

2.1

Two-Term and Three-Term Ratios

MathLinks 8, pages 46–54

Suggested Timing

80–100 minutes

Materials

- ruler
- coloured counters (optional)
- calculator (optional)
- coloured pencils
- grid paper

Blackline Masters

Master 8 Centimetre Grid Paper
 Master 19 Multiplication Chart
 BLM 2–3 Chapter 2 Warm-Up
 BLM 2–5 Section 2.1 Extra Practice
 BLM 2–6 Section 2.1 Math Link

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- Reasoning (R)
- Technology (T)
- Visualization (V)

Specific Outcomes

- N4** Demonstrate an understanding of ratio and rate.
N5 Solve problems that involve rates, ratios and proportional reasoning.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–6, 9, 11, 12, 19, Math Link
Typical	1–6, 9, 11–19, Math Link
Extension/Enrichment	1, 2, 20–22, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce material learned in previous sections.


As students discuss the photo and its enlargement, explain that when enlarging an image, the image produced is mathematically similar to the original image. This means that all of the dimensions of the original are multiplied by the same amount, and produces an image that has the same proportions as the original.

2.1

Two-Term and Three-Term Ratios

FOCUS ON...
 After this lesson, you will be able to...

- represent two-term and three-term ratios
- identify, describe, and record ratios from real-life examples
- represent a ratio as a fraction or as a percent
- solve problems using ratios




You use ratios to enlarge or reduce graphics and pictures. Discuss with a partner how you could use ratios to describe these two images.

Explore the Math

How can you compare an enlargement to its original image?

Jason made the following enlargement of a prairie dog. Kate asked how he enlarged it. Jason could not remember. Use ratios to help him answer Kate's question.



46 MHR • Chapter 2

Have students estimate how many times larger the enlargement is than the photograph. Ask what it means to enlarge a graphic to three times its original size.

You might explain that a ratio shows the relationship between two or more quantities. For example, three cans of water to one can of juice is a ratio of 3 to 1.

Explore the Math

In this exploration, students use ratios to compare an enlargement to its original image.

Method 1 Students may work individually or in pairs. It is important for students to estimate the multiplier prior to taking any measurements. Measurements can be horizontal, vertical, or diagonal. When using measurements to compare an enlargement to the original, emphasize the importance of measuring from exactly the same parts of each picture.

- How might you compare the original photo to its enlargement? Share your method with a classmate.
 - What might you compare on the two photos to give you a ratio?
 - How many measurements would you need to make?
 - Why would it be important to measure exactly the same parts of each photo?
- Suggest a multiplier that Jason would have used to enlarge the photo of the prairie dog. How did you arrive at your answer? Compare your answer with those of your classmates.

Reflect on Your Findings

- If the ratio comparing the image to the original is greater than one, what does this tell you?
 - If the ratio comparing the image to the original is less than one, what does this tell you?

Example 1: Represent Ratios

A bag contains 20 marbles.



- What is the **two-term ratio** of black to red marbles?
- Compare the number of red marbles to the total number of marbles. Write the ratio as an equivalent fraction in lowest terms.
- What marbles are represented by the ratio 6:10?
- Write the **three-term ratio** comparing the red, purple, and black marbles.

two-term ratio

- compares two quantities measured in the same units
- written as $a:b$ or a to b



blue:red is 6:4

three-term ratio

- compares three quantities measured in the same units
- written as $a:b:c$ or a to b to c



blue:red:brown is 6:4:10

2.1 Two-Term and Three-Term Ratios • MHR 47

Strategies Draw a Diagram

Solution

a) Method 1: Represent a Ratio Using a Drawing

There are 10 black and 4 red marbles. The drawing shows the ratio.



part-to-part ratio

- compares different parts of a group to each other
- 10:8 is the part-to-part ratio of brown to red beads.



part-to-whole ratio

- compares one part of a group to the whole group
- 5:23 is the part-to-whole ratio of blue to total number of beads.



Method 2: Represent a Ratio Using Symbols or Words

The ratio of black marbles to red marbles is 10:4 or 10 to 4.

- There are 4 red marbles out of a total of 20 marbles. The ratio can be expressed as 4:20. A **part-to-whole ratio** can be expressed as a fraction.

$$\frac{\text{red}}{\text{total}} \text{ is } \frac{4}{20}$$

Write the fraction as an equivalent fraction in lowest terms.

$$\frac{4}{20} = \frac{1}{5}$$

To express a fraction in lowest terms, divide the numerator and the denominator by the same number.

- There are 6 purple marbles and 10 black marbles. purple:black is 6:10

- You can compare red, purple, and black marbles using a three-term ratio. red:purple:black = 4:6:10 = 2:3:5

You can multiply or divide each term of a three-term ratio by the same number.

Show You Know

Use the design to help answer the following questions.



- What is the ratio of red tiles to total tiles? Express the answer three different ways.
- What could the ratio 4:6 represent?
- What is the ratio of red to black to white tiles?

48 MHR • Chapter 2

Have students share their solutions with each other, first in small groups, then as a class. Have them consider questions such as the following:

- Did you come to the same solution?
- If not, how did the solutions vary? Why?
- How and where did you measure? Why?
- How did you decide what multiplier to use?

Encourage discussion of what part of the prairie dog each student took the measurements from, what was similar about their measurements, and what was different. You may wish to have students test any conclusions by using the photograph in the section opener on page 46.

Consider having students discuss when they might use enlargements.

Method 2 In order to protect the student resource, you may wish to photocopy another visual from *MathLinks 8*, and then use the enlarging or reducing function on the photocopier to provide an enlarged and a reduced version of that picture.

Divide the class into an even number of smaller groups so that half the class works on the original and a reduction and the other half works on the original and an enlargement. Provide each group with a number of copies of the original and either

the reduction or the enlargement. Challenge the groups to consider how they can compare the enlarged or reduced photocopy to its original size.

- Have them brainstorm different strategies for making the comparison and then experiment with one or two of the strategies they develop.
- Have students consider what measurements they might take, where, and how many measurements they might need.
- Challenge students to consider whether or not they should always compare the same part of each picture, and why.

Have each group report their findings to the class and share the strategies used.

Example 1

Example 1 provides two methods for representing a ratio: using a drawing and using symbols or words. Using a drawing helps students visualize the relationships you are discussing. Using symbols or words is more abstract.

It is important that students state explicitly what each ratio represents. For example, in part a), discourage an answer of 10:4. Instead, model the answer as follows: black:red = 10:4 or $\frac{\text{black}}{\text{red}} = \frac{10}{4}$.

Example 2: Apply Ratios

Tamara has a recipe for fruit punch that calls for three cans of frozen orange juice concentrate, two cans of raspberry juice concentrate, and one can of lime juice concentrate. For each can of juice concentrate, the directions say to add three cans of water. All the cans are the same size. Tamara makes one recipe of fruit punch.

a) Copy and complete the following chart.

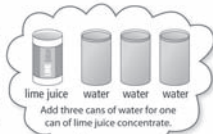
	Orange	Raspberry	Lime	Total
Juice Concentrate (cans)				
Water (cans)				
Total Punch (cans)				

- b) What is the ratio of orange juice to lime juice concentrate? Express the ratio two different ways.
- c) What is the ratio of lime to orange to raspberry juice concentrate?
- d) What is the ratio of water to juice concentrate?
- e) How many cans of punch does the recipe make?
- f) What is the ratio of orange, raspberry, and lime juice concentrate to total punch? Express the ratio as a fraction, a decimal, and a percent.

Solution

a)

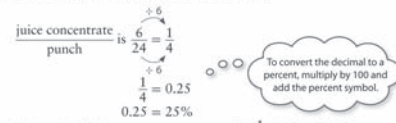
	Orange	Raspberry	Lime	Total
Juice Concentrate (cans)	3	2	1	6
Water (cans)	$3 \times 3 = 9$	$2 \times 3 = 6$	$1 \times 3 = 3$	18
Total Punch (cans)				



- b) three cans of orange juice and one can of lime juice concentrate
The ratio of orange to lime juice concentrate is 3:1 or 3 to 1.
- c) one can of lime juice, three cans of orange juice, and two cans of raspberry juice concentrate
The ratio of lime to orange to raspberry juice concentrate is 1:3:2.
- d) 18 cans of water, 6 cans of juice concentrate
The ratio of water to juice concentrate is 18:6 or 3:1.
- e) 18 cans of water + 6 cans of juice concentrate = 24 cans of punch
One recipe makes 24 cans of punch.



f) 6 cans of juice concentrate, 24 cans of punch



The ratio of juice concentrate to punch is $\frac{1}{4}$, 0.25, or 25%.

Show You Know

A recipe for trail mix calls for three cups of mini pretzels, two cups of roasted soy chips, one cup of raisins, and one cup of sunflower seeds. You make two batches of trail mix.

- a) What is the ratio of mini pretzels to raisins? Express the ratio two different ways.
- b) What is the ratio of roasted soy chips to sunflower seeds?
- c) How many cups of mix do two batches make?
- d) What is the ratio of soy chips and sunflower seeds to total trail mix? Express the ratio as a fraction, a decimal, and a percent.

Key Ideas

- A part-to-part ratio compares different parts of a group.
The ratio of red to black tiles is 6:3 or 6 to 3.
The ratio in lowest terms is 2:1 or 2 to 1.
- A part-to-whole ratio compares one part of a group to the whole group.
The ratio of red to total tiles is 6:12 or 6 to 12.
The ratio in lowest terms is 1:2 or 1 to 2. One out of every two tiles is red.
- A part-to-whole ratio can be written as a fraction, a decimal, and a percent.
The ratio of $\frac{\text{red}}{\text{total}}$ is $\frac{6}{12}$ or $\frac{1}{2}$, 0.5, 50%.



Make sure that students can differentiate between a part-to-part ratio and a part-to-whole ratio. Students should realize that they can use the processes they know for changing a fraction to lowest terms to change a ratio to lowest terms.

Three-term ratios are new to grade 8. Encourage students to identify real-life examples of three-term ratios. They may mention ratios used in sports (e.g., win:loss:tie), fertilizers (e.g., 10:20:30), and recipes (e.g., oatmeal:raisins:almonds).

Example 2

Example 2 illustrates using ratios in a real-world situation. Note that parts d) and f) provide opportunities for students to change ratios to lowest terms. Some students will automatically change their ratios to lowest terms; others will not. As a class, discuss the difference in the information provided by a ratio such as $\frac{18}{6}$ (not in lowest terms) and $\frac{3}{1}$ (in lowest terms). Students may say that the ratio $\frac{18}{6}$ is more representative of the problem, and this is correct. You could then remind them that ratios in lowest terms have numbers that make them easier to work with mentally.

Meeting Student Needs

- Consider having students redo the Explore using the large picture of the prairie dog as the original and the small picture as the reduction in order to help them answer #3b).
- Consider having concrete and kinesthetic learners place a 3×3 grid over each photo in the Explore the Math, and compare the enlargement to the original in terms of covered space on each grid.
- Provide students with additional examples of ratios with which they are familiar. For example, students who are familiar with two-stroke engines might determine the ratio of oil to gas needed to fill a gas tank for a two-stroke engine.
- For Example 2 part f), you may need to help reactivate some students' skills with a reminder of how to manipulate decimals and percents.

ELL

- English language learners may have difficulty with the term *multiplier*. Explain that a multiplier is the number used to multiply all of the dimensions of an object in order to enlarge it. For example, to double an image, multiply all of its dimensions by two.
- English language learners may have difficulty with terms such as *enlarge*, *reduce*, *prairie dog*, *width*, *widest*, *decimal form*, *height*, *diagonal*, *fruit punch*, *concentrate*, *trail mix*, *roasted soy chips*, and *sunflower seeds*. Have student add new terms to their dictionary.
- Encourage students to share their understanding of the Key Ideas during group discussion.

Common Errors

- Students may reverse the terms in a ratio.
- R_x** Use manipulatives such as coloured counters to show the ratio concretely. Then, use words and symbols to describe the ratio using the same order.
- Some students may confuse part-to-part and part-to-whole ratios.
- R_x** Have students carefully read the question. Identify the parts and the whole for the given situation together. Identify whether it is the parts and/or the whole that need to be determined. Have students record this information in the solution.

Answers

Explore the Math

- a) Answers will vary. Look for a variety of strategies such as visually estimating, taking measurements, and using ratios.
 - b) Answers will vary. Students may mention taking horizontal, vertical, or diagonal measurements. They may also mention measuring different parts of the prairie dog.
 - c) Answers will vary. A minimum of two measurements are needed, one from the same part of each picture. Some students may suggest taking two sets of measurements, such as length and width, in order to check that the ratios are the same.
 - d) in order to make an accurate comparison between the two photos
2. The multiplier is 2. Explanations will vary. Example: Comparing the measurements of the two heights indicates a 2 : 1 ratio.
3.
 - a) There is an enlargement.
 - b) There is a reduction or shrinking.

Show You Know: Example 1

- a) 4 : 21, 4 to 21, $\frac{4}{21}$
- b) red tiles to black tiles
- c) 4 : 6 : 11

Show You Know: Example 2

- a) 6 : 2, 6 to 2
- b) 4 : 2
- c) 14 cups
- d) $\frac{6}{14} \approx 0.4286 \approx 43\%$

Assessment	Supporting Learning
Assessment as Learning	
<p>Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.</p>	<ul style="list-style-type: none"> • Some students may benefit from being shown examples of copies in which the ratio of the copy to the original is greater than one, equal to one, and less than one. Ask them to describe the copy compared to the original in each case. • Help students remember the terms <i>less than</i>, <i>greater than</i>, and <i>equal to</i>. • Provide a different situation in which students compare an enlargement or a reduction to its original image. Have students respond orally. • Some students may benefit from using the class responses as springboards to prepare their own answer.
Assessment for Learning	
<p>Example 1 Have students do the Show You Know related to Example 1.</p>	<ul style="list-style-type: none"> • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students explicitly state the parts and the whole for each ratio in parts a) to c) orally and in written form. You might show students how to organize the information in a chart. • Have students identify the part-to-whole ratio for red, black, and white tiles. • Remind students that all of the parts add up to a whole, which equals 100% or 1.0. • Refer students to Example 1, which provides a similar question pattern. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.
<p>Example 2 Have students do the Show You Know related to Example 2.</p>	<ul style="list-style-type: none"> • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Consider modelling the situation using visuals of the concentrated juices and water to help students understand the recipe for punch. • Encourage students to use manipulatives or draw a diagram to model the situation. • Have students explicitly state the parts and the whole for the ratios in parts a), b), and d) orally and in written form. Students may benefit from using a chart to organize their information. • Have students identify the part-to-whole ratio for pretzels, soy chips, raisins, and sunflower seeds in the trail mix. • Have students calculate the total quantity of each ingredient in part c). Encourage students to record their answers using a chart. • Check for understanding by asking students to respond orally to questions like the following: What is the ratio of mini pretzels to sunflower seeds and soy chips? What is the ratio of mini pretzels and raisins to the total trail mix? • Allow students the time they need to explore changing fractions to decimals and percents. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.

f) 6 cans of juice concentrate, 24 cans of punch



The ratio of juice concentrate to punch is $\frac{1}{4}$, 0.25, or 25%.

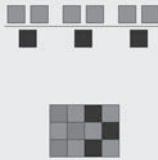
Show You Know

A recipe for trail mix calls for three cups of mini pretzels, two cups of roasted soy chips, one cup of raisins, and one cup of sunflower seeds. You make two batches of trail mix.

- What is the ratio of mini pretzels to raisins? Express the ratio two different ways.
- What is the ratio of roasted soy chips to sunflower seeds?
- How many cups of mix do two batches make?
- What is the ratio of soy chips and sunflower seeds to total trail mix? Express the ratio as a fraction, a decimal, and a percent.

Key Ideas

- A part-to-part ratio compares different parts of a group.
The ratio of red to black tiles is 6:3 or 6 to 3. The ratio in lowest terms is 2:1 or 2 to 1.
- A part-to-whole ratio compares one part of a group to the whole group.
The ratio of red to total tiles is 6:12 or 6 to 12. The ratio in lowest terms is 1:2 or 1 to 2. One out of every two tiles is red.
- A part-to-whole ratio can be written as a fraction, a decimal, and a percent.
The ratio of $\frac{\text{red}}{\text{total}}$ is $\frac{6}{12}$ or $\frac{1}{2}$, 0.5, 50%.



- A three-term ratio compares three quantities measured in the same units.

The ratio of red to black to blue tiles can be written as 6:3:3 or 6 to 3 to 3. The ratio in lowest terms is 2:1:1 or 2 to 1 to 1.



- A two-term ratio compares two quantities measured in the same units.

The ratio of black to total tiles can be written as 3:12 or 3 to 12. The ratio in lowest terms is 1:4 or 1 to 4. One out of every four tiles is black.

Communicate the Ideas

- Janine wants to write the ratio of oranges to apples. How does she know whether to write 3:4 or 4:3?
- Your friend missed the class when ratios were introduced. Use an example and draw a diagram to explain the difference between a part-to-part ratio and a part-to-whole ratio.
- Give two examples of how ratios are used in daily life. Share your ideas with a classmate.
- The fraction $\frac{2}{5}$ can be interpreted as two parts out of a total of five parts. Use a diagram to show an example of this part-to-whole ratio.



Check Your Understanding

Practise

For help with #5 to #8, refer to Example 1 on pages 47–48.

- Write each ratio using ratio notation. Do not write the answers in lowest terms.
 - \$2 compared to \$8.
 - The width of the cover of this book compared to its length, in centimetres.
 - In a class, 14 of 30 students are girls. What is the ratio of boys to girls to total students?
 - Your age compared to that of a 28-year-old person.
- Write each ratio in #5 as an equivalent ratio in lowest terms.
- Write each ratio in fraction form. Do not write the answers in lowest terms.
 - You spend \$4 out of \$10.
 - A team won three games and lost six games. What is the ratio of games won to games played?
 - A bag contains 12 red and 3 blue beads. Compare blue beads to total beads.
 - A pond contains 27 guppies and 33 goldfish. What is the ratio of guppies to total fish?

Key Ideas

The Key Ideas summarize two-term and three-term ratios with emphasis on part-to-part and part-to-whole ratios. Have students prepare their own summary of the Key Ideas and record them in the notes on their chapter Foldable.

Communicate the Ideas

Have students work individually or in groups to answer the questions. In #1, students write a ratio in the correct order. In #2, students explain the difference between a part-to-part and a part-to-whole ratio. In #3, students give examples of ratios used in daily life. In #4, students draw a diagram to show an example of a part-to-whole ratio. Have students share their answers for #3 in a class discussion.

Meeting Student Needs

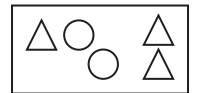
ELL

- Ensure students understand the term *units*.
- Help students learn the Key Words by using familiar examples. For example, students might use beads of different colours or shapes on beaded articles to express part-to-part and part-to-whole ratios.

Answers

Communicate the Ideas

- Janine needs to write the number of oranges first and the number of apples second in the ratio.
- Answers may vary. Example: There are three triangles and two circles on a mat.
 - A part-to-part ratio describes the number of circles to triangles. Example: 2:3.
 - A part-to-whole ratio describes the number of circles to all shapes. Example: 2:5.
- Answers may vary. Look for two examples like the following:
 - Ratios are used in recipes. Example: For salad dressing, use one part vinegar to two parts oil.
 - Ratios are used to describe populations. Example: A news article reports that two out of three Canadians are concerned about the environment.
- Answers may vary. Example: The forecast calls for thundershowers on two of the five days this work week.



Assessment as Learning

Communicate the Ideas

Have all students complete #1 and #2. Check their responses for conceptual understanding of ratios.

- Consider having students work in pairs or groups.
- Check each student's answers to #1 and #2. These are key questions; make sure students understand the concepts about ratios before proceeding.
- For #1, encourage students to use coloured counters to represent the ratio before using words and symbols to describe the ratio in the correct order. Have students identify each of the part-to-whole ratios.
- Some students may benefit from using an example from the classroom for #2. Have them complete #4 before going on.

- A three-term ratio compares three quantities measured in the same units.

The ratio of red to black to blue tiles can be written as 6:3:3 or 6 to 3 to 3. The ratio in lowest terms is 2:1:1 or 2 to 1 to 1.



- A two-term ratio compares two quantities measured in the same units.

The ratio of black to total tiles can be written as 3:12 or 3 to 12. The ratio in lowest terms is 1:4 or 1 to 4. One out of every four tiles is black.

Communicate the Ideas

- Janine wants to write the ratio of oranges to apples. How does she know whether to write 3:4 or 4:3?
- Your friend missed the class when ratios were introduced. Use an example and draw a diagram to explain the difference between a part-to-part ratio and a part-to-whole ratio.
- Give two examples of how ratios are used in daily life. Share your ideas with a classmate.
- The fraction $\frac{2}{5}$ can be interpreted as two parts out of a total of five parts. Use a diagram to show an example of this part-to-whole ratio.



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- Write each ratio in #5 as an equivalent ratio in lowest terms.
- Write each ratio in fraction form. Do not write the answers in lowest terms.
 - You spend \$4 out of \$10.
 - A team won three games and lost six games. What is the ratio of games won to games played?
 - A bag contains 12 red and 3 blue beads. Compare blue beads to total beads.
 - A pond contains 27 guppies and 33 goldfish. What is the ratio of guppies to total fish?

- Identify the missing number to make an equivalent fraction.

a) $\frac{1}{2} = \frac{\square}{8}$ b) $\frac{4}{5} = \frac{12}{\square}$
 c) $\frac{2}{7} = \frac{\square}{21}$ d) $\frac{\square}{4} = \frac{3}{12}$
 e) $\frac{21}{49} = \frac{3}{\square}$ f) $\frac{4}{\square} = \frac{12}{15}$

For help with #9 to #11, refer to Example 2 on pages 49–50.

- Use the data about wins and losses on school teams to answer the questions.

Sport	Wins	Losses
Hockey	9	6
Volleyball	10	5
Baseball	12	8

- Which sports have equivalent win-loss ratios? Show how you know.
 - What is the ratio of wins to total games played for hockey? Give your answer as a fraction, a decimal, and a percent.
- Tyler counted 20 cars in the school parking lot. Of these, 6 were red, 4 were green, and 1 was yellow.
 - Draw a diagram to represent the situation.
 - How many cars were not red, green, or yellow?
 - What is the ratio of yellow to green to red cars?
 - What is the ratio of red to total cars? Express the ratio as a fraction and a percent.
 - What tiles could be represented by each of the following ratios?
 - 1 to 5
 - 1:6:5
 - $\frac{1}{2}$
 - $\frac{11}{12}$



Apply

- In a class of 32 students, there are 24 girls.
 - What is the boys to total students ratio? Express the ratio as a fraction and a percent.
 - What is the girls to boys ratio? Use ratio notation to express the ratio.
- A soccer team played 28 games and won 4 out of every 7 games. There were no tied games.
 - How many games did they lose?
 - What was the team's win-loss ratio? Explain how you got your answer.
 - If this trend continues, how many losses would you expect the team to have once they have won 20 games?
- Three eighths of the 96 adults in the McGregor clan are less than 150 cm tall.
 - Draw a diagram to represent the statement.
 - How many adults in the McGregor clan are less than 150 cm tall? Show your work using equivalent ratios.
 - How many adults are 150 cm or taller? Explain your thinking.
- Diana and John are making three-cheese lasagna. The recipe calls for 100 g of Romano, 300 g of mozzarella, and 250 g of cottage cheese.
 - Write a ratio in lowest terms to compare the amounts of the three cheeses. State the order of the cheeses.
 - What amounts of Romano and cottage cheese do you need to make lasagna that contains 900 g of mozzarella cheese? Hint: Use equivalent ratios to help you.

Check Your Understanding

Practise

Question 5 emphasizes part-to-part ratios, while #7 emphasizes part-to-whole ratios. Students could be given a choice between #9 and #10. Question 9 relates to wins and losses in sports while #10 relates to the colour of cars.

Apply

The Apply questions provide a wide range of contexts in which ratios are used. The most straightforward applications are found in #12, #16, #18, and #19. Students need to use equivalent ratios to find the solutions to #13, #14, #16, and #18b). For #19, consider

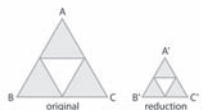
allowing students to use a calculator for the entire question. Question 20 involves a three-term ratio.

Literacy Link Direct students to the Literacy Link for #16 that describes *A prime*.

Extend

Students are introduced to the golden ratio in #21. In #22, the concept of slope is introduced and students predict what happens to slope as the rise or run is increased or decreased. In #22, consider allowing students to use a drawing program such as *The Geometer's Sketchpad*® to test their predictions.

16. Heather used a copier to make the following 50% reduction.



- a) Write the ratio of the length of $A'B'$ compared to the length of AB .

- b) Write the ratio of the length of $A'C'$ compared to the length of AC .

- c) Use your knowledge of ratio and percent to explain the meaning of a 50% reduction.

Library Link
A' is read as 'A prime'. A' labels the point in the reduction that corresponds to point A.

17. There are 48 passengers on a transit bus. At the next stop, 16 passengers got off and 12 others got on the bus.

- a) What is the ratio of the passengers who got off the bus compared to the original number on the bus? Show the ratio in lowest terms.

- b) What is the ratio of the passengers who got on the bus at the stop compared to the new total then on the bus? Write your answer as a fraction, a decimal, and a percent.

18. The ratio of the width to the length of the Canadian flag is 1:2.



- a) The flag on the cover of an atlas is 12 cm wide. How long is it?

- b) A large flag outside a Calgary school is 3 m long. What is its width?

WWW Web Link

To practise solving ratio problems, go to www.mathlinks.ca and follow the links.

19. The table gives the lengths of some rivers in Western Canada.

River	Length (km)
Churchill	1608
Fraser	1368
MacKenzie	1800
North Saskatchewan	1392
Thelon	904

- a) Write a ratio in fractional form to compare the length of the Churchill River and the Mackenzie River. Express the ratio as a fraction in lowest terms.

- b) Write a ratio in decimal form to compare the length of the North Saskatchewan River and the Fraser River. Express the ratio as a decimal correct to the nearest hundredth.

- c) The calculator keying sequence that compares the length of the Thelon River and the Churchill River is $\boxed{904} \div \boxed{1608} = 0.5621890547$. Write the decimal to the nearest hundredth. What comparison statement can you make about the length of the two rivers?

20. A 30-kg bag of fertilizer is labelled 15–20–10. This means that it contains 15% nitrogen, 20% phosphorus, and 10% potassium by weight. How many kilograms of nitrogen, phosphorus, and potassium are in the bag?

Did You Know?

Fertilizer ratios indicate the percent of nitrogen, phosphorus, and potassium. This bag of fertilizer contains 15% nitrogen, 20% phosphorus, and 10% potassium by weight. The remaining 55% is made up of other micronutrients and filler.



2.1 Two-Term and Three-Term Ratios • MHR 53

Extend

21. A golden rectangle has a length to width ratio called the golden ratio, which is approximately 1.62.

- a) Which of the following dimensions of rectangles are examples of golden rectangles?

- $24 \text{ m} \times 38.9 \text{ m}$
- $52 \text{ cm} \times 120.5 \text{ cm}$
- $348 \text{ mm} \times 565 \text{ mm}$

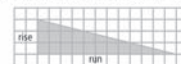
- b) If the width of a golden rectangle is 6.4 m, what is its length? Give your answer to the nearest tenth of a metre.

Did You Know?

The golden rectangle is used often in art and architecture. For example, the front of the Parthenon, a temple in Athens, Greece fits into a golden rectangle.



22. The side view of a ramp is shown.



- a) Express the ratio of rise to run in lowest terms. This ratio describes the slope of the ramp.

- b) Express the slope ratio as a fraction, a decimal, and a percent.

- c) Predict what effect each of the following would have on the slope of the ramp:

- increasing the rise
- decreasing the rise
- increasing the run
- decreasing the run

Did You Know?

The ratio $\frac{\text{rise}}{\text{run}}$ is called slope. Slope describes the steepness of roads, ramps, and ski runs.

WWW Web Link

For more information about the golden ratio in nature, architecture, art, poetry, and music, go to www.mathlinks.ca and follow the links.

MATH LINK

Plan an invitation for your international meal. Create a logo as part of the front of the invitation. A logo is an identifying symbol used in advertising.

- a) Design your logo using colours or measurements to show each of the following ratios

4:3 2:3:4

For example, if you use a rectangle in your logo, you could show that the length to width ratio is 4:3.

- b) Draw the logo on a 36 cm² section of centimetre grid paper.

- c) Identify the ratios used in your logo.

54 MHR • Chapter 2

Math Link

The Math Link provides students with an opportunity to apply their understanding of ratio by creating a logo. Remind them that they need the logo to help them complete the Wrap It Up! at the end of the chapter. Emphasize using each of the following ratios in the logo design: 4:3 and 2:3:4. Have students use coloured pencils and grid paper to create a logo appropriate for the invitation to the international meal (e.g., food theme). The logo can be as simple or complex as students desire. You may wish to have students share their logos with the class and identify the ratios.

Meeting Student Needs

- Providing students with choices can help improve their attitude to mathematics. Consider letting them choose to do #9 or #10, and possibly any five questions from #11 to #19.
- For #8, consider providing **Master 19 Multiplication Chart** to students who may benefit from using a multiplication table.
- Provide **BLM 2–5 Section 2.1 Extra Practice** to students who would benefit from more practice.

ELL

- English language learners may have difficulty with terms such as *won*, *lost*, *guppies*, *equivalent*, and *McGregor clan*. Have students add new terms to their dictionary.
- For #10b), clarify what the negative means. Explain that the question does not ask how many cars are red, white, or yellow, but rather how many cars have a colour different from red, white, or yellow.

Gifted and Enrichment

- Challenge students to prepare a proportional drawing of their home using a ratio of 1:20. Have them add the actual dimensions as a check for accuracy.
- Have students research and report on examples of the golden ratio found in nature, architecture, art, poetry, and music. Encourage them to use visuals as part of their report. Have students present their findings in a format of their choice.

Common Errors

- For #12, students may give the ratio of girls (instead of boys) to total number of students.

R_x Have students explicitly state each of the parts and the whole and the value for each. Doing so will reduce the chances for misinterpretation.



For activities in a bakery that may help students with proportions for their logo in the Math Link, go to www.mathlinks8.ca and follow the links.

Assessment	Supporting Learning
Assessment for Learning	
<p>Practise and Apply Have students do #5, #6, #9, #11, and #12. Students who have no problems with these questions can go on to the rest of the Apply questions.</p>	<ul style="list-style-type: none"> • Provide additional coaching with Example 1 to students who need help with #5 and #6. Have students write each of the parts and the whole for #5a) to c), and then clarify any misunderstandings. For #6, you might have students state out loud the multiplier or divisor needed to make each equivalent fraction. Coach students through #5 and #6, then have them complete #7 and #8 on their own. Check back with them several times to make sure that they understand the concepts. • Provide additional coaching with Example 2 to students who need help with #9. Help reactivate students' skills in reducing fractions to lowest terms. Coach students through #9, and then assign #10. Check back with them several times to make sure that they understand the concepts. • Help students with #11 by referring them to the Show You Know on page 48 and the Key Ideas in section 2.1. • For #12a) and b), have students write the parts and the whole for each.
<p>Math Link The Math Link on page 54 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 73.</p>	<ul style="list-style-type: none"> • Have students look for logos using various sources such as school materials, team logos, and print sources, such as magazines. Consider having students choose one sample logo and discuss how ratios were used in the logo. • You may wish to have students use different colours to help them visualize the ratios. • Students who need help getting started could use BLM 2–6 Section 2.1 Math Link, which provides scaffolding.
Assessment as Learning	
<p>Math Learning Log Have students answer the following questions:</p> <ul style="list-style-type: none"> • Give two examples of how you use ratios. • Think of a context and use it to show an example of a part-to-part ratio and a part-to-whole ratio. 	<ul style="list-style-type: none"> • As a class, discuss where ratios are used in daily life. • Encourage concrete and kinesthetic learners to use manipulatives, and then draw diagrams to show their part-to-part and part-to-whole ratios. • Depending on students' learning style, have them provide oral or written answers. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.