Chapter 8

8.1 Exploring Integer Multiplication, pages 291–292

5. a) $(+5) \times (+1)$ **b)** $(+2) \times (-6)$
6. a) $(+3) \times (+7)$ **b)** $(+4) \times (-4)$
7. a) $(+8) + (+8) + (+8)$
b) $(-6) + (-6) + (-6) + (-6) + (-6)$
8. a) $(+2) + (+2) + (+2) + (+2) + (+2) + (+2)$
b) $(-9) + (-9) + (-9) + (-9)$
9. a) $(+2) \times (+4)$ **b)** $(+4) \times (-2)$
10. a) $(+7) \times (+2)$ **b)** $(+6) \times (-1)$
11. a) $(-3) \times (-2)$ **b)** $(-3) \times (+3)$
12. a) $(-1) \times (+7)$ **b)** $(-2) \times (-5)$
13. a) $(+4) \times (+6) = 24$ **b)** $(+7) \times (-2) = -14$
c) $(-1) \times (+5) = -5$ **d)** $(-8) \times (-2) = 16$
14. a) $(+6) \times (+2) = 12$; The temperature increased 12 °C in 6 h. **b)** $(+4) \times (+8) = 32$; Ayesha repaid a total of $32.
15. $(+12) \times (-3) = -36$; The aircraft descends 36 m.
16. a) 40 m **b)** 12 m
17. 16 m
18. No. Doubling a negative integer results in an integer of lesser value.
19. a) 3 **b)** Yes; -6 **c)** The easiest solution is to multiply each integer in part a) by -4. Many other solutions are possible. Example:

-4	-22	14
-4	4	-12
-4	6	-14

20. Many solutions are possible. Example:

a)

1	1	−1
1	−1	1
−1	1	1

b)

1	−1	1
−1	−1	−1
1	−1	1

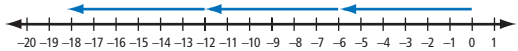
8.2 Multiplying Integers, pages 297–298

4. a) (+2) × (+4) **b)** (+3) × (−5)

5. a) (+2) × (−6) **b)** (+4) × (+4)

6. a) 

; 25

b) 

; −18

7. a) −28 **b)** +18

8. a) 40 **b)** −30 **c)** −35 **d)** 32

9. a) 36 **b)** 54 **c)** −24 **d)** 0

10. Estimates may vary. **a)** −408 **b)** 814 **c)** −1080 **d)** 1316

11. Estimates may vary. **a)** 252 **b)** −1326 **c)** 1188 **d)** 3025

12. $180

13. 1500 m

14. a) The first week, the value of her shares dropped by 4500¢ or $45.00. **b)** The second week, the value of her shares rose by 6375¢ or $63.75. **c)** Over the two-week period her shares rose by 1875¢ or $18.75.

15. 2400 m

16. −23 and −18

17. The least product is (+99) × (−82). The first two products have factors with like signs. Therefore, only the third product is negative.

18. Answers may vary. Example: **a)** Explain to your friend that the products are decreasing by five. So, as the pattern continues, the missing numbers are −5, −10, −15, and so on. **b)** Change the order of the factors to (+6) × (−2). Then, use the following pattern: (+6) × (+3) = 18, (+6) × (+2) = 12, (+6) × (+1) = 6, (+6) × 0 = 0, (+6) × (−1) = −6, (+6) × (−2) = −12.

19. a) Yes; (+2) × (+2) = 4 and (−2) × (−2) = 4. **b)** No. Since −4 is negative, the two factors must have opposite signs. Therefore, these factors cannot be equal.

20. a) (+6) × (+3) = +18 **b)** (+5) × (−2) = −10 **c)** (−4) × (+3) = −12 **d)** (−4) × (−4) = +16

21. a) (−1) × (−10), (−2) × (−5), (+1) × (+10), (+2) × (+5), (−10) × (−1), (−5) × (−2), (+10) × (+1), (+5) × (+2) **b)** (−1) × (+16), (+16) × (−1), (+1) × (−16), (−16) × (+1), (+2) × (−8), (−8) × (+2), (−2) × (+8), (+8) × (−2), (+4) × (−4), (−4) × (+4) **c)** (+1) × (−24), (−1) × (+24), (+2) × (−12), (−2) × (+12), (+3) × (−8), (−3) × (+8), (+4) × (−6), (−4) × (+6), (−24) × (+1), (+24) × (−1), (−12) × (+2), (+12) × (−2), (−8) × (+3), (+8) × (−3), (−6) × (+4), (+6) × (−4)

22. −9 and 4

23. Answers may vary. Example: The temperature dropped 6 °C/h over a 5-h period. What was the temperature at the end of the 5-h period if the original temperature was 0 °C?

24. Answers may vary. Example: A mine elevator descends at a rate of 2 m/s. How far would it descend in 5 min? Answer: (−2) × (+5) × (+60) = −600. The elevator would descend 600 m.

25. Descriptions may vary. Example: **a)** Each number is the previous number multiplied by 3. The next three numbers are 81, 243, and 729. **b)** Each number is the previous number multiplied by −2. The next three numbers are −16, +32, and −64. **c)** Each number is the previous number multiplied by 2. The next three numbers are −32, −64, and −128. **d)** Each number is the previous number multiplied by −4. The next three numbers are 512, −2048, and 8192.

26. a) One of the integers is 1. **b)** One of the integers is −1. **c)** The two integers have different signs, and neither integer is a 1, 0, or −1. **d)** Both integers are less than −1 or both integers are greater than +1.

27. a) −1, 0, 1 **b)** −2, −1, 0, 1, 2 **c)** No. Explanations may vary. Example: The sum of consecutive integers is zero only if the integers include zero and pairs of integers with opposite signs. Therefore, the number of integers must be odd.

28. a) −216 **b)** Yes; −1728 **c)** No.

29. a) The product of an even number of positive integers is positive. **b)** The product of an odd number of positive integers is positive. **c)** The product of an even number of negative integers is positive. **d)** The product of an odd number of negative numbers is negative.

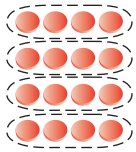
8.3 Exploring Integer Division, pages 304–305

3. a) (+10) ÷ (+2) = +5 **b)** (−16) ÷ (−4) = +4 **c)** (−14) ÷ (+2) = −7

4. a) (−4) ÷ (−2) = +2 **b)** (+9) ÷ (+3) = +3 **c)** (−12) ÷ (+6) = −2

5. a) (+14) ÷ (+2) = +7; (+14) ÷ (+7) = +2 **b)** (−10) ÷ (−2) = +5; (−10) ÷ (+5) = −2

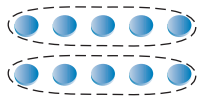
6. a) (+15) ÷ (+5) = +3; (+15) ÷ (+3) = +5 **b)** (−18) ÷ (−9) = +2; (−18) ÷ (+2) = −9

7. a)  (+16) ÷ (+4) = +4

b)  (−7) ÷ (+7) = −1

c)  (−12) ÷ (−6) = +2

8. a)  (−20) ÷ (−10) = +2

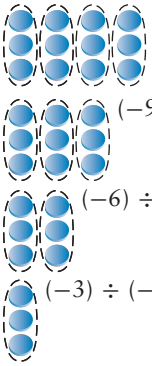
b)  (−10) ÷ (+2) = −5

c)  (+4) ÷ (+2) = +2

9. 7 min

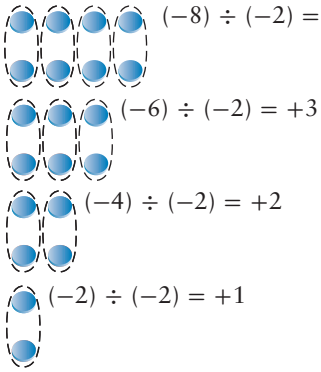
10. a) The temperature fell −18 °C **b)** −3 °C/ h; Assume that the rate of change was constant.

11. $2

12. a) 

Descriptions may vary. Example: The quotients are decreasing consecutive integers starting with +4.

b) (+6) ÷ (−3) = −2

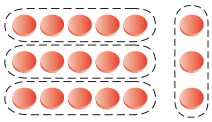
13. a) 

Descriptions may vary. Example: The quotients are decreasing consecutive integers starting with +4.

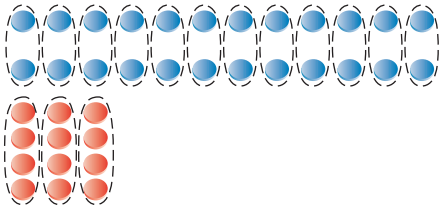
b) (+4) ÷ (−2) = (−2)

14. a) (−2000) ÷ (−500) = (+4) **b)** Answers may vary. Example: Let each chip represent 100 m. **c)** +4

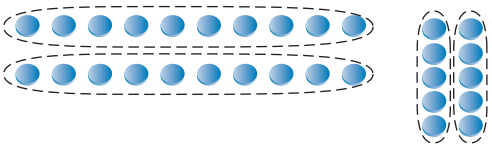
15. a) 15 positive chips are grounped by 5s into 3 groups. 3 positive chips are grouped by 3s into 1 group. Quotient is 1.



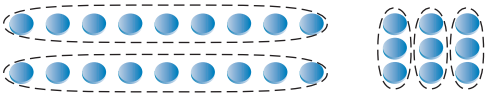
b) 24 negative chips are grouped by 2s into 12 groups. These 12 groups represent (−24) ÷ (−2) = 12. 12 positive chips are grouped by 4s into 3 groups. Quotient is 3.



c) 20 negative chips are separated into 2 groups of 10 chips. These 10 chips are grouped by 5s into 2 groups. Quotient is 2.



d) 18 negative chips are separated into 2 groups of 9 chips. These 9 chips are grouped by 3s into 3 groups. Quotient is −3.



16. In 3 h, the temperature will be −17 °C. Assume that the temperature continues to drop at the constant rate of 2 °C/h.

8.4 Dividing Integers, pages 310–311

5. a) (+18) ÷ (+9) = +2; (+18) ÷ (+2) = +9

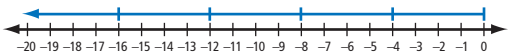
b) (−12) ÷ (−3) = +4; (−12) ÷ (+4) = −3

6. a) (−10) ÷ (+5) = −2; (−10) ÷ (−2) = +5

b) (+16) ÷ (+2) = +8; (+16) ÷ (+8) = +2

7. a) 

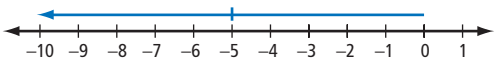
The quotient is +2.

b) 

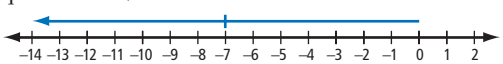
The quotient is +5.

c) 

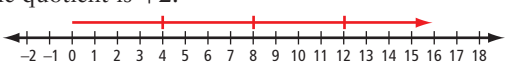
The quotient is −2.

d) 

The quotient is +2.

8. a) 

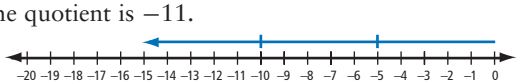
The quotient is +2.

b) 

The quotient is +4.

c) 

The quotient is −11.

d) 

The quotient is +3.

9. a) +4; (+4) × (+5) = +20 **b)** −6; (−6) × (−6) = +36

c) −3; (−3) × (+19) = −57 **d)** +2; (+2) × (−42) = −84

10. a) +1 **b)** −19 **c)** 0 **d)** −4

11. 4 months

12. a) 6 m/min **b)** 8 m/min

13. 7

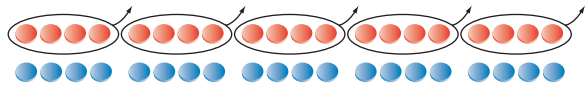
14. The drill cut through the floor at a rate of 3 cm/min. Assume that the cutting rate was constant.

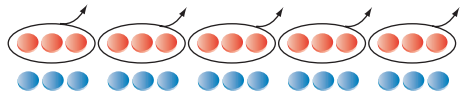
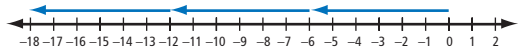
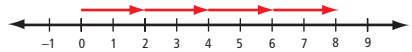
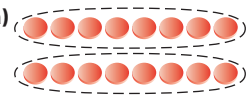
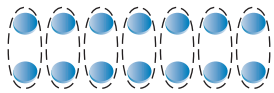
15. $12
16. $(+2408) \div (-43)$. In the first two expressions, the two integers have the same sign, so both these quotients are positive. In the third expression, the two integers have different signs, so the quotient is negative.
17. -16
18. a) $(+72) \div (+8) = +9$ **b)** $(-120) \div (+12) = -10$
c) $(+143) \div (-13) = -11$ **d)** $(-84) \div (-14) = +6$
19. A pump draws 80 L of water from a storage tank in 16 s. By how much does the volume of water in the tank change in 1 s?
20. Answers may vary. Example: Yvette borrows $80 from her brother and pays him back in 16 equal weekly payments. How much does she pay her brother each week?
21. a) Each number in the sequence is the previous number divided by 5. The next three terms are +200, +40, and +8. **b)** Each number in the sequence is the previous number divided by −2. The next three terms are −32, +16, and −8. **c)** Each number in the sequence is the previous number divided by 10. The next three numbers are −100, −10, and −1. **d)** Each number in the sequence is the previous number divided by −3. The next three numbers are +18, −6, and +2.
22. +30 and −10.

8.5 Applying Integer Operations, pages 315–317

4. a) +17 **b)** −11 **c)** +21
5. a) −3 **b)** 0 **c)** +4
6. a) 6 **b)** −14 **c)** −10
7. a) −32 **b)** 41 **c)** 3
8. −2 °C
9. a) a decrease of two subscribers per month **b)** 195
10. a decrease of 1000 people
11. −55
12. a) 12 strokes below par, or −12 **b)** 276
13. 90 °C
14. 12 weeks
15. 8 h
16. a) an increase of 50 m **b)** 10 m/min
17. 7:54 a.m.
18. a) $6000/month **b)** $12 000/month.
19. a) Rohana spent $150, saved $90, and still owes her sister $40. **b)** 4
20. a) $20 - 3 \times (-8) = 44$ **b)** $4 \times 5 + (-2) \times (-3) = 26$
c) $-62 \div (-11 + 9) = 31$
d) $[-3 + (-5)] \times 3 \div (-4) - 13 = -7$
21. a) $2 \times 3 - 4 \times 5 = -14$ **b)** $3 \times [14 + (-2)] - 30 = 6$
22. −21 and −13
23. a) 130 **b)** 65%
24. Answers may vary. Example:
$(-2) \div (-2) + (-2) \div (-2) = 2$;
$[-2 + (-2) + (-2)] \div (-2) = 3$;
$-2 \times (-2) \times (-2) \div (-2) = 4$;
$-2 \times (-2) + (-2) \div (-2) = 5$;
$-2 - (-2) \times (-2) \times (-2) = 6$;
$-2 \times (-2) + (-2) \times (-2) = 8$

Chapter Review, pages 318–319

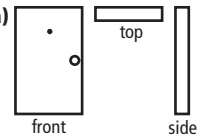
1. zero
2. the operation (subtraction) within the brackets
3. zero pair
4. a) $(+2) \times (-5)$ **b)** $(-4) \times (+2)$
5. a)  The product is +9.
b) 
The product is −20.
c)  The product is +2.

d) 
The product is −15.
6. The sloth climbed down 18 m.
7. a) 
The product is −18.
b) 
The product is +8.
8. a) −56 **b)** 108
9. Estimates may vary. **a)** 770 **b)** −637
10. Possible answers are −3 and +33, +3 and −33, −9 and 11, 9 and −11, −1 and 99, and 1 and −99.
11. a) 5×52 **b)** $260
12. a) $(+10) \div (+2) = +5$; $(+10) \div (+5) = +2$
b) $(-8) \div (-2) = +4$; $(-8) \div (+4) = -2$
13. a)  The quotient is +2.
b)  The quotient is +7.
c)  The quotient is −1.
14. Answers may vary. Example: The value of a share of Orange Computers Limited fell $14 in 7 h. How much did the value fall per hour if the rate of fall was constant?
15. 
The quotient is +6.
16. $(-247) \div (-13)$. The quotient of two integers with the same sign is positive. The quotient of two integers with different signs is negative. Therefore, only $(-247) \div (-13)$ is positive.
17. a) +3 **b)** −8 **c)** −17 **d)** +8
18. Answers may vary. Example: The two integers are identical except for having different signs.
19. $18
20. a) −6 **b)** 4

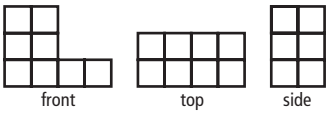
21. a) -7 **b)** No. Different sets of integers can have the same mean. Example: $\{-20, -16, 18, -25, 22, -21\}$ and $\{-62, 9, -2, 1, 6, 6\}$ both have a sum of -42 and a mean of -7.
22. a decrease of 2341 people per year
23. 70 s
24. \$570

Chapters 5–8 Review, pages 324–326

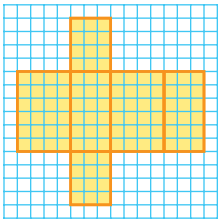
1. a)



b)



2.



3. 150.4 cm^2
4. a) area with bottom: 8.25 m^2
b) area without bottom: 5.73 m^2
5. 83 m^2
6. 187.68 cm^2
7. cylinder A: about 14 130 cm^2; cylinder B: about 39 564 cm^2
8. a) 21 days **b)** 14 days
9. $\frac{1}{10}$ of a cake
10. $\frac{1}{2}$ of the lifespan of a bison
11. $\frac{1}{3}$ of Earth's surface
12. 54 cm
13. a) $1\frac{2}{5}$ **b)** $\frac{5}{7}$
14. winner: \$450; runner-up: \$300; third-place: \$150
15. 40 km/h
16. *Method 1*: Since $\frac{1}{5}$ of the flagpole is 2 m long, the remaining $\frac{4}{5}$ must be four times as long, which is 8 m.
Method 2: Since $\frac{1}{5}$ of the flagpole is 2 m long, the length of the whole flagpole is 5×2 m, which equals 10 m. The length of portion above ground is 10 m $-$ 2 m, which equals 8 m.
17. 0.196 m^3
18. a) 1331 cm^3 **b)** about 1020 cm^3
19. 756.6 kg
20. a) 4630 cm^3 **b)** 5898 cm^3
21. 52 cm^3
22. a) $(+5) \times (+3) = +15$ **b)** $(-14) \times (-2) = +28$
c) $(-4) \times (+8) = -32$ **d)** $(-6) \times (+4) = -24$
23. Estimates may vary. **a)** -308 **b)** 598
24. 1 and -20; -1 and 20; 2 and -10; -2 and 10; 4 and -5; -4 and 5
25. a) $(+20) \div (+4) = +5$ **b)** $(-22) \div (-11) = +2$
c) $(-24) \div (+8) = -3$ **d)** $(-21) \div (-3) = +7$
26. -1
27. a) Yes. Multiplication is repeated addition. Since the sum of any set of integers is an integer, the product of two integers is also an integer. **b)** No. Division of an integer by most other integers gives parts that do not contain a whole number of units. For example, the quotient of 5 divided by any integer greater than 5 or less than -5 is not an integer.
28. \$200
29. -12 °C
30. 186 L
31. a) -2 **b)** 5
32. -20