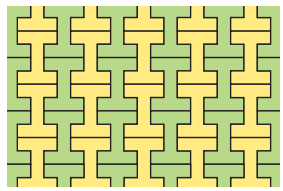
Chapter 12

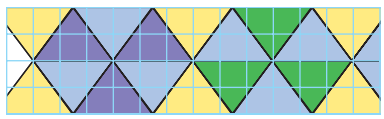
12.1 Exploring Tessellations With Regular and Irregular Polygons, pages 450–451

4. a) Yes. Each angle of a regular hexagon is 120°. The sum of three vertices of a regular hexagon is 360°.
b) No. Each angle of a regular heptagon is about 128.6°. Any number of vertices of this shape will not have the interior angle measures total exactly 360°.

5. Answers may vary. Example:

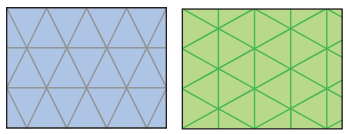


6. Answers may vary. Example:



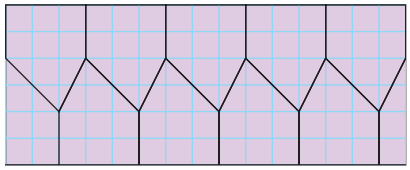
7. Answers may vary. Example: The tessellations of rectangular bricks on walls, of rectangular shingles on roofs, and of square tiles on floors.

8. Answers may vary. Example: One tessellation is drawn on triangular dot paper and the other is drawn on the same triangular dot paper rotated 90°.

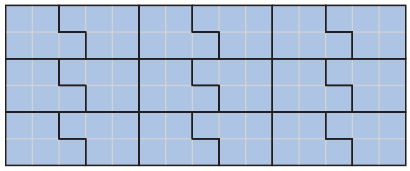


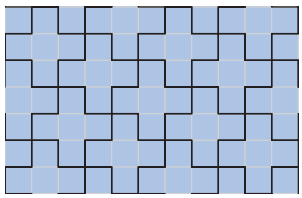
9. Answers may vary. For example,

10. a) Answers may vary. Example: A pentagon made of a rectangle and a triangle can tessellate the plane. At the point where the vertices meet, the sum of the interior angles measures is the sum of the three angles of the triangle (180°) and the two right angles of the rectangle (180°), which is 360°.



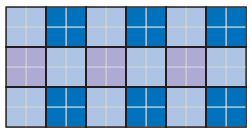
11. Answers may vary. Example:



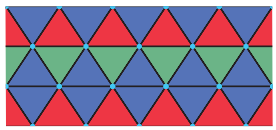


Yes, each of these two pentominos makes a tessellation because the plane is completely covered by repeated patterns of each shape without any overlap or gaps.

12. Answers may vary. Example:
Using a square on grid paper:

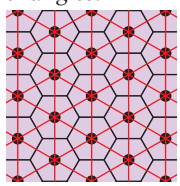


Use an equilateral triangle on triangular dot paper:

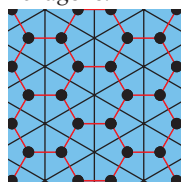


13. a) The dual is a translation of the original tessellation, so it also tessellates the plane. If the square is a unit square, the translation is half unit right and half unit down.

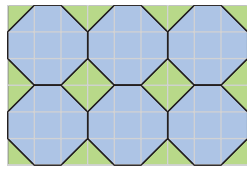
b) The dual is a tessellation of congruent equilateral triangles.



c) The dual is a tessellation of congruent regular hexagons.

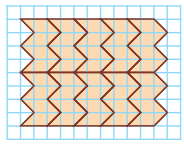


14. Answers may vary. Example: A regular octagon and a square can be used together to create a tessellation.

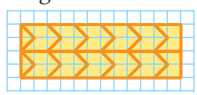


12.2 Constructing Tessellations Using Translations and Reflections, pages 455–456

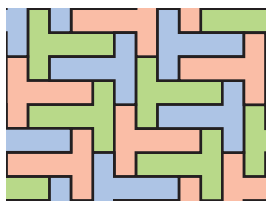
3. a) regular hexagon and equilateral triangle **b)** square and equilateral triangle **c)** regular octagon and square
4. Answers may vary. Example: **a)** translation or reflection **b)** reflection **c)** translation or reflection
5. a) The sum of the interior angle measures at the point where the vertices of the dodecagons meet is 360°.
b) Answers may vary. Example:

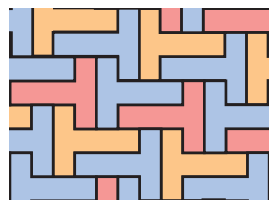


c) The sum of the interior angle measures at the point where the vertices of the decagons meet is 360°.
d) Answers may vary. Example:

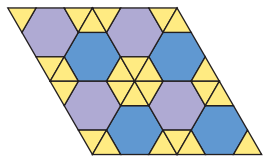


e) The sum of the interior angle measures at the point where the vertices of the hexagons meet is 360°.
6. Answers may vary. Example:



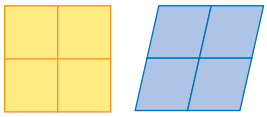


7. Answers may vary. Example:



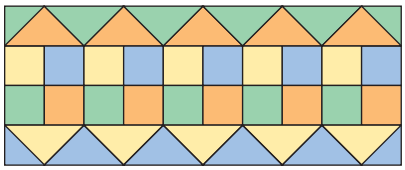
8. No. Each angle of the regular pentagon is 108° and each angle of the equilateral triangle is 60°. There is no combination of 108° vertices and 60° vertices that will have the interior angle measures total 360°.

9. The shapes a) and b) are reptiles. Answer may vary. Example:

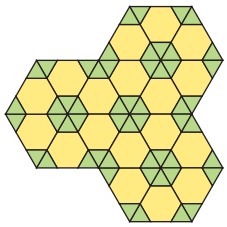


12.3 Constructing Tessellations Using Rotations, pages 459–460

3. a) square **b)** regular octagon and triangle
c) a cross shape and square
4. Answers may vary. Example: **a)** Rotate the square 90° about one of its vertices until a full turn is made. Then, rotate the larger square formed 90° about one of its vertices until a full turn is made. **b)** Rotate the square shape formed by a regular octagon and four isosceles triangles 90° about one of its vertices until a full turn is made. Then, translate the larger square horizontally to the right two times.
c) Rotate the shape formed by the cross shape with 4 small squares 90° about the free corner of the small square until a full turn is made. Then, translate the resulting shape horizontally to the right and vertically up and down.
5. a) Answers may vary. Example:
• Start with the top piece of the stained glass that is a 45° sector, or one-eighth, of the circle. Reflect it along a line making 45° with the horizontal.
• Reflect the resulting larger piece along the x-axis.
• Reflect the resulting larger piece along the y-axis.
b) Answers may vary. Example: Trim the edge of the 45° sector to make a right-angled triangle with the right angle touching the line of reflection. The resulting shape will be a square that tiles the plane.
6. Answers may vary. Example: Translate the combined shape of 4 squares and 4 isosceles triangles in four different colours horizontally to form the pattern.



7. Answers may vary. Example: The following tessellation is made using regular hexagons and equilateral triangles.

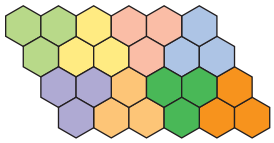


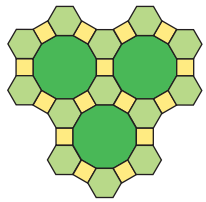
8. Shapes A, B, C, D, and G tessellate. Answers may vary. Example: A and B are quadrilaterals and all quadrilaterals tessellate the plane. C and D can tessellate by horizontal translation, fitting the part sticking out of the shape into the space going into the shape. G can tessellate by horizontal and vertical translations, fitting the parts sticking out into spaces going in.

9. a) and **b)** Answers may vary. Example: A combination of three regular hexagons (6, 6, 6) and a combination of one square, one regular hexagon, and one dodecagon will work.

Tessellations Involving Three Regular Polygons	Shape 1	Shape 2	Shape 3	Shape 4
Triangle (60°)	1	0	0	0
Square (90°)	0	1	0	1
Pentagon (108°)	0	0	0	0
Hexagon (120°)	0	0	3	1
Octagon (135°)	0	2	0	0
Dodecagon (150°)	2	0	0	1
Number of sides	(3, 12, 12)	(4, 8, 8)	(6, 6, 6)	(4, 6, 12)
Sum of angles	60° + 2(150°) = 360°	90° + 2(135°) = 360°	3(120°) = 360°	90° + 120° + 150° = 360°

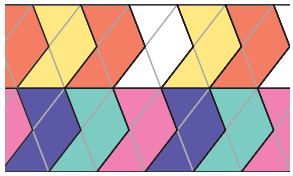
c) Answers may vary. Example:



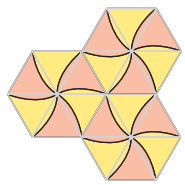


12.4 Creating Escher-Style Tessellations, pages 464–465

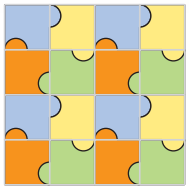
4. a) translation **b)** rotation
5. a) hexagon **b)** triangle
6. a) rotation and reflection **b)** rotation and translation
7. a) parallelogram **b)** square
8. Answers may vary. Example:



9. Answers may vary. Example:



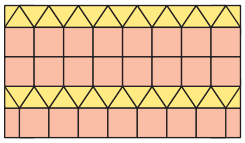
10. Answers may vary. Example:



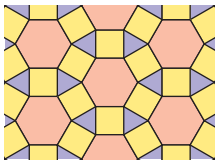
11. a) Answers may vary. Example: Staircases that appear to be upside down, people that appear to be walking right side up and upside down and sideways.

Chapter 12 Review, pages 466–467

1. tiling the plane **2.** plane
3. tessellation **4.** transformation
5. a) regular hexagon and equilateral triangle
b) rhombus, isosceles triangle, and regular hexagon
c) regular hexagon and equilateral triangle
d) regular hexagon, parallelogram, and equilateral triangle
6. The regular hexagons and equilateral triangles in #5 are regular polygons, while the isosceles triangles, rhombuses, and parallelograms are not. Regular polygons have equal interior angle measures and equal side lengths.
7. No. Answers may vary. Example: Each interior angle of a regular octagon is 135°, which is not a factor of 360°. However, two octagons and a square can tile the plane.
8. Answers may vary. Example: **a)** Translation or rotation of the combined shape. **b)** Translation of the dodecagon and reflections of the hexagon and rectangle.
9. Answers may vary. For example,



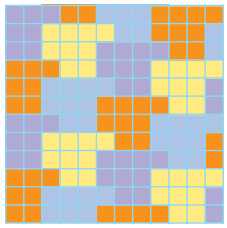
10. Answers may vary. Example:



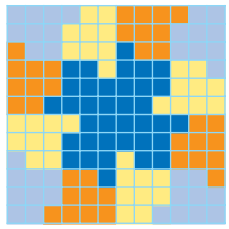
11. Answers may vary. Example: **a)** translation and reflection **b)** translation and reflection
12. Answers may vary. Example: A square that has the same side length as the shorter side of the irregular polygon.

13. a quadrilateral
14. a rotation about the centre of the regular hexagon

15. Answers may vary. Example:
Translation:

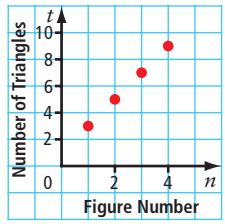


Rotation:



Chapters 9–12 Review, pages 473–475

1. a) 9 triangles
b)



c) Yes. When the four points are connected, they form a straight line.

2. Answers may vary. Example: **a)** I might have purchased hamburgers or sandwiches. The cost of one item is $3.
b) For every additional item purchased, the cost increases by $3.
c)

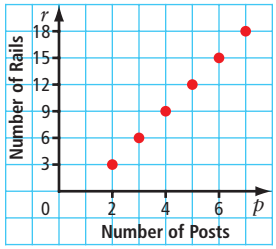
Quantity	2	3	4	5	6	7
Cost ($)	3	6	9	12	15	18

Use q for quantity and c for cost; q represents the quantity purchased and c represents the cost of purchase.
d) $c = 3q$ **e)** The cost is $24.

3. a)

Number of Posts, p	2	3	4	5	6	7
Number of Rails, r	3	6	9	12	15	18

b) Answers may vary. Example:



The relationship appears to be linear because the six points seem to lie on a straight line.
4. Answers may vary. Example: **a)** $y = 2x - 3$

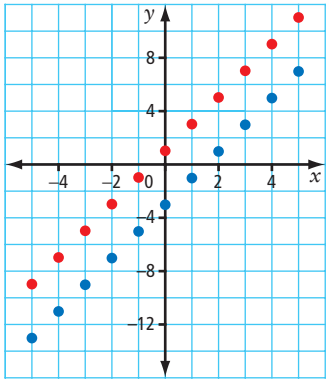
x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y	−13	−11	−9	−7	−5	−3	−1	1	3	5	7

$y = 2x + 1$

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y	−9	−7	−5	−3	−1	1	3	5	7	9	11

These values for x are easy to graph.

b)



c) Similar: The points for the two graphs form parallel lines. Different: For the same x-value, the y-value on the graph of $y = 2x + 1$ is 4 greater than the corresponding y-value on the graph of $y = 2x - 3$.

5. a) $4x = 12$ **b)** $x = 3$

6. a) $s = -10$ **b)** $x = 3$ **c)** $v = 16$ **d)** $x = 3$

7. a) $x = -28$ **b)** $x = 8$ **c)** $x = -18$ **d)** $x = 5$

8. a) $10 = \frac{1}{3}x - 3$ **b)** Jason's father is 39 years old.

9. $40(x + 2) = 960$; $x = 22$
Elijah's regular hourly wage is \$22/h.

10. a) Answers may vary. Example: Use a table.

		Die 2					
		1	**2**	**3**	**4**	**5**	**6**
Die 1	**1**	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

b) $P(\text{both even}) = \frac{9}{36} = \frac{1}{4}$

c) $P(\text{sum} \geq 6) = \frac{26}{36} = \frac{13}{18}$

11. a) $P(\text{odd number}) = \frac{2}{5}$ **b)** $P(\text{even number}) = \frac{2}{5}$

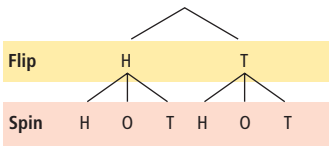
c) $P(\text{odd, then even}) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

12. 12 options

13. a) $P(\text{H on disk}) = \frac{1}{2}$; $P(\text{H is spun}) = \frac{1}{3}$

b) $P(\text{H on disk, H is spun}) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

c) Answers may vary. Example: Use a tree diagram.



From the tree diagram, $P(\text{H on disk, H is spun}) = \frac{1}{6}$.

14. a) Answers may vary. Example: The twins could use a spinner divided into four equal regions labelled with the four colours of the spinning tops. They can spin the spinner twice in each trial for at least 20 trials to find the probability of spinning blue in both spins.

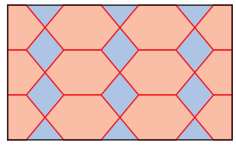
b) They need to assume that the spinning tops are identical.

c) $P(\text{experimental}) = \frac{1}{20}$ or 5%

d) $P(\text{theoretical}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ or 6.25%

15. Polygon A, a triangle, can tile the plane because two congruent triangles form a parallelogram, which is a quadrilateral. A quadrilateral can tile the plane because the sum of the interior angle measures is 360° at the point where the vertices of the quadrilaterals meet.
Polygon B, a regular hexagon, can tile the plane because each interior angle measure is 120°, which can total 360° at the point where the vertices of the hexagons meet.
Polygon C, a regular pentagon, cannot be used to tile the plane because each interior angle measure is 108°, which cannot total 360° at the point where the vertices of the pentagons meet.

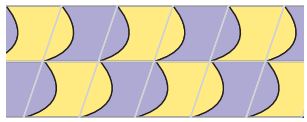
16. Answers may vary. Example:



The pattern is made up of squares and irregular hexagons. Translation is used to create the pattern.

17. Answers may vary. Example: A tessellating tile is made from a square by removing a piece from the bottom and left side of the square and translating these pieces to the opposite sides of the square. The tessellating tile is then translated horizontally and vertically to create the tessellation.

18. Answers may vary. Example:



A tessellating tile is made by removing a piece from the left side of a parallelogram and adding the piece to the other side. The tessellation is created by translating the tessellating tile horizontally and vertically.