2 Ratios, Rates, and Proportional Reasoning

General Outcomes

- Develop number sense.

Specific Outcomes

N4 Demonstrate an understanding of ratio and rate.
N5 Solve problems that involve rates, ratios and proportional reasoning.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
2.1	✓ represent two-term ratios
	✓ represent three-term ratios
	✓ identify, describe, and record ratios from real-life examples
	✓ represent a ratio as a fraction
	✓ represent a ratio as a percent
	✓ solve problems using ratios
2.2	✓ express rates using words and symbols
	✓ identify, describe, and record rates from real-life examples
	✓ solve problems using rates
2.3	✓ solve problems using proportional reasoning
	✓ use more than one method to solve proportional reasoning problems

Assessment	Supporting Learning
Assessment *for* Learning	
Method 1: Use the Math Link introduction on page 45 in *MathLinks 8* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal to explain what they personally know about ratios and rates, including how they are similar, and how they differ.	• **BLM 2–1 Chapter 2 Math Link Introduction** provides scaffolding for the Math Link introduction. • Have students use the What I Need to Work On section of the chapter Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete the Get Ready materials available on **BLM 2–2 Chapter 2 Get Ready**, in the *MathLinks 8 Practice and Homework Book*, and at the www.mathlinks8.ca book site.
Assessment *as* Learning	
Literacy Link (page 43) At the beginning of the chapter, work with students to model the use of a Frayer model for the term *ratio*.	• Encourage students to use the glossary starting on page 517 to help them. • Students who computerize their model may wish to access the *MathLinks 8* online glossary by going to www.mathlinks8.ca and following the links. • At the end of section 2.1, have students revisit their Frayer model for *ratio* in order to make additions and improvements.
Chapter 2 Foldable As students work on each section in Chapter 2, have them keep track of any problems they are having in the What I Need to Work On section of their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled.
Assessment *for* Learning	
BLM 2–3 Chapter 2 Warm-Up This BLM includes three warm-ups, one to be used at the beginning of each section. Each warm-up provides cumulative review questions for the entire student resource to that point, as well as mental math practice.	• As students complete questions from previous chapters, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing mental math calculations.

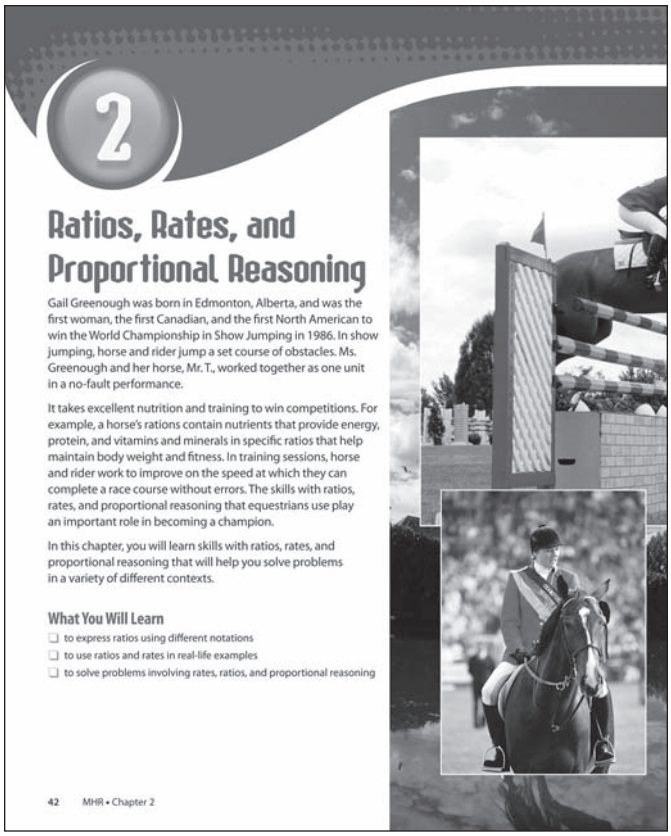
Problems of the Week

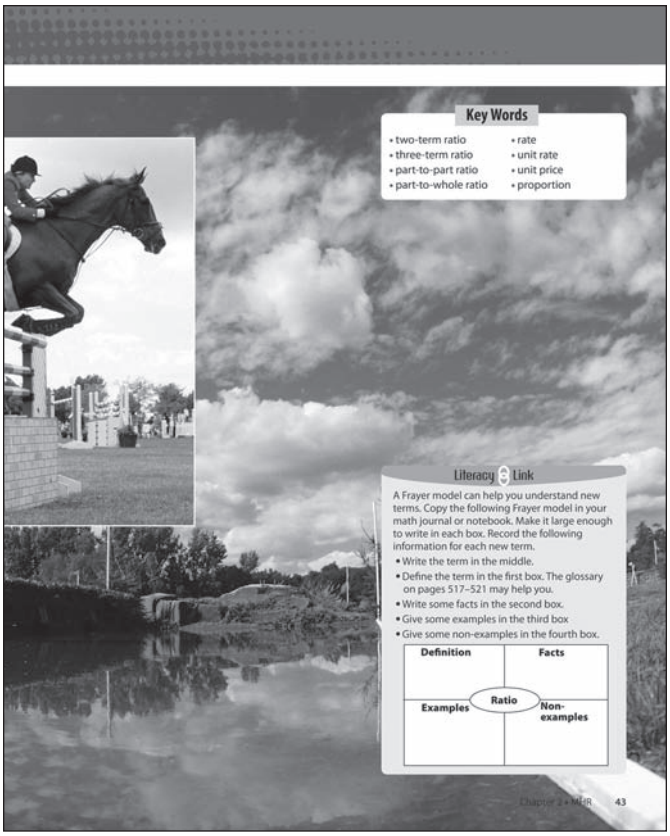
Have all students try at least one of the problems on **BLM 2–4 Chapter 2 Problems of the Week**. Many of these problems require students to think outside the box and experiment with a variety of approaches. Some have definitive answers; others can be answered in more than one way.

Students can take the problems home and consult with parents or guardians, work with other students when their work is completed, or try the problems on their own. The questions take a varying amount of time to solve, depending on the particular student and the problem itself. You may wish to give out these problems at the beginning of the chapter and discuss the solutions at appropriate times throughout your work on the chapter.

Chapter 2 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Extra Support	Assessment		
						Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 40–50 minutes (TR page 51)	Students should be familiar with • division by whole numbers • writing ratios more than one way	• 11 × 17 sheet of paper • ruler	Master 17 Frayer Model BLM 2–1 Chapter 2 Math Link Introduction BLM 2–2 Chapter 2 Get Ready BLM 2–4 Chapter 2 Problems of the Week		Online Learning Centre	TR page 50 Chapter 2 Foldable, TR page 50	TR page 50	
2.1 Two-Term and Three-Term Ratios • 80–100 minutes (TR page 56)	Students should be familiar with • meaning of a fraction • equivalent fractions • conversion of fractions to decimals and percents	• ruler • coloured counters (optional) • calculator (optional) • coloured pencils • grid paper	Master 8 Centimetre Grid Paper Master 19 Multiplication Chart BLM 2–3 Chapter 2 Warm-Up BLM 2–5 Section 2.1 Extra Practice BLM 2–6 Section 2.1 Math Link	**Essential:** 1–6, 9, 11, 12, 19, Math Link **Typical:** 1–6, 9, 11–19, Math Link **Extension/Enrichment:** 1, 2, 20–22, Math Link	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 60, 62 Math Learning Log, TR page 64 Chapter 2 Foldable, TR page 64	TR pages 60, 64	
2.2 Rates • 80–100 minutes (TR page 65)	Students should be familiar with • division of decimals • equivalent fractions • rounding	• ruler • standard paper clips • jumbo paper clips • flyers for products showing unit pricing information • calculator (optional)	Master 17 Frayer Model BLM 2–3 Chapter 2 Warm-Up BLM 2–7 Compare a Ratio and a Rate BLM 2–8 Section 2.2 Extra Practice BLM 2–9 Section 2.2 Math Link	**Essential:** 1–4, 6, 7, 8, 10, 13, Math Link **Typical:** 1–4, 6, 7, 8, 10–15, 16, Math Link **Extension/Enrichment:** 1, 2, 3c), 14–19	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 69, 70 Math Learning Log, TR page 72 Chapter 2 Foldable, TR page 72	TR pages 69, 72	
2.3 Proportional Reasoning • 80–100 minutes (TR page 73)	Students should be familiar with • equivalent fractions • substituting into a formula	• ruler • computer access • short story • stopwatch • sticky notes • access to recipes in print material and/or on the Internet	Master 2 Two Stars and One Wish Master 17 Frayer Model Master 19 Multiplication Chart BLM 2–3 Chapter 2 Warm-Up BLM 2–10 Section 2.3 Extra Practice BLM 2–11 Section 2.3 Math Link	**Essential:** 1, 3, 4, 6, 8, 10, 11, 14, 15, 18, Math Link **Typical:** 1, 3, 4, 6, 8, 10, 11, 14, 15–18, 21–23, 25, Math Link **Extension/Enrichment:** 1, 3, 25–28, Math Link	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	Master 2 Two Stars and One Wish TR pages 77, 79 Math Learning Log, TR page 81 Chapter 2 Foldable, TR page 81	TR pages 77, 81	
Chapter 2 Review • 40–50 minutes (TR page 82)			BLM 2–5 Section 2.1 Extra Practice BLM 2–8 Section 2.2 Extra Practice BLM 2–10 Section 2.3 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	*MathLinks 8 Practice and Homework Book* *MathLinks 8 CAB*	Chapter 2 Foldable, TR page 82	TR page 83	
Chapter 2 Practice Test • 40–50 minutes (TR page 84)		• ruler	BLM 2–12 Chapter 2 Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1–3, 6, 9, 11, 12	*MathLinks 8 CAB*	TR page 85		TR page 85 BLM 2–12 Chapter 2 Test
Chapter 2 Wrap It Up! • 80–100 minutes (TR page 86)		• access to recipes in print material and/or on the Internet • recipe from section 2.3 Math Link • ruler • logo design from section 2.1 Math Link • grid paper (optional) • art materials to make invitations (including coloured pencils or markers)	Master 1 Project Rubric Master 8 Centimetre Grid Paper BLM 2–1 Chapter 2 Math Link Introduction BLM 2–6 Section 2.1 Math Link BLM 2–9 Section 2.2 Math Link BLM 2–11 Section 2.3 Math Link BLM 2–13 Chapter 2 Wrap It Up!		Online Learning Centre			TR page 87 Master 1 Project Rubric
Chapter 2 Math Games • 30 minutes (TR page 89)		• three dice per pair of students • calculator					TR page 89	
Chapter 2 Challenge in Real Life • 40–50 minutes (TR page 90)		• map of Northwest Territories • 30-cm ruler • compass • calculator	Master 1 Project Rubric BLM 2–14 Map of the Northwest Territories BLM 2–15 Chapter 2 BLM Answers		Online Learning Centre		TR page 91	TR page 91 Master 1 Project Rubric





***MathLinks 8*, pages 42–45**

Suggested Timing
40–50 minutes

Materials
- 11 × 17 sheet of paper
- ruler

Blackline Masters
Master 17 Frayer Model
BLM 2–1 Chapter 2 Math Link Introduction
BLM 2–2 Chapter 2 Get Ready
BLM 2–4 Chapter 2 Problems of the Week

Key Words

two-term ratio	rate
three-term ratio	unit rate
part-to-part ratio	unit price
part-to-whole ratio	proportion

What's the Math?

In this chapter, students solve problems that involve rates, ratios, and proportional reasoning. They start by learning to describe and record two- and three-term ratios using real-life examples, and then solve problems using ratios. Students describe and record rates using real-life examples. They solve problems using unit rates and unit prices to help determine the best buy for various items. Students then learn to use different methods to solve proportional reasoning problems.

Planning Notes

Tell students that they will learn about ratios and rates and different strategies for solving problems involving ratios and rates. Ask them to recall what they know about ratio notation and have them identify examples of ratios in their daily life (e.g., using two parts water to one part rice when cooking rice). Then, ask students how they think a rate is different from a ratio. Encourage them to talk about what they know and to give real-world examples. Try to elicit ideas from all class members.

As a class, read the information, draw students' attention to the visuals, and ask students to share their experiences of equestrian events. Discuss how equestrian riders might use ratios and rates. For example, explain that *ration* comes from *ratio*, which means calculation. A ration is a fixed portion of food allowed per person or animal.

The main horse feed is pasture or hay. Ideally, horses receive a minimum of 1% of their body weight in high-quality hay or pasture daily. That is a ratio of 1 kg forage to 100 kg body weight. Many horses, including performance horses, also receive a concentrated feed that consists of grain, soybean, molasses, minerals, and other ingredients. Many of these feeds are administered using ratios. For example, horses may receive 0.5 kg of concentrate per 100 kg of body weight.

Horse owners also concern themselves with the ratios of various nutrients in their horses' diets. For example, the best rations contain calcium, phosphorus, and magnesium in a 2 : 1 : 1 ratio. Improper ratios in a horse's diet will diminish its ability to thrive and perform.

Literacy Link Frayer models provide a useful strategy for assessing students' understanding of terms. They also help students deepen their understanding of a term by analysing its essential characteristics and by communicating examples and non-examples.

At the beginning of the chapter, use the term *ratio* to demonstrate how to use a Frayer model. Consider using an overhead copy of **Master 17 Frayer Model**. Explain the purpose of each part of the model.

- Definition – Work with students to develop a clear definition. They may wish to check the *MathLinks 8* glossary.
- Facts – Ask students to record what they already know about ratios. Some students may be familiar with rations as described in the chapter opener. Mr. T's food ration included nutrients that provided energy, protein, and minerals in specific ratios that helped him maintain his weight and fitness level.
- Examples – Most students have probably made juice from concentrate. Have them record the ratio of concentrate to water.
- Non-examples – Help students think of things that look like ratios but are not. For example, a mark of 75% on a test is not a ratio.

During the chapter, have students use the Frayer model to show their understanding of rates and proportions. Have them develop a Frayer model showing what they already know about rates at the beginning of section 2.2 and what they know about proportions at the beginning of section 2.3. You may wish to have them revisit their Frayer models at the end of each section.

Post samples of the three Frayer models to help students differentiate between ratios, rates, and proportions.

Meeting Student Needs

- Some students may need help to recall what they know about fractions, decimals, and percent before beginning this chapter. Consider having them complete the questions on **BLM 2–2 Chapter 2 Get Ready** to activate the prerequisite skills for this chapter.
- Consider providing web sites about national and international equestrian events such as those held at Spruce Meadows in Calgary, Alberta and the Olympics for students who are not familiar with these equestrian events.
- You might have students discuss how ratios and rates are used in local rodeo events such as the Kikino Silver Birch Rodeo held annually near Lac La Biche in Alberta.
- Some students may not be familiar with any equestrian events. Consider discussing ratio and rates in a more familiar context, such as sports (e.g., BMX riding) or daily life. For ratio, you might compare the distance from home to school to the distance from home to the ice rink or a community centre or store. For rate, discuss how weather or traffic affects the rate at which students travel to school. For proportional reasoning, compare the time it takes to walk or bike to school compared to the time spent riding in a vehicle.
- You may wish to invite a person who works with patterns and sewing, such as a sewer or tailor, to talk to the class about how ratios and proportional reasoning are used in making clothing. For example, you might have an Inuit seamstress talk about sewing traditional clothing without using patterns.

See *Sinews of Survival: The Living Legacy of Inuit Clothing* by Betty Kobayashi Issenman (UBC Press, 1997).

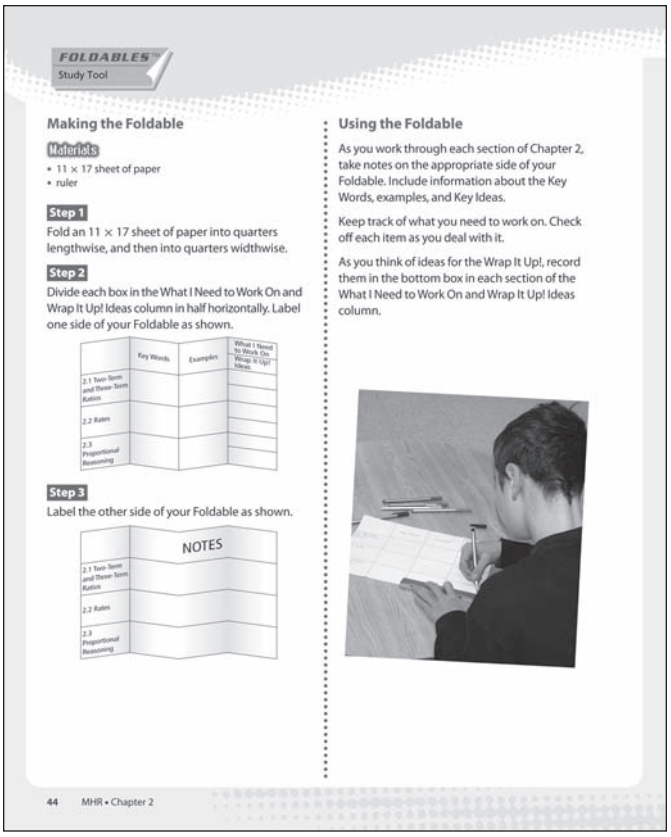
- Consider having students brainstorm and then develop a mind map about ratios, including ways to represent ratios, such as using manipulatives, drawings, symbols, or words, and strategies they have used to solve ratio problems.
- Have students use small cards for their Frayer models. If students make a Frayer model for each key word in the chapter, they can develop a deck of cards that will be useful for review.

ELL

- English language learners may have difficulty with terms such as *show jumping*, *rider*, *obstacles*, *no-fault performance*, *nutrition*, *competitions*, *rations*, *nutrients*, *energy*, *protein*, *vitamins*, *minerals*, *specific ratios*, *body weight*, *fitness*, *speed*, *errors*, *equestrians*, and *champion*. Have students add any new terms to their dictionary.
- Visuals of show jumping may help students develop a better sense of the sport.
- Consider displaying Key Words on a math word wall. Encourage students to create their own vocabulary/picture dictionary. Matching a picture with a key word and its definition helps reinforce students' understanding of vocabulary.

Gifted and Enrichment

- Have students research the key ratios of various minerals in the ideal human or horse diet and find out why these ratios are important. Individuals or groups could check the following ratios:
  - calcium : phosphorus : magnesium
  - copper : zinc : manganese
  - copper : iron
  - potassium : sodium



FOLDABLES™
Study Tool

Making the Foldable

Materials
- 11 × 17 sheet of paper
- ruler

Step 1
Fold an 11 × 17 sheet of paper into quarters lengthwise, and then into quarters widthwise.

Step 2
Divide each box in the What I Need to Work On and Wrap It Up! Ideas column in half horizontally. Label one side of your Foldable as shown.

	Key Words	Examples	What I Need to Work On/ Wrap It Up! Ideas
2.1 Two-Term and Three-Term Ratios			
2.2 Rates			
2.3 Proportional Reasoning			

Step 3
Label the other side of your Foldable as shown.

	NOTES
2.1 Two-Term and Three-Term Ratios	
2.2 Rates	
2.3 Proportional Reasoning	

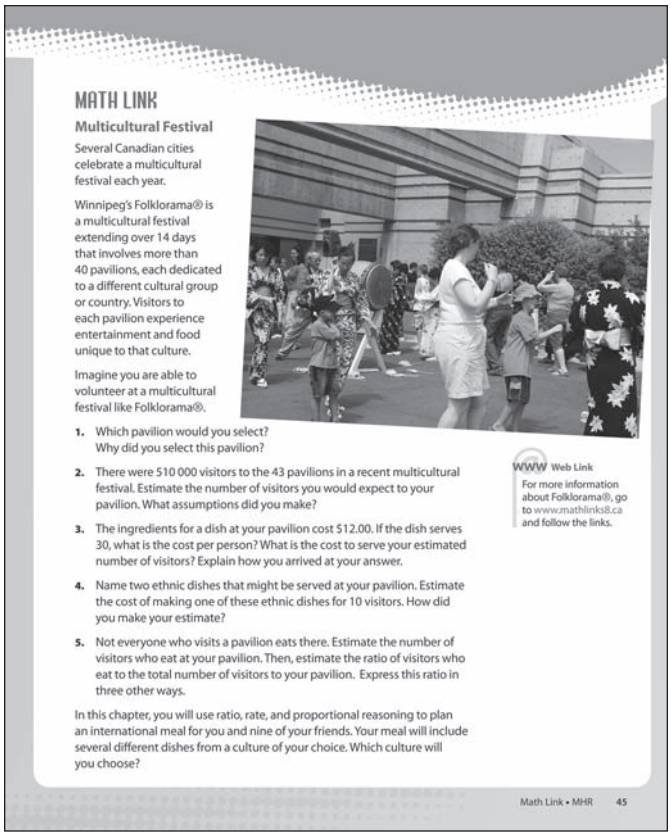
Using the Foldable

As you work through each section of Chapter 2, take notes on the appropriate side of your Foldable. Include information about the Key Words, examples, and Key Ideas.

Keep track of what you need to work on. Check off each item as you deal with it.

As you think of ideas for the Wrap It Up!, record them in the bottom box in each section of the What I Need to Work On and Wrap It Up! Ideas column.

44 MHR • Chapter 2



MATH LINK

Multicultural Festival

Several Canadian cities celebrate a multicultural festival each year.

Winnipeg's Folklorama® is a multicultural festival extending over 14 days that involves more than 40 pavilions, each dedicated to a different cultural group or country. Visitors to each pavilion experience entertainment and food unique to that culture.

Imagine you are able to volunteer at a multicultural festival like Folklorama®.

1. Which pavilion would you select? Why did you select this pavilion?
2. There were 510 000 visitors to the 43 pavilions in a recent multicultural festival. Estimate the number of visitors you would expect to your pavilion. What assumptions did you make?
3. The ingredients for a dish at your pavilion cost $12.00. If the dish serves 30, what is the cost per person? What is the cost to serve your estimated number of visitors? Explain how you arrived at your answer.
4. Name two ethnic dishes that might be served at your pavilion. Estimate the cost of making one of these ethnic dishes for 10 visitors. How did you make your estimate?
5. Not everyone who visits a pavilion eats there. Estimate the number of visitors who eat at your pavilion. Then, estimate the ratio of visitors who eat to the total number of visitors to your pavilion. Express this ratio in three other ways.

WWW Web Link
For more information about Folklorama®, go to www.mathlinks8.ca and follow the links.

In this chapter, you will use ratio, rate, and proportional reasoning to plan an international meal for you and nine of your friends. Your meal will include several different dishes from a culture of your choice. Which culture will you choose?

Math Link • MHR 45

Foldables Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter. Have them record the Key Ideas for each section in the Notes on the back of the Foldable. Filling in the What I Need to Work On section as they progress through the chapter will assist them in identifying and solving any difficulties with concepts, skills, and processes.

Have students store the Foldable in a binder by folding their folder in half and punching holes along one of the long sides. You may also wish to provide students with a plastic envelope that fits into their binder.

Note that there is no room on this Foldable for the Math Links throughout the chapter. You may wish to have students keep track of this work in their math portfolio or slip it into the plastic envelope mentioned above.

Math Link

Read the Math Link introduction on page 45 and ask students to describe any multicultural festivals that they have attended. Have students answer and then discuss the questions.

The Math Links for this chapter are about planning an international meal. As an alternative, students may want to plan a meal with a different theme. For example, they might decide to research foods traditional to a specific First Nations culture and plan a traditional feast.

Consider having students share the different ethnic groups represented in the class to help choose a culture to research. Students with multiple ethnic identities might choose one of them.

Have students read the Wrap It Up! problem on page 73 to give them a sense of where the Math Link introduction is heading. The Wrap It Up! problem is a summative assessment. As students work through the chapter, they need to complete at least the related Math Links in sections 2.1 and 2.3. These Math Links will assist them in doing the Wrap It Up! problem. Have students who experience difficulty with rates and ratios complete the Math Link in section 2.2.

Meeting Student Needs

- Consider creating the chapter Foldable ahead of time to use as a model. Consider giving students the option of using different-coloured pens to record their notes for What I Need to Work On and Wrap It Up! Ideas, instead of dividing each section into two cells.
- Have students work individually, with a partner, or as a class to complete the Math Link.
- To help them to get started, some students may benefit from using **BLM 2–1 Chapter 2 Math Link Introduction**, which provides scaffolding for this activity.

ELL

- English language learners may have difficulty with the terms *multicultural*, *pavilions*, and *ethnic*. Have students add any new terms to their dictionary.

Web Link

For more information about equestrian events at Spruce Meadows, go to www.mathlinks8.ca and follow the links.

For more information about Winnipeg's Folklorama®, go to www.mathlinks8.ca and follow the links.

Answers

Math Link

1. Answers will vary.
2. Answers may vary. Example: $500\,000 \div 40 \approx 12\,000$. An estimated 12 000 people visited each pavilion. Look for at least one assumption. Example: Approximately the same number of people visited each pavilion.
3. Answers may vary. Example: Divide \$12 by 30. The cost per person is \$0.40. $12\,000 \times 4 = 4800$. The estimated cost to serve 12 000 people is \$4800. Multiply the average number of visitors by the cost per person.
4. Answers will vary. Look for the names of 2 dishes and the estimated cost of serving 1 dish to 10 visitors.
5. Answers may vary. Example: 10 000 : 12 000 or 5 : 6; $\frac{5}{6}$, 0.83, 83%