapter 8: 8.1–8.5 Wrap It Up! Math Games Challenge in Real Life: Running a Small Business Challenge in Real Life: The Earth’s Core Chapters 5–8 Review	pp. 286–317 p. 321 p. 322 p. 323  p. 405 pp. 324–326

Strand/Outcome	Chapter/Section	Pages
<b>Strand: Patterns and Relations (Patterns)</b>		
<b>General Outcome</b> <i>Use patterns to describe the world and solve problems.</i>	Chapter 9	pp. 328–365
<b>Specific Outcome</b>		
<b>1.</b> Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V]	Chapter 9: 9.1–9.3 Wrap It Up! Math Games Challenge in Real Life: Comparing Wages Challenge in Real Life: The Earth’s Core Chapters 9–12 Review	pp. 332–359 p. 363 p. 364 p. 365 p. 405 pp. 472–474
<b>Strand: Patterns and Relations (Variables and Equations)</b>		
<b>General Outcome</b> <i>Represent algebraic expressions in multiple ways.</i>	Chapter 10	pp. 366–405
<b>Specific Outcome</b>		
<b>2.</b> Model and solve problems using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b, a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c, a \neq 0$ • $a(x + b) = c$ concretely, pictorially and symbolically, where $a, b$ and $c$ are integers. [C, CN, PS, V]	Math Games Chapter 10: 10.1–10.4 Wrap It Up! Math Games Challenge in Real Life: The Earth’s Core Chapters 9–12 Review	p. 364 pp. 370–399 p. 403 p. 404 p. 405 pp. 472–474
<b>Strand: Shape and Space (Measurement)</b>		
<b>General Outcome</b> <i>Use direct or indirect measurement to solve problems.</i>	Chapter 3 Chapter 5 Chapter 7	pp. 76–117 pp. 160–193 pp. 242–281
<b>Specific Outcomes</b>		
<b>1.</b> Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]	Chapter 3: 3.2, 3.4–3.5 Wrap It Up! Challenge in Real Life: Building a Staircase Chapters 1–4 Review Task: Test the Efficiency of a Ramp	pp. 88–94, 101–111 p. 115 p. 117 pp. 156–158 p. 159
<b>2.</b> Draw and construct nets for 3-D objects. [C, CN, PS, V]	Chapter 5: 5.2–5.4 Wrap It Up! Challenge in Real Life: Design a Bedroom Chapters 5–8 Review Task: Fraction Cubes	pp. 170–187 p. 191 p. 193 pp. 324–326 p. 327
<b>3.</b> Determine the surface area of: • right rectangular prisms • right triangular prisms • right cylinders to solve problems. [C, CN, PS, R, V]	Chapter 5: 5.3–5.4 Wrap It Up! Math Games Challenge in Real Life: Design a Bedroom Chapters 5–8 Review	pp. 176–187 p. 191 p. 192 p. 193 pp. 324–326
<b>4.</b> Develop and apply formulas for determining the volume of right prisms and right cylinders. [C, CN, PS, R, V]	Chapter 7: 7.1–7.4 Wrap It Up! Math Games Challenge in Real Life: Create a Storage Container Chapters 5–8 Review	pp. 246–275 p. 279 p. 280 p. 281 pp. 324–326

Strand/Outcome	Chapter/Section	Pages
<b>Strand: Shape and Space (3-D Objects and 2-D Shapes)</b>		
<b>General Outcome</b> <i>Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.</i>	Chapter 5	pp. 160–193
<b>Specific Outcomes</b>		
<b>5.</b> Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V]	Chapter 5: 5.1 Wrap It Up! Challenge in Real Life: Design a Bedroom Challenge in Real Life: Create a Storage Container Chapters 5–8 Review	pp. 164–169 p. 191 p. 193 p. 281 pp. 324–326
<b>Strand: Shape and Space (Transformations)</b>		
<b>General Outcome</b> <i>Describe and analyze position and motion of objects and shapes.</i>	Chapter 12	pp. 442–471
<b>Specific Outcomes</b>		
<b>6.</b> Demonstrate an understanding of tessellation by: <ul style="list-style-type: none"> <li>• explaining the properties of shapes that make tessellating possible</li> <li>• creating tessellations</li> <li>• identifying tessellations in the environment.</li> </ul> [C, CN, PS, T, V]	Chapter 12: 12.1–12.4 Wrap It Up! Math Games Challenge in Real Life: Border Design Chapters 9–12 Review Task: Put Out a Forest Fire	pp. 446–465 p. 469 p. 470 p. 471 pp. 472–474 p. 475
<b>Strand: Statistics and Probability (Data Analysis)</b>		
<b>General Outcome</b> <i>Collect, display and analyze data to solve problems.</i>	Chapter 1	pp. 2–41
<b>Specific Outcomes</b>		
<b>1.</b> Critique ways in which data is presented. [C, R, T, V]	Chapter 1: 1.1–1.3 Wrap It Up! Math Games Challenge in Real Life: Keep Your Community Green Chapters 1–4 Review	pp. 6–35 p. 39 p. 40 p. 41 pp. 156–158
<b>Strand: Statistics and Probability (Chance and Uncertainty)</b>		
<b>General Outcome</b> <i>Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.</i>	Chapter 11	pp. 406–441
<b>Specific Outcomes</b>		
<b>2.</b> Solve problems involving the probability of independent events. [C, CN, PS, T]	Chapter 11: 11.1–11.3 Wrap It Up! Math Games Challenge in Real Life: Treasure Hunt Chapters 9–12 Review Task: Put Out a Forest Fire	pp. 410–435 p. 439 p. 440 p. 441 pp. 472–474 p. 475

## TIME LINES FOR MATHLINKS 8

The chart below shows estimated times, in minutes, for covering the material in *MathLinks 8*. Please note that times will vary depending on your particular class and its individual students. Field-testing shows that many classes can do some of this material in much less time than is outlined here, while it takes others more time. The chart shows an average. In most cases, the full course can be handled in 160 classes.

Also note that there are alternative ways to cover and assess many outcomes. For example, student achievement of chapter outcomes can be checked by having students do the **chapter review**, **practice test**, and **chapter test**, *or*, more holistically, by having students complete a related **Challenge in Real Life**, *or* by doing a combination of these things. Similarly, some of the exercise questions can be replaced by a **Math Games** activity, which provides a motivating way for students to do extra practice.

In a similar manner, you may wish to have some advanced students do the **Challenge in Real Life** for a particular chapter while other students work on the sections. In other chapters, the **Challenge in Real Life** may provide additional motivation for all students. Questions from the **cumulative review** could be used for extra practice for students who need it.

Chapter	1	2	3	4	5	6	7	8	9	10	11	12
Chapter Opener	50–60	40–50	40–50	40–50	40–50	50–60	40–50	40–50	40–50	40–50	40–50	40–50
Section 1	80–100	80–100	80–100	80–100	80–100	50–60	80–100	50–60	100–120	80–100	80–100	50–60
Section 2	80–100	80–100	80–100	80–100	80–100	50–60	80–100	50–60	100–120	80–100	80–100	50–60
Section 3	80–100	80–100	80–100	80–100	80–100	50–60	80–100	50–60	100–120	80–100	80–100	50–60
Section 4			80–100	80–100	80–100	50–60	80–100	50–60		80–100		50–60
Section 5			80–100			60–75		50–60				
Section 6						50–60						
Chapter Review	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50
Practice Test	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50
Wrap It Up!	40–50	80–100	80–100	80–100	80–100	40–50	80–100	80–100	80–100	80–100	60–90	80–100
Chapter Game	30–40	30	30–40	15–20	20–40	20–30	20–30	30–40	20–30	30–40	30–40	40–50
Challenge in Real Life	40–50	40–50	40–50	80–100	80–100	40–50	80–100	40–50	80–100	80–100	40–50	80–100
Cumulative Review				60–75				60–75				60–75
Task				80–100				60–75				100–120

# AN INTRODUCTION TO MATHLINKS 8

## TEACHER'S RESOURCE

The teaching notes for each chapter have the following structure:

### Opening Matter and Charts

- These are provided on a four-page foldout immediately after each chapter tab.
- These pages provide:
  - an overview of the chapter outcomes and the concepts, skills, and processes that will be assessed.
  - assessment suggestions for the use of the **Literacy Link** in the chapter opener, the **Math Link** introduction, the **Foldable**, and the section **Warm-Ups**
  - an introduction to the **Problems of the Week**
- The **Chapter Planning Chart** provides
  - suggested timing for the numbered sections, chapter review, practice test, Wrap It Up!, games, and challenge
  - a list of prerequisite skills for each section
  - suggested assignments for most students
  - a list of related blackline masters available on the CD-ROM
  - a list of materials and technology tools needed for each lesson
  - the location of Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning opportunities in the chapter
  - suggested sources for extra support

### Chapter Opener

The Chapter Opener includes:

- a description of the math that will be covered in the chapter
- suggestions for introducing students to the chapter's topics
- ideas about how to introduce the Math Link
- suggestions for how students could use the Foldable™ most effectively



### Numbered Sections

The opening page lists:

- **Specific Outcomes and Mathematical Processes** that the section covers in whole or in part
- **Materials** needed for the section
- **Suggested Timing** for the section
- **Blackline Masters** useful for extra practice, assessment, and adaptations. This includes a Warm-Up master that provides exercises for reinforcing material in previous sections of the student resource, as well as mental math skills.
- a **question planning chart** that specifies the questions to be assigned.
  - Essential: the minimum, usually knowledge and skill questions, that all students should be able to complete to address the outcomes
  - Typical: questions that most students should be fairly successful with
  - Extension/Enrichment: questions that extend the concepts horizontally and provide additional challenge

## Teaching Notes

The key items include the following:

- Answers for the **Explore the Math** questions let you know the expected outcome of these activities.
- **Planning Notes** give insights about and suggestions for the three parts of the lesson: **Explore the Math**, **Key Ideas**, and **Check Your Understanding**.
- Sample responses for the **Communicate the Ideas** questions provide the type of answers students are expected to give.
- **Assessment** boxes give a variety of short assessment strategies and related supported learning for Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning. These boxes are provided for each of the activities in the student resource numbered sections.
- A **Math Link** box describes what students will achieve with the Math Link activity and provides strategies for students to complete it successfully.

## End of Chapter Items

- The chapter sections are followed by a **chapter review** and a **practice test**.
- The chapter problem is finalized in a **Wrap It Up!** Related notes provide ideas for handling this assessment opportunity. A rubric and suggested assessment notes are provided.
- **Math Games** appear after the practice test. These can be used for additional reinforcement, alternative assessment, or end-of-chapter review.
- The final page in each chapter is a **Challenge in Real Life**. This activity can be used as a holistic assessment tool, as an extra activity for gifted and enriched students, and/or by all students as a motivating activity related to real life. A rubric and suggested assessment notes are provided.
- **Cumulative reviews** reinforce the previous four chapters.
- The cumulative reviews are followed by a **Task**. This activity can be used as a holistic assessment tool for cross-strand work or as an extra activity for gifted and enriched students. A rubric and suggested assessment notes are provided.

The Teacher's Resource CD-ROM also provides editable masters:

- **Generic Masters** such as grid paper
- **Blackline Masters** related to each chapter:
  - an open-ended diagnostic assessment opportunity
  - **Warm-Up** questions that provide exercises for reinforcing material in previous sections and mental math skills
  - **Problems of the Week**, including innovative problems that require students to think outside the box and experiment with a variety of approaches
  - **Extra Practice** questions for each section
  - scaffolding for each **Math Link** and **Wrap It Up!** for students who need supported learning
  - a **chapter test**
  - answers for blackline master questions

## CHARACTERISTICS OF MCGRAW-HILL RYERSON'S MATHLINKS PROGRAM

McGraw-Hill Ryerson's *MathLinks* program is based on a view that all students can be successful in mathematics and should have the opportunity and support to learn mathematics for depth and understanding (NCTM, 2000). The goal is to assist students in becoming more responsible, thoughtful, and active learners. The program is built on principles of effective practice and on research about how early adolescents learn—prerequisites for achieving a balanced approach to instruction in mathematics.

### Mathematics: Making Links


Throughout the *MathLinks* student resource, students are given the opportunity to see the links between real life and mathematics.

- Every chapter is introduced with a **Math Link** problem that models mathematics in the real world, engages students' interest, and gives students a meaningful purpose for learning the mathematics presented in the chapter. The Math Link provides an important foundation for the concepts and skills developed throughout the chapter. The problem is designed to engage students by making links between the mathematics in the chapter and students' personal experiences, as well as between mathematics and the real world.

#### MATH LINK

##### Music Industry

Music producers sell hundreds of millions of recordings each year. Although music is popular, predicting the sales of a new release can be challenging due to new technology. Will a new release be a hit or a flop? Music producers collect information to help them predict sales. For example, is the artist new? Is the artist currently touring? Who does the music appeal to? How could you organize the information that music producers gather?



1. The circle graph shows the music preferences of young Canadian adults between the ages of 14 and 19.

- What was the favourite type of music? What is the least favourite type of music?
- Research the music preferences of young adults between the ages of 14 and 19 in your province or territory. Does this circle graph provide a good representation of preferences where you live? Explain.

2. The double bar graph shows Canadian sales of music in different formats.

- What were the sales for DVDs in 2006?
- Compare the sales for CDs in 2006 and 2007.
- How do you see this data changing over time? Explain your reasoning.

3. The table shows the music market shares for several music producers.

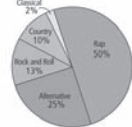
Music Producer	Market Share (%)
Sony BMG	26
Universal Music Group	32
Warner Music Group	15
EMI Group	9
Independent Labels	18

- Represent the data using a bar graph and a circle graph.
- Which graph do you prefer? Explain.

In this chapter, you will collect, analyse, and display data about the music industry. What is your favourite type of music?

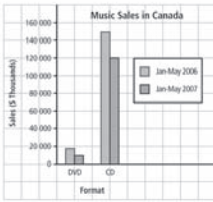
Math Link • MHR 5

**Music Preferences in Young Canadian Adults (Ages 14 to 19)**



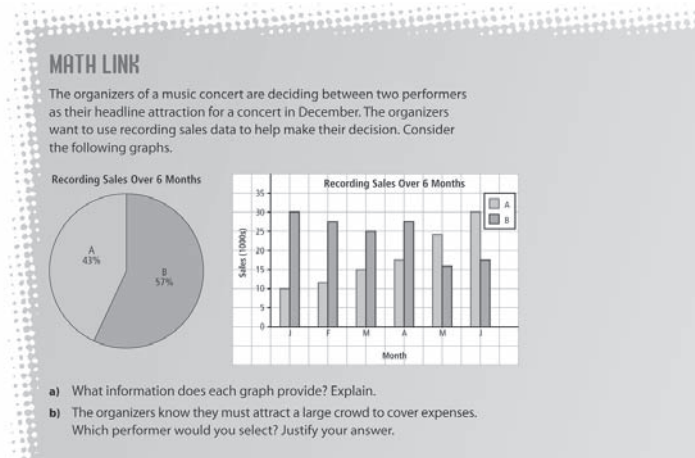
Genre	Percentage
Rap	59%
Alternative	25%
Country	10%
Rock and Roll	7%
Classical	2%

**Music Sales in Canada**



Format	Jan-May 2006 (\$ Thousands)	Jan-May 2007 (\$ Thousands)
DVD	~10,000	~15,000
CD	~140,000	~120,000

- The **Math Link** is revisited at the end of most lessons. This provides students with the opportunity to apply newly acquired concepts and skills in the context of the original problem.



- At the end of the chapter, the **Wrap It Up!** offers an open-ended assessment opportunity for students to demonstrate their understanding by solving the problem introduced at the beginning of the chapter.

**WRAP IT UP!**

Suppose you are a reporter for the school's newspaper. Search the Internet, magazines, or newspapers for data about a topic related to the music industry. For example, you might want to use data about sales, favourite artists, attendance at tours, or popularity of different types of music.

a) Make a table that displays your data.

b) Decide on two different ways to represent the data by drawing one graph that represents the data accurately and another graph that misrepresents the data.

c) You have been asked to write articles that relate to each of your graphs. What will the headlines of your articles be? Explain your thinking.

- Most concepts or procedures in the chapter are introduced in a real-life context.
- **What You Will Learn** at the beginning of each chapter lists in student-friendly language the outcomes of the mathematics curriculum that are covered in the chapter. These outcomes may be from different strands that naturally fit together and further illustrate how the program makes important links among concepts within the discipline and with the real world.
- Connections with other curriculum areas, such as science, geography, and art, are evident in a number of lessons in several of the chapters. The Teacher's Resource also identifies the various ways that concepts developed in the chapter are linked to concepts in the different strands.



## Procedural Fluency and Conceptual Understanding

The three-part lesson structure in McGraw-Hill Ryerson's *MathLinks* program is designed to engage students in learning that develops their conceptual understanding and procedural fluency. The three parts are described below.

### 1. Explore the Math

- begins with a focus question that identifies the learning objective of the lesson
- provides an opportunity for students to generalize learning about the key concepts and to answer the original focus question in the **Reflect on Your Findings**
- provides worked **examples** of the mathematics being modelled, often with multiple approaches to a solution
- makes use of commonly available concrete materials and mathematics manipulatives
- provides opportunities for students to check their understanding of concepts, through **Show You Know** questions, before proceeding to the next example

#### Explore the Math

**How does the area of the base of a right prism relate to its volume?**

- Use centimetre cubes to build models of four different right rectangular prisms.
- What is the area of the **base** for each model? Record your data.
- What is the **height** of each model? Record your data.


### 2. Key Ideas and Communicate the Ideas

- summarizes the key concepts or big ideas of the lesson


**Key Ideas**

- The volume of a right cylinder or a right prism can be determined by multiplying the area of the base by the height of the cylinder or prism.

Volume = area of base  $\times$  height of cylinder  
 $V = 20 \times 8$   
 $V = 160$   
 The volume of the cylinder is  $160 \text{ cm}^3$ .

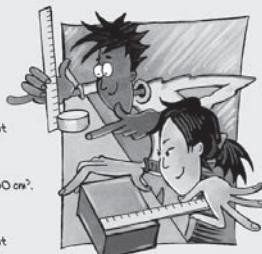


Volume = area of base  $\times$  height of prism  
 $V = 17 \times 10$   
 $V = 170$   
 The volume of the triangular prism is  $170 \text{ cm}^3$ .




**Communicate the Ideas**

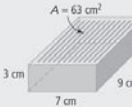
- Evan calculated the volume of a right cylinder. Charlotte calculated the volume of a right rectangular prism. Did either of them make an error in their solutions? Explain how you know.



Volume = area of base  $\times$  height  
 $V = 15 \times 2$   
 $V = 30$   
 The volume of the cylinder is  $30 \text{ cm}^3$ .



Volume = area of base  $\times$  height  
 $V = 63 \times 7$   
 $V = 441$   
 The volume of the rectangular prism is  $441 \text{ cm}^3$ .



- consolidates student learning through questions that include explaining or comparing concepts, identifying and correcting errors, and discussing as a group

### 3. Check Your Understanding (Practise/Apply/Extend)

- allows practice of new skills and application of learning to different situations
- provides opportunities for solving problems in a variety of contexts and using multiple approaches
- provides opportunities for students to extend their thinking (e.g., synthesizing, analysing, evaluating) by using what was discussed in the chapter in a different context or a different way

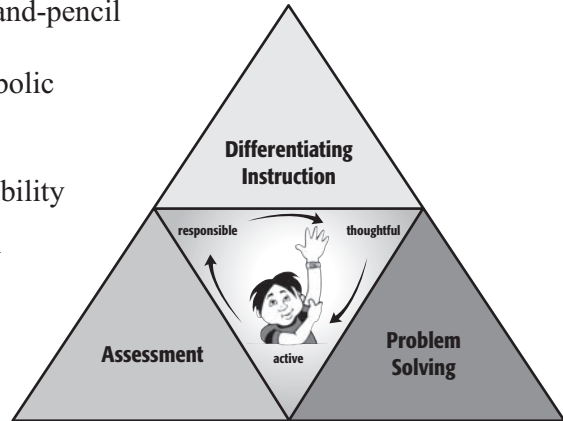
## Check Your Understanding

*MathLinks* balances:

- procedural fluency and conceptual understanding
- mental mathematics, paper-and-pencil arithmetic, and technology
- concrete, pictorial, and symbolic representations
- practice and application
- student and teacher responsibility

This approach is embedded in three cornerstones of the *MathLinks* program:

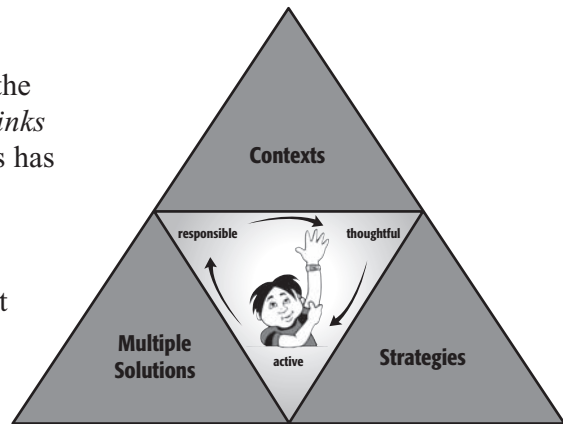
- problem solving
- differentiating instruction
- assessment



## Problem Solving


Problem solving is central to the McGraw-Hill Ryerson *MathLinks* program. Significant emphasis has been placed on incorporating problems that:

- have a range of contexts
- can be solved using different problem solving strategies
- may have multiple solutions




A variety of problem solving experiences are provided throughout the lessons:

- A four-step **problem solving model** is outlined at the beginning of the student resource: Understand, Plan, Do It!, Look Back.
- **Problem solving strategies** are reinforced at the beginning of the student resource. These pages serve as a reference for students as they solve problems within the chapters.

Problem Solving	
<p>People solve mathematical problems at home, at work, and at play. There are many different ways to solve problems. In <i>MathLinks 8</i>, you are encouraged to try different methods and to use your own ideas. Your method may be different but it may also work.</p> <p><b>A Problem Solving Model</b> Where do you begin with problem solving? It may help to use the following four-step process.</p> <p><b>Understand</b> Read the problem carefully.  <ul style="list-style-type: none"> <li>• Think about the problem. Express it in your own words.</li> <li>• What information do you have?</li> <li>• What further information do you need?</li> <li>• What is the problem asking you to do?</li> </ul> </p> <p><b>Plan</b> Select a strategy for solving the problem. Sometimes you need more than one strategy.  <ul style="list-style-type: none"> <li>• Consider other problems you have solved successfully. Is this problem like one of them? Can you use a similar strategy? Strategies that you might use include: <ul style="list-style-type: none"> <li>– Draw a Diagram</li> <li>– Make an Organized List or Table</li> <li>– Work Backwards</li> <li>– Guess and Check</li> <li>– Look for a Pattern</li> <li>– Decide whether any of the following might help. Plan how to use them. <ul style="list-style-type: none"> <li>– tools such as a ruler or a calculator</li> <li>– materials such as grid paper or a number line</li> </ul> </li> </ul> </li> </ul> </p> <p><b>Do It!</b> Solve the problem by carrying out your plan.  <ul style="list-style-type: none"> <li>• Use mental math to estimate a possible answer.</li> <li>• Do the calculations.</li> <li>• Record each of your steps.</li> <li>• Explain and justify your thinking.</li> </ul> </p> <p><b>Look Back</b> Examine your answer. Does it make sense?  <ul style="list-style-type: none"> <li>• Is your answer close to your estimate?</li> <li>• Does your answer fit the facts given in the problem?</li> <li>• Is the answer reasonable? If not, make a new plan. Try a different strategy.</li> <li>• Consider solving the problem a different way. Do you get the same answer?</li> <li>• Compare your methods with those of your classmates.</li> </ul> </p>	
<p>Here are several strategies you can use to help solve problems. Your ideas on how to solve the problems might be different from any of these.</p> <p><b>Problem 1</b> Candice has a rectangular vegetable garden that measures 4 m by 6 m. She wants to divide the garden into three equal sections to plant three different vegetables. What is the area of each section?</p> 	
<p><b>Strategy</b></p> <p><b>Use a Variable</b></p>	<p><b>Example</b></p> <p>The garden is a rectangle with a length of 6 m and a width of 4 m.  <math>A = l \times w</math>  <math>A = 6 \times 4</math>  <math>A = 24</math>  The area of the garden is 24 m<sup>2</sup>.</p> <p><b>Model It</b></p> <p>Use 24 square tiles to model the garden. Each tile represents 1 m<sup>2</sup>.</p> <p>Divide the tiles into three equal groups to represent the three sections.</p> <p>There are eight tiles in each group. The area of each section is 8 m<sup>2</sup>.</p> <p><b>Use a Variable</b></p> <p>The garden is a rectangle with a length of 6 m and a width of 4 m.  <math>A = l \times w</math>  <math>A = 6 \times 4</math>  <math>A = 24</math>  The area of the garden is 24 m<sup>2</sup>.</p> <p>Let <math>S</math> represent the area of each section.  <math>S = \text{area of garden} \div \text{number of sections}</math>  <math>S = 24 \div 3</math>  <math>S = 8</math>  The area of each section is 8 m<sup>2</sup>.</p>

- Examples throughout the *MathLinks 7* student resource show the problem solving model and strategies being used in context. In *MathLinks 8* and *MathLinks 9*, the examples continue to show strategies in context.

**Example 1: Use Diameter to Find Circumference**  
 Traffic circles, or roundabouts, are used in some neighbourhoods to slow down traffic. Vehicles enter the circle and drive around in a counterclockwise direction.



**a)** Estimate the circumference of this traffic circle.  
**b)** What is the circumference of the traffic circle, to the nearest tenth of a metre?  
**c)** Is your estimate reasonable?

**Solution**  
 You are given the diameter of the traffic circle. You need to find the circumference.  
 $C = \pi d$ ,  $d = 5.2$  m

Use the formula  $C = \pi \times d$ . Use an approximate value for  $\pi$  to estimate and calculate the circumference. Substitute the diameter into the formula.

**a)** When estimating, use 3 as an approximate value for  $\pi$ .  
 The diameter of the traffic circle is about 5 m.  
 $C = \pi \times d$   
 $C \approx 3 \times 5$   
 $C \approx 15$   
 The circumference of the traffic circle is approximately 15 m.  
 The actual value should be higher because you estimated using numbers smaller than the actual numbers.



**b)** When calculating, use 3.14 as an approximate value for  $\pi$ .  
 $C = \pi \times d$   
 $C \approx 3.14 \times 5.2$   
 $C \approx 16.3$   
 The circumference of the traffic circle is approximately 16.3 m.

**c)** The answer of 16.3 m is close to but a bit higher than the estimate of 15 m. The estimate of 15 m is reasonable.

**Tech Link**  
 If your calculator has a  $\pi$  key, you can use the  $\pi$  key instead of the value 3.14.

**Check that you rounded your answer to the correct number of decimal places. Remember to use the proper units in your final answer.**

**Show You Know**  
 Estimate and calculate the circumference of each circle, to the nearest tenth of a unit.

**a)**  **b)** 

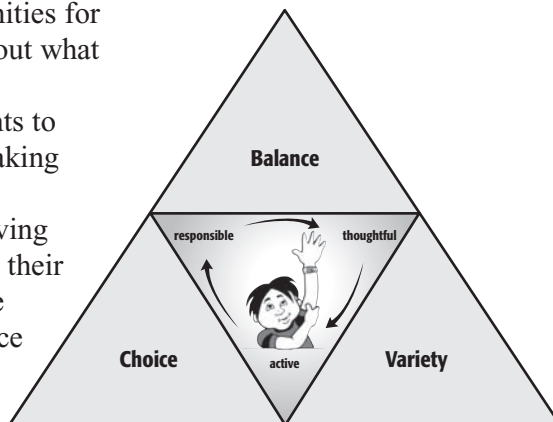
8.2 Circumference of a Circle • MHR 291

- Students are frequently asked to discuss their methods for solving problems. Doing so reinforces thinking and helps students realize that there may be multiple methods for solving a problem.
- The **Math Link** at the beginning of each chapter activates student knowledge of skills and concepts related to the topic in the chapter. The Math Link models a mathematics problem from the real world. This problem is wrapped up at the end of the chapter in the form of a performance task.
- A problem provides the focus for learning in **Explore the Math**, often making use of concrete materials.
- Students are challenged to higher levels of thinking and to extend their thinking in the **Extend** section of the exercises, the **Problems of the Week** blackline master for each chapter, the **Extended Response** section in the practice test, the **Math Games**, and the **Challenge in Real Life**.

## Differentiating Instruction

Differentiating instruction provides educators with the tools needed to create a learning environment where students are actively involved and working together. Hands-on activities engage students and help to meet their diverse needs. Significant emphasis has been placed on:

- variety — provides opportunities for students to be thoughtful about what and how they learn
- choice — encourages students to develop responsibility by making good personal decisions
- balance — is essential in having students actively involved in their learning. Students' needs are best met when they experience a variety of ways to develop and understand concepts.



Care has been taken in the McGraw-Hill Ryerson *MathLinks* program to ensure that all students—including special needs students (with learning disabilities or gifted), students at risk, English language learners, or students from different cultures—can access the mathematics and experience success.

- Visuals that illustrate how to carry out explorations accompany the instructions. These visuals help the student to “see” the process. They also aid in the acquisition of mathematical language.
- Visuals and graphics are paired with questions and content in other strategic locations in the student resource.

3.4

### Area of a Parallelogram

**Focus on...**  
After this lesson, you will be able to...

- develop the formula for the area of a parallelogram
- calculate the area of a parallelogram

**parallelogram**  
• a four-sided figure with opposite sides parallel and equal in length

**Materials**

- centimetre grid paper
- ruler
- scissors
- tape

**Explore the Math**

**How do you determine the area of a parallelogram?**

- On centimetre grid paper, draw a rectangle that is 6 cm long and 4 cm wide. Cut out the rectangle with scissors.

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- A **Literacy Link** at the beginning of each chapter provides tips to help students read and interpret the chapter content.
- Other Literacy Links throughout the chapters provide students with strategies for how to read and understand mathematical language.

**Literacy Link**

A break in the y-axis of a graph means the length of the axis has been shortened. The break can be shown as

or

**Literacy Link**

You can use a concept map to visually organize your understanding of a math concept such as percent.

Copy the concept map below into your math journal or notebook. Make each shape large enough to write in. Write what you already know about percents.

- Definition: What is a percent?
- Comparisons: What can you compare percents to?
- Facts: What are some facts or characteristics you know about percents?
- Examples: What are some examples of percents?

Share your ideas with a classmate. You may wish to add to or correct what you have written.

- The Teacher's Resource provides strategies and blackline master support for accommodating different learning styles, special needs, English language learners, First Nations, Métis, Inuit, and at-risk students.
- The **Get Ready** materials in the *MathLinks* Practice and Homework Books and on the *MathLinks* book site activate student knowledge and concepts related to the topic in the upcoming chapter.

**Get Ready**

**Plot Integers on a Number Line**

Integers include positive numbers, negative numbers, and zero.

Integers can be shown on a number line.

0 is neither positive nor negative

+1 is read as "positive one."  
-1 is read as "negative one."

- For each letter on the number lines, identify the integer.
  - 
  -
- Draw and label an integer number line by 2s. Plot the following integers on it: 6, 0, -1, 9, -11, -6, 1.
  - List the integers in a) in order from greatest to least.

**Find the Distance Between Points on a Number Line**

The distance between two points on a number line can be determined by counting.

- What is the distance between the two numbers placed on a number line?
 

a) 4 and 10	b) -2 and -8
c) -12 and -3	d) +7 and -3
e) -12 and 12	f) 0 and -11
- Draw an integer number line.
  - Mark the point that is four less than zero. Label it A.
  - Mark the point that is three more than zero. Label it B.
  - Mark the point that is 6 less than +3. Label it C.
  - Mark the point that is 7 more than -2. Label it D.

**Plot Points on a Coordinate Grid**

The points A(1, 3), B(5, 2), C(6, 4), D(4, 0), and E(0, 1) can be plotted on a coordinate grid. Each point is named with an ordered pair.

x-coordinate      y-coordinate

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- Support for combined grades situations appears on the *MathLinks* book site at [www.mathlinks.ca](http://www.mathlinks.ca).
- The Teacher’s Resource, *MathLinks* Practice and Homework Books, and *MathLinks* Online Learning Centre offer further support in the form of concrete activities, additional practice, and diagnostic strategies to support students who may have gaps in their learning.
- The open-ended nature of many of the problems and tasks accommodate the needs of all students by allowing for multiple entry points.

### Challenge in Real Life

**Running a Small Business**

Small business owners need to keep track of their finances—both the money they take in from customers and the money they pay out to suppliers.

You be the small business owner! Assume that you own a games store. Part of your job is to keep track of your financial accounts.

The table below shows information about some of the games you carry.

- You buy them from a supplier at one price.
- You sell them to customers at a higher price.

1. Choose a + or – sign to place beside each value in the table. Choose the sign by considering how each value affects your account (money in or money out). Justify your choice of signs.

2. Show how the multiplication or division of integers can be used to model each situation below. Justify your choices.

- You buy 12 copies of Game Z from the supplier.
- You spend \$72 to buy these copies of Game Z from the supplier.
- You sell three copies of Game Y to customers.
- A customer returns two copies of Game X for a refund.
- You find that four copies of Game Y have defects. You return them to the supplier.

3. If you buy 36 copies of Game X from the supplier, how many will you have to sell to *break even* on them? Show your thinking.

4. Create a scenario for a typical week of buying, selling, returns, and so on. Design a table that summarizes your transactions for the week. Your table should demonstrate your understanding of how multiplication and division of integers can be applied.

Buy from Supplier	Game X	\$14
	Game Y	\$10
	Game Z	\$6
Sell to Customers	Game X	\$24
	Game Y	\$15
	Game Z	\$11

**Money - Link**

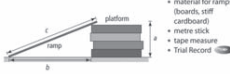
When you break even on something, you neither gain nor lose money.

Challenge in Real Life • MHR 323

### Task

**Test the Efficiency of a Ramp**

Civil engineers design and build structures such as bridges, roads, and ramps. Before doing the actual construction, they test the design for strength and efficiency. Your team’s task is to design a ramp that allows a vehicle to travel the farthest.



**Materials**

- toy vehicles, such as Hot Wheels
- material for platform (books, chair)
- material for ramps (boards, stiff cardboard)
- metric stick
- tape measure
- Trial Record

- Use books, a chair, or other material to create a platform with a height of your choice. Round the height to the nearest centimetre. Height,  $a$ .
- Design a ramp so that a vehicle can roll down without falling off the side.
  - Record the length of the ramp from the edge of the platform to the floor to the nearest centimetre. Length of ramp,  $c$ .
- Does your ramp design use a right triangle? Using the method of your choice, calculate the length of the base in your triangle,  $b$ . Justify your response.
- Test your ramp by placing your vehicle at the top of the ramp, with its front wheels even with the edge of the platform. Let go of the vehicle without pushing it. Measure the distance the vehicle travels from the foot of the ramp to where it stops. You may wish to do three trials and take the average distance.
- Repeat steps 3 and 4 for at least two different lengths of ramp. Complete the chart provided to you.
- The most efficient ramp is the one that allows the vehicles to travel the farthest.
  - Based on your result, what is the ratio of  $a$  to  $b$  distances that resulted in the most efficient design? Express your answer as a percent.
  - Compare your ratio to those found by other teams. Explain any similarities and differences.

Task • MHR 329

- **Did You Know?** boxes present interesting information related to the math or context of the lesson.

**Did You Know?**

Exposure to sounds above 85 dB for a long time can lead to hearing loss.

## Ten Needs of the Learner

Anna Sfard (2003) has identified ten needs of the intermediate learner. She claims these needs are the driving force behind learning and must be fulfilled if the learning is to be successful. The needs are universal, but may be expressed differently in different individuals and at different ages.

### 1. The Need for Meaning

Learners look for order, logic, and causal dependencies behind things, events, and experiences. This approach requires students to actively engage in generating the meaning for themselves. It also directs students’ thinking so no time is lost investigating incorrect paths. While students are developing meaning for new concepts, they are guided to develop patience, persistence, and tolerance in the face of insufficient clarity.

## **2. The Need for Structure**

This need follows from the need for meaning. Meaning involves relationships among concepts. The connections among concepts already learned and new concepts being introduced should be an integral part of instruction. Such connections must include not only real-world applications and relevance, but also assistance in building mathematical abstractions, so students can see how the results can be transferred from one context to another (Wu, 1997). The more connections that exist among facts, ideas, and procedures, the better students' conceptual understanding.

## **3. The Need for Repetitive Action**

A person who has created meaning and structure for a mathematical concept is aware of a repetitive, constant structure in certain actions. A reasonable level of mastery of basic skills is an important element in constructing mathematical knowledge (Fuson and Briars, 1990; Fuson and Kwon, 1992; Hiebert and Wearne, 1996; Siegler, 2003; Stevenson and Stigler, 1992). If students are to reflect on the processes of mathematics, they must first master those processes to a sufficient degree. This does not mean a focus on rote repetition. Rather, it should be a process of reflective practice, where mastery of the action leads to reflection on the meaning of that action, which leads to further understanding and learning.

## **4. The Need for Difficulty**

True learning implies coping with difficulties. The goal of learning is to advance students from abilities they now possess to those they have not yet developed. The best way to accomplish this goal is to present students with tasks that are demanding but still within their reach.

## **5. The Need for Significance and Relevance**

Significance means linking new knowledge to existing knowledge, so this need also stresses the importance of helping students build connections. Significance and relevance do not come from only the concrete and the real; they also come from problems that are more abstract. Focusing only on real-life applications would lead to a fragmented, incomplete picture of mathematics.

## **6. The Need for Social Interaction**

There is an inherent social nature to learning and making meaning. Jerome Bruner states that the fundamental vehicle of education is social interaction not “solo performance” (Bruner, 1985). The most obvious forms are student–teacher or student–student exchanges, but even interaction with a textbook is a form of social interaction (Sfard, 2003). Cooperative learning is another form of learning interaction in which the teacher does not have the central role.

## **7. The Need for Verbal–Symbolic Interaction**

Interaction in learning means communication, and communication means using language (speech) and symbols (written language as well as mathematical symbols) to convey thoughts. If mathematical learning is to take place in an interactive setting, students must be encouraged to “talk” mathematics.

### **8. The Need for a Well-Defined Discourse**

Discourse goes beyond the idea of a conversation. It refers to all communication practices of the classroom, both written and verbal. Discourse implies that the resulting communication follows specific rules. Researchers now recommend that rules be adjusted to the needs and potential of the learning child (Siegler, 2003). This does not mean giving up the need for rigour, but it does mean carefully choosing which rules we use and which rules we modify, and making these rules clear to students.

### **9. The Need for Belonging**

The desire to belong and be counted as a member of a particular social group is a powerful force behind our actions. Learning by participation requires us to be a part of a learning community. Students need to feel respected and free to speak their mind in the classroom. However, the extent to which students value mathematics is influenced by the value given to mathematics by the wider community (Comiti and Ball, 1996). Thus, it may be difficult to instill a desire to embrace mathematics in an environment where mathematics is not valued. The most promising directions for improvement seem to be those that incorporate historical context in the mathematics content, portray mathematics as something unique in our world, and present it as something to be valued for its own sake (Sfard, 2003).

### **10. The Need for Balance**

To meet learners' wide range of needs, the pedagogy must be variegated and rich in possibilities. The need for balance suggests an advantage in searching for the good in all theories. It does not imply that old and new are mutually exclusive. The reality is that there must be a bit of everything in the classroom: problem solving as well as skills practice, teamwork as well as individual learning and teacher exposition, real-life problems as well as abstract problems, and learning by talking as well as silent learning.



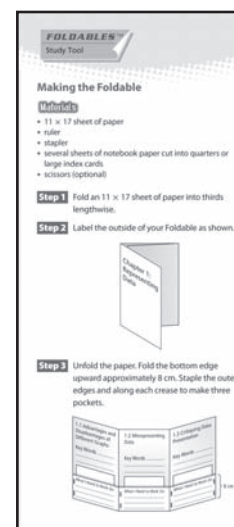
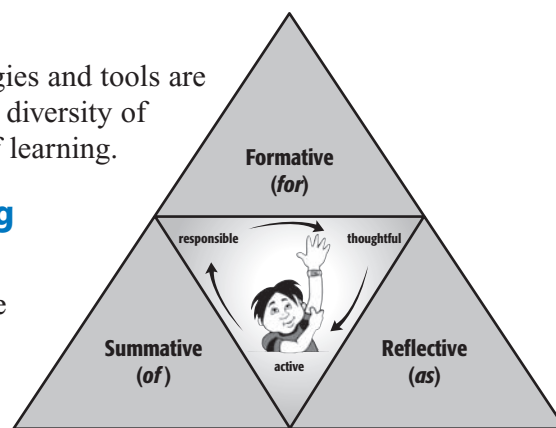
## ASSESSMENT

A variety of assessment strategies and tools are employed to accommodate the diversity of students' abilities and styles of learning.

### Assessment *as* Learning (Diagnostic)

These assessment tools include student reflection. They are provided throughout the *MathLinks* student resource and Teacher's Resource to assist the teacher in programming by identifying student weaknesses and gaps.

- The **Foldables™** activity in each chapter gives students a way to organize their learning and provides them with opportunities to express their understanding in their own words. A unique part of each chapter Foldable asks students to keep track of what they need to work on, allowing them to be self-directed learners.
- The **Reflect on Your Findings** questions in each Explore the Math provide early opportunities for students to construct knowledge about the section content.
- The **Communicate the Ideas** questions allow students to explore their initial understandings of a concept.
- The **Warm-Up** exercises, journaling questions, and **Math Learning Log** suggestions in the Teacher's Resource provide additional support in identifying and facilitating student learning.
- The suggested assignments, questions, **Problems of the Week**, and activities in the **Meeting Student Needs** boxes in the Teacher's Resource address a variety of learner needs, including those of English language learners and gifted and enrichment students.
- Diagnostic support in the form of introductory questions designed to open discussion in the classroom and in the form of exploration activities are provided in the Teacher's Resource, where appropriate.



### Assessment *for* Learning (Formative)

Formative assessment tools are provided throughout the *MathLinks* student resource and the Teacher's Resource.

- The **Math Link** and the **Literacy Link** at the beginning of each chapter activate learning necessary for students' success in the upcoming chapter.
- The *MathLinks* Practice and Homework Books and the *MathLinks* book site include a **Get Ready** section designed to provide teachers with an opportunity to activate student knowledge and assess the understanding that students should have to begin the chapter. The alternative, open-ended assessments for the Get Ready are provided as blackline masters. These assessments focus on determining if students possess both procedural knowledge and conceptual understanding of prerequisite concepts.

- Additional support in the Teacher’s Resource and on the *MathLinks* book site provides assistance for identifying and supporting weaknesses in students’ learning.
- The *MathLinks* Teacher’s Resource provides **blackline masters** that complement the student resource in areas where formative assessment indicates students may need further support.
- The **Reflect on Your Findings** and **Communicate the Ideas** questions provide an opportunity to determine students’ understanding of concepts through conversations and/or written work.
- The **Show You Know** questions target key skills of a section.
- Students can use the **Practise** assignments in each section to check their understanding.
- The **Math Links** at the end of most sections allow students to apply their understanding of the lesson’s concepts to a problem that is linked to the **Wrap It Up!** at the end of each chapter.
- The **chapter reviews** and **cumulative chapter reviews** provide opportunities to assess knowledge/understanding, applications, communications, mental math, and problem solving.

## Assessment of Learning (Summative)

Summative assessment is provided in the following ways:

- **Practice tests** are provided at the end of the chapters in the student resource, and **chapter tests** are provided as blackline masters in the Teacher’s Resource.

2

Practice Test

For #1 to #4, choose the best answer.

1. The ratio of Jared’s stamps to Paulo’s stamps is 4:7. If Jared has 36 stamps how many stamps does Paulo have?
 

A 21
B 63

C 84
D 99
2. A robot can make 27 toy cars in 9 min. Which of the following is the unit rate for this robot?
 

A 27 cars/9 min
B 3 cars

C 3 cars per min
D  $\frac{1}{3}$  car/min
3. In the school choir, the ratio of girls to boys is 17:8. What percent of the school choir are boys?
 

A 8%
B 17%

C 25%
D 32%
4. The picture shows the ratio of the cost of a shirt to the cost of a hat and the ratio of the cost of the hat to the cost of a pair of jeans. What is the ratio of the cost of the shirt to the cost of the jeans?
 

A 1 to 1
B 1:2

C 2 to 1
D 9:7

Complete the statements in #5 and #6.

5. A currency exchange requires \$500 Canadian to receive \$600 Australian. For \$300 Canadian, you would receive
 

■ Australian.
6. Canadians buy five loaves of brown bread for every two loaves of white bread and one loaf of rye bread. A large bakery makes 20 000 loaves of brown bread. The bakery should make
 

■ loaves of white and ■ loaves of rye.

**Short Answer**

7. Randi made nine scarves from 4 m of fabric. How many scarves can she make from 28 m of the same fabric?

8. Tank A has a capacity of 20 L. It is half filled with maple syrup. The ratio of the volume of maple syrup in Tank A compared to the volume of maple syrup in Tank B is 2:5. How much maple syrup is in Tank B?

9. The circle graph shows the favourite pets for a class of 32 grade 8 students.

a) How many students selected a pet other than a dog, cat, or a rabbit?

b) Write a ratio to represent the number of students who selected a cat compared to a rabbit. Write an equivalent ratio.

10. The lengths of A, B, and C are in the ratio 8:2:3, respectively. The length of side C is 24 cm. What is the area of the top of the box?

**Extended Response**

11. Kyra is shopping for ketchup. Her favourite brand is available in two sizes.

a) Estimate which is the better buy. Show your work.

b) Calculate to find the better buy. Show your work.

12. Peter runs 200 m in 30 s, while his sister Eva runs 300 m in 36 s.

a) Who is the faster runner? Explain how you can tell.

b) At the same rate, how far will each runner go in 2 min?

c) How long should it take for each runner to travel 1 km? State any assumptions that you must make.

13. Each week, Karen earns \$420 for 35 h of work at a factory. Her friend Liam makes \$440 for 40 h of work at a store.

a) Who has the greatest hourly rate of pay?

b) How much does Liam earn in an 8-h shift?

14. The actual height of the goat shown here is 1.14 m. Measure the height of the goat in the picture in centimetres.

a) What is the ratio of the height of the goat in the picture compared to the actual height of the goat? Explain what this ratio means.

b) Use ratios to find the actual length of one of the mountain goat’s horns. Show your work.

**WRAP IT UP!**

Plan an international meal that will serve 10 people. Include at least one dish from each of the following categories:

- a soup, salad, or appetizer
- a main course
- a dessert

Create your meal plan.

a) Finalize your invitation to the meal. Ensure that your logo design has an area of 36 cm<sup>2</sup> and uses colours or measurements to show each of the following ratios: 4:3    2:3:4

b) Record your three recipes. Beside each recipe, write the amount of each ingredient you need to serve 10 people.

c) Justify your calculations for one recipe in part b).

d) Calculate the total cost of serving one of your dishes to your guests. Show your work.

WWW Web Link  
To discover some international food recipes go to [www.mathlinks.ca](http://www.mathlinks.ca) and follow the links.

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Practice Test • MHR 73

- The **Wrap It Up!** at the end of each chapter provides teachers with an opportunity to check whether students have synthesized the concepts and procedures. A rubric for each Wrap It Up! is included in the Teacher’s Resource. Student exemplars are on the *MathLinks* book site.

- **Math Games** at the end of each chapter give students and teachers another opportunity for assessment. These games are linked to concepts studied in the chapter. Some games also review outcomes in previous chapters.


## Math Games

### Rolling Ratios

1. Play Rolling Ratios with a partner. These are the rules:

- Each player rolls one die to decide who will play first. If there is a tie, roll again.
- In one round, each partner takes a turn.
- For each turn, roll all three dice.
- Record the ratio of the least value to the sum of the rolled values, in fraction form.




2. Modify the rules of the game. For example, change the number of dice or choose a different ratio. Play your modified version of the game.

#### Materials

- three dice per pair of students
- calculator per student

I rolled a 2, a 4, and a 5. The sum of the rolled values is  $2 + 4 + 5 = 11$ , so the ratio of the least value to the sum of the values is  $2:11$  or  $\frac{2}{11}$ .

So,  $\frac{2}{11} = 0.18$ , to the nearest hundredth.



- Express the fraction as a decimal. If necessary, use a calculator and round to the nearest hundredth.
- Add the decimals from your turns. The first player to reach 2.5 or higher wins.
- If both players reach 2.5 in the same round, the player with the higher total wins. If the totals are tied, the players continue playing until one of them pulls ahead.

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- A **Challenge in Real Life** is provided at the end of every chapter and accompanied by a rubric and suggested scoring in the Teacher's Resource. Student exemplars are on the *MathLinks* book site.

- A **Task** is included after each cumulative review. An accompanying rubric and suggested scoring can be found in the Teacher’s Resource. Student exemplars are provided on the *MathLinks* book site.

Task

Test the Efficiency of a Ramp

Civil engineers design and build structures such as bridges, roads, and ramps. Before doing the actual construction, they test the design for strength and efficiency. Your team’s task is to design a ramp that allows a vehicle to travel the farthest.

1. Use books, a chair, or other material to create a platform with a height of your choice. Round the height to the nearest centimetre. Height,  $a$  ■
2. a) Design a ramp so that a vehicle can roll down without falling off the side.  
b) Record the length of the ramp from the edge of the platform to the floor to the nearest centimetre. Length of ramp,  $c$  ■
3. Does your ramp design use a right triangle? Using the method of your choice, calculate the length of the base in your triangle,  $b$ . Justify your response.
4. Test your ramp by placing your vehicle at the top of the ramp, with its front wheels even with the edge of the platform. Let go of the vehicle without pushing it. Measure the distance the vehicle travels from the foot of the ramp to where it stops. You may wish to do three trials and take the average distance.
5. Repeat steps 3 and 4 for at least two different lengths of ramp. Complete the chart provided to you.
6. The most efficient ramp is the one that allows the vehicles to travel the farthest.
  - a) Based on your result, what is the ratio of  $a$  to  $b$  distances that resulted in the most efficient design? Express your answer as a percent.
  - b) Compare your ratio to those found by other teams. Explain any similarities and differences.

**Materials**

- toy vehicles, such as Hot Wheels®
- material for platform (books, chair)
- material for ramps (boards, stiff cardboard)
- metre stick
- tape measure
- Trial Record

Task • MHR 159

- A **Computerized Assessment Bank (CAB)** offers a database of additional questions. The database includes a variety of question types (True/False, Multiple Choice, Completion, Matching, Short Answer, and Problems) and levels of difficulty (easy, average, and difficult).

### Portfolio Assessment

Student-selected portfolios provide a powerful platform for assessing students’ mathematical thinking. Portfolios provide the following benefits:

- help teachers assess students’ growth and mathematical understanding
- give insight into students’ self-awareness about their own progress
- help parents/guardians understand their child’s growth

*MathLinks 8* has many components that provide ideal portfolio items. Including any or all of the following chapter items is a non-threatening, formative way to gain insight into students’ progress:

- student responses to the chapter opener
- answers to the **Reflect on Your Findings** questions, which give students early opportunities to construct knowledge about the section content
- answers to the **Communicate the Ideas** questions, which allow students to explore their initial understanding of concepts
- journal and **Math Learning Log** responses, which show student understanding of the chapter skills and processes
- student responses to the **Wrap It Up!** assignments
- **Task and Challenge in Real Life** assignments, which show student understanding, usually across several chapters and strands

## Master 1 Project Rubric

The **Master 1 Project Rubric** may be used for all assessments of **Wrap It Up!** assignments, **Challenge in Real Life** activities, and **Tasks**. This unique rubric includes

- a Score/Level grade ranging from 1 to 5 (Beginning to Standard of Excellence)
- a Holistic Descriptor for each grade range, describing the level of understanding and communication skills
- Specific Question Notes, which provide suggested solutions typical of each grade range. These notes are meant to represent what the majority of students display. They are by no means exhaustive of all possible solutions. Teachers are encouraged to continually refer to both the specific and holistic pieces of the rubric.

Score/Level	Holistic Descriptor	Specific Question Notes
<b>5</b> (Standard of Excellence)	<input type="checkbox"/> Applies/develops <b>thorough</b> strategies and mathematical processes making <b>significant</b> comparisons/connections that demonstrate a <b>comprehensive</b> understanding of how to develop a complete solution <input type="checkbox"/> Procedures are <b>efficient and effective</b> and may contain a <b>minor mathematical error</b> that does not affect understanding <input type="checkbox"/> Uses <b>significant</b> mathematical language to explain their understanding and provides <b>in-depth</b> support for their conclusion	<ul style="list-style-type: none"> <li>• provides a complete and correct solution</li> </ul>
<b>4</b> (Above Acceptable)	<input type="checkbox"/> Applies/develops <b>thorough</b> strategies and mathematical processes for making <b>reasonable</b> comparisons/connections that demonstrate a <b>clear</b> understanding <input type="checkbox"/> Procedures are <b>reasonable</b> and may contain a <b>minor mathematical error</b> that may hinder the understanding in one part of a complete solution <input type="checkbox"/> Uses <b>appropriate</b> mathematical language to explain their understanding and provides <b>clear</b> support for their conclusion	<ul style="list-style-type: none"> <li>• provides a complete and correct response to #2, #3, and #4 based on an incorrect #1 <i>or</i></li> <li>• provides a correct and complete response with one calculation error <i>or</i></li> <li>• provides a correct and complete response with weak justification</li> </ul>
<b>3</b> (Meets Acceptable)	<input type="checkbox"/> Applies/develops <b>relevant</b> strategies and mathematical processes making <b>some</b> comparisons/connections that demonstrate a <b>basic</b> understanding <input type="checkbox"/> Procedures are <b>basic</b> and may contain a <b>major error or omission</b> <input type="checkbox"/> Uses <b>common</b> language to explain their understanding and provides <b>minimal</b> support for their conclusion	<ul style="list-style-type: none"> <li>• provides a correct and complete #1, #2, and #3 <i>or</i></li> <li>• provides a correct and complete #1, #2, and #4 <i>or</i></li> <li>• provides a correct #1, #2, and partial response to #3 and #4 <i>or</i></li> <li>• provides answers only, with no work shown, to #1, #2, #3, and #4</li> </ul>
<b>2</b> (Below Acceptable)	<input type="checkbox"/> Applies/develops <b>some relevant</b> mathematical processes making <b>minimal</b> comparisons/connections that lead to a <b>partial solution</b> <input type="checkbox"/> Procedures are <b>basic</b> and may contain <b>several major mathematical errors</b> <input type="checkbox"/> Communication is <b>weak</b>	<ul style="list-style-type: none"> <li>• provides a correct #1 and #2, with no proportions shown in #2 <i>or</i></li> <li>• provides a correct and complete #2 for an incorrect #1 <i>or</i></li> <li>• provides a correct and complete #3 or #4</li> </ul>
<b>1</b> (Beginning)	<input type="checkbox"/> Applies/develops an <b>initial start</b> that may be <b>partially correct</b> or could have led to a correct solution <input type="checkbox"/> Communication is <b>weak or absent</b>	<ul style="list-style-type: none"> <li>• provides a correct #1 <i>or</i></li> <li>• provides a correct #1 and an initial start to #2, #3, or #4</li> </ul>

Teachers are encouraged to share the rubric with students early in the year. This will help them become active participants in their own assessment and program planning. Discussing and building the Specific Question Notes with students allows them to engage actively in their learning.

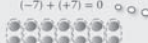
# CONCRETE MATERIALS

The McGraw-Hill Ryerson *MathLinks* program engages students in a variety of worthwhile mathematical tasks that span the continuum from concrete to abstract.

Where appropriate, concept development in the program begins with students working with concrete materials. Most **Explore the Math** activities have students using commonplace materials and conventional mathematical manipulatives in a hands-on approach. Pictorial images of the materials support the text and accommodate the stages of investigations in the absence of concrete materials. After an appropriate number of hands-on opportunities, students move from the pictorial to the symbolic in the **examples, Show You Know, and Check Your Understanding** exercises.



An example of how students move through the continuum of learning can be seen in the development of the concept of addition of integers. Students begin the exploration using positive and negative integers. Pictorial images of addition with the chips are paired with the text in the initial stages of the exploration. Eventually, the pictorial images are removed and the student is presented only with the symbolic.

**Key Ideas**

- You can use integer chips to represent integer addition.
- A zero pair, which includes one +1 chip and one -1 chip, represents 0.
- The sum of any two opposite integers is zero.  
 $(-7) + (+7) = 0$   








*-7 and +7 are opposite integers.*

**Communicate the Ideas**

- Do the integer chips in the diagram represent a sum of +3 or -3? How do you know?  

- What addition statement do the integer chips in the diagram represent? Explain your reasoning.  

- Suppose that the sum of two integers is represented by equal numbers of red and blue chips. Can you state the sum without knowing how many chips there are? Explain.
- David asked his classmate Avril to show him why  $(+1) + (-1) = 0$ . She modelled the addition by climbing up one step and then climbing down it again. Explain how her model shows that  $(+1) + (-1) = 0$ .

**Practise**

For help with #5 to #8, refer to Example 1 on page 329–330.

- What addition statement does each diagram represent?  
 a)       b)   
 c) 
- What addition statement does each diagram represent?  
 a)   
 b)   
 c) 

9.1 Explore Integer Addition • MHR 331

- Add using integer chips. Have a partner check your chips. Then copy and complete the addition statement.  
 a)  $(+3) + (+4) = \blacksquare$   
 b)  $(-2) + (-4) = \blacksquare$   
 c)  $(+5) + (-2) = \blacksquare$   
 d)  $(-8) + (+8) = \blacksquare$
- Add using integer chips. Then copy and complete the addition statement.  
 a)  $(-4) + (-1) = \blacksquare$   
 b)  $(+2) + (+6) = \blacksquare$   
 c)  $(-7) + (+4) = \blacksquare$   
 d)  $(+8) + (-3) = \blacksquare$

**Apply**

For help with #9 to #12, refer to Example 2 on page 330.

- Use the sum of two integers to represent each situation.  
 a) Sharon found \$10 and then lost \$4. How much did she have left?  
 b) A snail slid 7 cm down a stalk and climbed 5 cm back up. How far was the snail below its original position?  
 c) In one game, the Rockies girls' soccer team scored 4 goals and had 1 goal scored against it. How many goals did the team win by?  
 d) A scuba diver dove 4 m under the water and then went down another 8 m. What was the diver's final depth under the water?
- Miguel spent \$6 on Saturday morning and another \$9 on Saturday afternoon. How much less money did he have at the end of the day than at the beginning? Use integer addition to determine your answer.
- The temperature on the Moose Lake Reserve in Manitoba was  $+6^{\circ}\text{C}$ . The temperature dropped by  $10^{\circ}\text{C}$  to reach the overnight low temperature. What was the overnight low temperature? Use integer addition to determine your answer.

**Did You Know?**

The Celsius temperature scale is named after Anders Celsius (1701–1744), a Swedish astronomer. In 1742, he divided the temperature difference between the freezing point and boiling point of fresh water into 100°. However, his scale was upside down. It had  $0^{\circ}$  at the boiling point and  $100^{\circ}$  at the freezing point. Two years later, a Swedish botanist named Carl Linnaeus (1707–1778) switched these values.

- Use the sum of two integers to represent each situation. What is each sum? Explain the meaning of each numerical answer.  
 a) Nadia had 6 world-music CDs and then bought another 2 world-music CDs.  
 b) The temperature went down by  $5^{\circ}\text{C}$  and then went up by  $8^{\circ}\text{C}$ .  
 c) Parminder took 4 steps forward and 4 steps backward.  
 d) Joe caught 6 char in his net, but 2 got away as he pulled the net in.
- Copy and complete the table.  

$(+2) + (+3) = \blacksquare$	$(+3) + (+2) = \blacksquare$
$(-1) + (-4) = \blacksquare$	$(-4) + (-1) = \blacksquare$
$(+2) + (-2) = \blacksquare$	$(-2) + (+2) = \blacksquare$
$(+4) + (-7) = \blacksquare$	$(-7) + (+4) = \blacksquare$

 b) Compare the two addition statements on each row of the completed table. What can you conclude about the order in which you can add two integers? Test your conclusion on some other integer additions.

332 MHR • Chapter 9

## TECHNOLOGY

Where appropriate, lessons are designed to provide students with the opportunity to develop their skills in the use of calculators and spreadsheets, but not to rely on this technology to think mathematically. Students are also asked to use the Internet to research information related to problems they are required to solve.

The student resource provides technology learning that matches technology requirements for curriculum expectations.


**Example 2: Draw a Circle Graph Using Technology**

James surveyed all the students in his grade to find their favourite type of milk to drink. His results are shown in the table.

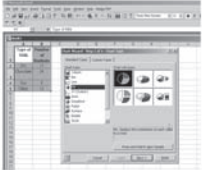
Type of Milk	Number of Students
2%	15
Chocolate	24
Soy	8
Other	13

Draw a circle graph to display the data.

**Solution**  
Enter the data from the table into two columns of a spreadsheet.




Select the Chart Wizard.  
Select Pie Chart.



*Pie chart is another name for a circle graph.*

Follow the instructions in the Chart Wizard.  
Enter a title for the graph.  
Choose to display Category Name and Percentage from the Data Labels page.



**Tech Link**  
Use the spreadsheet software available on your computer to create the circle graph shown here.

310 MHR • Chapter 8


**Show You Know**

Repeat the experiment in Example 1 using two coins that you flip 100 times. Use a tally chart to keep track of your results.

- What is your experimental probability of making two right turns?
- What is the theoretical probability of making two right turns?
- Compare the experimental probability with the theoretical probability.

**Example 2: Compare Experimental and Theoretical Probability Using Technology**

A group of medical students wanted to determine the probability of having a girl and a boy in a two-child family. They used a random number generator to give them results for 20 families.



**Tech Link**  
A random number generator on a computer or calculator can be used to generate a large number of outcomes for a probability experiment.



- What is the experimental probability of getting children of two different genders?
- What is the theoretical probability of getting children of two different genders?
- Compare the experimental probability with the theoretical probability.

**Solution**

- On the spreadsheet, a family with two different genders appears as either 0, 1 or 1, 0.  

$$\text{Experimental } P(\text{boy and girl}) = \frac{11}{20}$$

$$= 0.55$$

$$= 55\%$$

- 

$$\text{Theoretical } P(\text{boy and girl}) = \frac{10}{20}$$

$$= 0.50$$

$$= 50\%$$
- 55% > 50%. The experimental probability is greater than the theoretical probability.

5.5 Conduct Probability Experiments • MHR 195

**Blackline masters** of text-based technology activities that can easily be used in a computer laboratory are also included in the Teacher's Resource when grade-specific outcomes suggest these are needed. The masters include directions for using a number of different softwares common in many classrooms.

## CAPITALIZING ON DIVERSITY AND REAL LIFE

Throughout the student resource, students are given opportunities to see how mathematics connects to real life by engaging in meaningful problem solving situations. Chapters are introduced with problems that model real life. Visual images used to introduce lessons, as well as those in the **Explore the Math** and in the exercise sets, depict the cultural diversity within classrooms. Examples of mathematics from other cultures are evident throughout the text. Names used in the lessons and exercises also reflect the diversity of Canadian society.

**Practise**

For help with #4, refer to Example 1 on pages 284–285.

4. Using string, draw a circle with a radius the length of each line segment.

a) \_\_\_\_\_  
b) \_\_\_\_\_  
c) \_\_\_\_\_

For help with #5 to #8, refer to Example 2 on pages 285–286.

5. Use a compass to draw a circle with each radius.

a) 3 cm      b) 5.5 cm      c) 70 mm

6. What is the diameter of a circle with each radius?

a) 5 cm      b) 8 cm      c) 95 mm

7. What is the radius of a circle with each diameter?

a) 4 cm      b) 7 cm      c) 86 mm

8. Draw a circle with each diameter.

a) 15 cm      b) 20 cm      c) 110 mm

**Apply**

9. Plot the following coordinates on a grid. Draw a line connecting points A and B. Use a compass to draw a circle with centre A and passing through point B. What does the length of line segment AB represent?



a) A(5, 0) and B(8, 4)  
b) A(-2, 1) and B(4, 5)

10. Without drawing the circles, determine which circle is bigger. How do you know?  
Circle A with  $r = 25$  cm  
or  
Circle B with  $d = 45$  cm

11. Consider the following statement.  
*If the radius of a circle is doubled, the diameter is also doubled.*  
Which of the following best describes the statement? Use examples to support your answer.

A Always true  
B Sometimes true  
C Never true

12. Mandalas are used in many cultures. A mandala is thought to bring happiness and good luck to its owner. Draw a circle with a radius of 10 cm. Design your own mandala to hang in your room.



**Did You Know?**  
The word *mandala* is Sanskrit for “circle.” The mandala is an old and universal symbol that stands for peace. Many African cultures have used variations of the mandala in their art and culture to show the connections between people and their environment.

8.1 Construct Circles • MHR 287

## Grouping

There are multiple opportunities throughout the program for teachers to use different types of student groupings. The **Explore the Math** sections lend themselves to group work, but teachers are free to choose student groupings that meet their needs. Additional suggestions are also provided in the Teacher’s Resource.



## Home Connections

The design of the McGraw-Hill Ryerson *MathLinks* program recognizes that students' learning in mathematics also takes place outside of the classroom as they complete their homework, work with parents/guardians, and employ their mathematical skills in everyday life. The following features support learning outside of the classroom:

- **Key Ideas** provide summaries and worked examples to serve as references for students and parents/guardians when doing homework.
- Visuals and **Key Ideas** allow investigations to be easily followed independently.
- Opportunities for bringing mathematics activities home are provided through **Practise/Apply/Extend, Math Links, Math Games, Challenge in Real Life,** and **Tasks.**
- The ***MathLinks* Practice and Homework Books** provide additional opportunities for parents/guardians to assist students in developing needed skills.
- Additional activities, as well as games and puzzles, are available on the McGraw-Hill Ryerson **Online Learning Centre**, which includes a **Parent Centre.**



## COOPERATIVE LEARNING

There are multiple opportunities throughout the program for teachers to use different types of classroom groupings. The Explore the Math explorations lend themselves to being completed in groups, but teachers are free to choose class groupings that meet the needs of their students. Additional suggestions are also provided in this Teacher’s Resource.

Students learn effectively when they are actively engaged in the process of learning. Most sections of *MathLinks 8* begin with a hands-on activity that fosters this approach. These activities are best done through cooperative learning during which students work together—either with a partner or in a small group of three or four—to complete the activity and develop generalizations about the topic or process.

Group learning such as this is an important aspect of a constructivist educational approach. It encourages interactions and increases chances for students to communicate and learn from each other.<sup>1</sup>

**Teachers’ Role**—In classrooms where students are adept at cooperative learning, the teacher becomes the facilitator, guide, and progress monitor. Until students have reached that level of group cooperation, however, the teacher will need to coach them in how to learn cooperatively. This may include

- making sure that the materials are at hand and directions perfectly clear, so that students know what they are doing before starting group work
- carefully structuring activities so that students can work together
- coaching how to provide peer feedback in a way that allows the listener to hear and attend
- constantly monitoring student progress and providing assistance to groups having problems with either group cooperation or the math at hand

**Group Composition**—The size of group may vary from activity to activity. Small-group settings allow students to take risks that they might not take in a whole class.<sup>2</sup> Research suggests that small groups are fertile environments for developing mathematical reasoning.<sup>3</sup>

Results of international studies suggest that groups of mixed ability work well in mathematics classrooms.<sup>4</sup> If your class is new to cooperative learning, you may wish to assign students to groups according to the specific skills of each individual. For example, pair a student who is talkative but weak in number sense and numeration with a quiet student who is strong in those areas. Pair a student who is weak in many parts of mathematics but has excellent spatial sense with a stronger mathematics student who has poor spatial sense. In this way, student strengths and weaknesses complement each other, and peers have a better chance of recognizing the value of working together.

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<sup>1</sup>Sternberg, R.J., and W.M. Williams, *Educational Psychology* (Boston, MA: Allyn & Bacon, 2002).

<sup>2</sup>Van De Walle, J., *Elementary and Middle School Mathematics: Teaching Developmentally*, 4th ed. (Boston, MA: Addison Wesley Longman, 2000).

<sup>3</sup>Artzt, A.F., and S. Yaloz-Femia, “Mathematical Reasoning During Small-Group Problem Solving,” in L. Stiff and F. Curcio (eds.), *Developing Mathematical Reasoning in Grades K–12* (Reston, VA: National Council of Teachers of Mathematics, 1999), 115–26.

<sup>4</sup>Kilpatrick, J., J. Swafford, and B. Findell, *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: National Academy Press, 2001).

**Cooperative Learning Skills**—When coaching students about cooperative learning, consider task skills and working relationship skills.

Task Skills	Working Relationship Skills
<ul style="list-style-type: none"> <li>• following directions</li> <li>• communicating information and ideas</li> <li>• seeking clarification</li> <li>• ensuring that others understand</li> <li>• actively listening to others</li> <li>• staying on task</li> </ul>	<ul style="list-style-type: none"> <li>• encouraging others to contribute</li> <li>• acknowledging and responding to the contributions of others</li> <li>• checking for agreement</li> <li>• disagreeing in an agreeable way</li> <li>• mediating disagreements within the group</li> <li>• sharing</li> <li>• showing appreciation for the efforts of others</li> </ul>

Use class discussions, modelling, role-plays, and drama to provide positive task skills. For example, role-play different ways to provide feedback and have a class discussion on which ones students like and why. Discuss common group roles and how group members can use them. Make sure students understand that the same person can play more than one role.

Role	Job	Sample Comment
Leader	<ul style="list-style-type: none"> <li>• makes sure the group is on task and everyone is participating</li> <li>• pushes group to come to a decision</li> </ul>	<p>Let's do this. Can we decide ... ? This is what I think we should do ...</p>
Recorder	<ul style="list-style-type: none"> <li>• manages materials</li> <li>• writes down data collected or measurements made</li> </ul>	<p>This is what I wrote down. Is that what you mean?</p>
Presenter	<ul style="list-style-type: none"> <li>• presents the group's results and conclusions</li> </ul>	<p>This is what the group thinks ...</p>
Organizer	<ul style="list-style-type: none"> <li>• watches time</li> <li>• keeps on topic</li> <li>• encourages getting the job done</li> </ul>	<p>Let's get started. Where should we start? So far we've done the following ... Are we on topic? What else do we need to do?</p>
Clarifier	<ul style="list-style-type: none"> <li>• checks that members understand and agree</li> </ul>	<p>Does everyone understand? So, what I hear you saying is ... Do you mean that ... ?</p>

## Types of Groups

Three group types are commonly used in the mathematics classroom.

**Think/Pair/Share**—This consists of having students individually think about a concept and then pick a partner to share their ideas. For example, students might work on the **Communicate the Ideas** questions and then choose a partner to discuss the concepts with. Working together, the partners could expand on what they understood individually. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

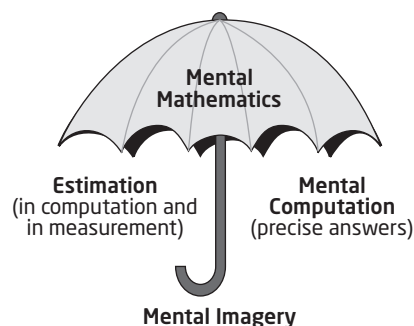
**Cooperative Task Group**—Task groups of two to four students can work on activities in the **Explore the Math** sections. As a group, students can share their understanding of what is happening during the activity and how that relates to the mathematics topic, at the same time as they develop group cooperation skills.

**Jigsaw**—Another common cooperative learning group is called a jigsaw. In this technique, individual group members are responsible for researching and understanding a specific area of information for a project. Individual students then share what they have learned so that the entire group gets information about all areas being studied. For example, during data management, this type of group might have “experts” in the advantages of a particular type of graph. Group members could then coach each other on the best graph(s) to use a particular application.

Another way of using the jigsaw method is to assign “home” and “expert” groups during a large project. For example, students researching recipes from a particular culture might have a home group in which each member is responsible for researching recipes from one of four cultures. Individual members could then move to expert groups. Expert groups would include all of the students responsible for researching each of the cultures. Each of the expert groups would research appetizers, salads, main courses, and desserts in their particular culture. Once the information had been gathered and prepared for presentation, individual members of the expert group would return to their home group and teach other members about recipes from the culture they researched.

## MENTAL MATHEMATICS

A major goal of mathematics instruction for the twenty-first century is for students to make sense of the mathematics in their lives. The development of all areas of mental mathematics is a major contributor to this comfort and understanding. Mental mathematics is the mental manipulation of knowledge dealing with numbers, shapes, and patterns to solve problems.



The diagram above shows the various components under the umbrella of Mental Mathematics. All three are considered mental activities and interact with each other to make the connections required for mathematics understanding. Estimation and mental math are not topics that can be isolated as a unit of instruction; they must be integrated throughout the study of mathematics.

### Estimation

*Estimation* refers to the approximate answers for calculations, a very practical skill in today's world. The development of estimation skills helps refine mental computation skills, enhances number sense, and fosters confidence in math abilities, all of which are key in problem solving. Over 80% of out-of-school problem solving situations involve mental computation and estimation.<sup>5</sup>

Estimation does not mean guessing at answers. Rather, it involves a host of computational strategies that are selected to suit the numbers involved. The goal is to refine these strategies over time with regular practice, so that estimates become more precise. The ultimate goal is for students to estimate automatically and quickly when faced with a calculation. These estimations allow for recognition of errors on calculator displays, provide learners with a strategy for checking the reasonableness of their calculations, and give students a strategy for finding an answer when only an approximation is necessary.

### Mental Imagery

*Mental imagery* in mathematics refers to the images in the mind when one is doing mathematics. It is this mental representation, or conceptual knowledge, that needs to be developed in all areas of mathematics. Capable math students "see" the math and are able to perform mental manoeuvres in order to make connections and solve problems. These images are formed

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<sup>5</sup>Reys, B. J., and R.E. Reys, "One Point of View: Mental Computation and Computational Estimation—Their Time Has Come," *Arithmetic Teacher* (Vol. 33, No. 7, 1986), 4–5.

when students manipulate objects, explore numbers and their meanings, and talk about their learning. Students must be encouraged to look into their mind’s eye and “think about their thinking.”

Asking, “What do you see in your mind’s eye?” when asked to visualize, as in the exercises below, forces students to think about the images they are using to help them solve problems. Students are often surprised when fellow students share their personal images; the discussion generated is very worthwhile.

Try these mental imaging exercises with students.

<b>Example 1:</b> Draw the mental image you have for each of the following: <ul style="list-style-type: none"><li>• <math>\frac{2}{3}</math></li><li>• 75% of the questions on the page</li><li>• a <math>175^\circ</math> angle</li></ul>	<b>Example 2:</b> Use mental imagery to answer the following: <ol style="list-style-type: none"><li>1. How many edges does a cube have?</li><li>2. If I am facing east, what direction is to my left?</li><li>3. What is the perimeter of a 90 cm <math>\times</math> 30 cm shelf?</li></ol>
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## Mental Computation

*Mental computation* refers to an operation used to obtain the precise answer for a calculation. Unlike traditional algorithms, which involve one method of calculation for each operation, mental computations include a number of strategies—often in combination with each other—for finding the exact answer. As with estimation, strategies for mental computation develop in quantity and quality over time. A thorough understanding of, and facility with, mental computation allows students to solve complicated multi-step problems without spending needless time figuring out calculations and is a valuable prerequisite for proficiency with algebra. Students need regular practice in these strategies.

### Some Points Regarding Mental Mathematics

- Students must have a knowledge of the basic facts (addition and multiplication) in order to estimate and calculate mentally. They learn the many strategies for fact learning in elementary school. With practice, they eventually commit these facts to memory. Without knowing the basic facts, it is unlikely that students will ever attempt to employ any estimation or mental math strategies, as these will be too tedious.
- The various estimation and mental calculation strategies must be taught and are best developed in context; opportunities must be provided for regular practice of these strategies. Having students share their various strategies is vital, as it provides possible options for classmates to add to their repertoire.
- Unlike the traditional paper-and-pencil algorithms, there are many mental algorithms to learn. With the learning, however, comes a greater facility with numbers. Key to the development of skills in mental math is the understanding of place value (number sense) and the number operations. This understanding is enhanced when students make mental math a focus as they calculate.

- Mental math strategies are flexible; the student needs to select one that is appropriate for the numbers in the computation. Practice should be in the form of practising the strategy itself, selecting appropriate strategies for a variety of computation examples, and using the strategies in problem solving situations.
- Although students should not be pressured with time constraints when first learning a mental math strategy, it is beneficial to provide timed tests once they have some facility with mental computation. If too much time is provided, many students will resort to the traditional algorithm and will not use mental strategies.
- Mental math algorithms are used with whole numbers, fractions, and decimal numbers.
- Sometimes mental math strategies are used in conjunction with paper-and-pencil tasks. The questions are rewritten to make the calculation easier.
- The ultimate goal of mental mathematics is for students to estimate for reasonableness and to look for opportunities to calculate mentally.
- Encourage students to refer to the strategies by their name (e.g., front-end strategy). Once the strategies have been taught, post them around the room. Have students write problems in which a mental strategy would be the appropriate computation. Share these problems with the class.
- Students need to identify why particular procedures work; they should not be taught computation “tricks” without understanding.
- Those who are skilled in using mental mathematics will be able to transfer, relate, and apply mental strategies to paper-and-pencil tasks.

### Keep in Mind

Practice in classrooms has traditionally been in the form of asking students to write the answers to questions presented orally. This is particularly challenging for students who are primarily visual learners. Although we are sometimes faced with computations of numbers we cannot see, most often the numbers are written down. This makes it easier to select a strategy. In daily life, we see the numbers when solving written problems (e.g., when checking calculations on a bill or invoice, when determining what to leave for tips, when calculating discounted prices from a price tag). Provide students with mental math practice that is sometimes oral and sometimes visual.

