# Rates

# 2.2

#### MathLinks 8, pages 55-62

# Suggested Timing

80–100 minutes

#### Materials

- ruler
- standard paper clips
- jumbo paper clips
- flyers for products showing unit pricing information
- calculator (optional)

#### **Blackline Masters**

Master 17 Frayer Model BLM 2–3 Chapter 2 Warm-Up BLM 2–7 Compare a Ratio and a Rate BLM 2–8 Section 2.2 Extra Practice BLM 2–9 Section 2.2 Math Link

#### **Mathematical Processes**

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)

- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- 🖌 Technology (T)
- Visualization (V)

#### **Specific Outcomes**

N4 Demonstrate an understanding of ratio and rate.N5 Solve problems that involve rates, ratios and proportional reasoning.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 7, 8, 10, 13, Math Link
Typical	1–4, 6, 7, 8, 10–15, 16, Math Link
Extension/Enrichment	1, 2, 3c), 14–19

# **Planning Notes**

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce material learned in previous sections.



As a class, read and discuss the information about rates. Have students examine the photo and discuss similarities and differences between taking a horse's heart rate and taking their own heart rate. Ask them to explain how heart rate is different from a ratio. Students may say that although ratios and rates both compare quantities, a ratio uses the same units whereas a rate uses different units. Ask them to share what they know about other rates and the units used to measure them (e.g., speed, postage, currency exchange, interest).

**Literacy Link** Have students use **Master 17 Frayer Model** to develop a Frayer model showing what they already know about rates at the beginning of section 2.2. You may wish to have them revisit their Frayer models at the end of the section.



# Explore the Math

In this exploration, students compare measurements made using different units and then determine the conversion rate.

Students should work in pairs. Provide each pair of students with standard and jumbo paper clips to make chains. The length of chain will depend on the number of paper clips available. Shorter chains mean that students will be able to measure only smaller objects.

Encourage students to measure a number of objects, including large and small ones. As students work, you may wish to guide those who need it using questions such as the following:

- What will you measure first?
- What units of measure are you using?
- How will you record the measurement?
- How might you organize your data?
- Now that you have data on a number of objects, how might you use that data to help you determine a multiplier that could describe the number of standard clips to the number of jumbo clips it takes to measure each object?

- What term could you use for this multiplier? (You may want to provide a sample for struggling students. For example, you could work together to develop a conversion rate for centimetres to large paper clips, which is 4.7:1, then have them use that skill to develop the conversion rate for small paper clips to large paper clips.)
- Compare your conversion rate with the conversion rate of another group. Explain any similarities or differences.
- Do you think that the conversion rate between these two paper clips (hold the ones you have been working with) will always be the same? Explain your thinking.

Encourage students to share the different strategies they have used to develop a conversion rate. Discuss as a class what other areas of math have helped individuals and pairs develop conversion strategies.

# **Example 1**

Example 1 illustrates the process of determining unit rates by estimating and then calculating to the nearest hundredth the speed of a hummingbird and a butterfly. If students use a calculator, make sure they understand that giving an answer to the nearest hundredth requires rounding to two decimal places.



# Example 2

Example 2 illustrates the use of unit rates to compare the prices of different-sized containers of juice. It involves finding three unit prices and then comparing them to find the best buy. One container is measured in litres and the others are measured in millilitres. Make sure students understand that all of the units must be the same in order to compare rates. In this example, rate can be expressed in dollars/millilitre, cents/millilitre, dollars/litre, or cents/litre. The solution uses dollars/millilitre. Explain that stores often express unit prices per 100 mL or per 100 g. These units are more familiar to consumers and using them eliminates small decimal values. You might consider showing some samples of unit price labels frohm food products and having students discuss how unit prices make it easier for consumers to compare costs.

# **Meeting Student Needs**

• Help students understand heart rate by having them participate in an activity to measure their own heart rate at rest, during exercise, and during recovery. Have students measure their heart rate at rest by placing their second and third fingers on the artery just inside the wrist bone of the other hand, counting their heart beats in 15 seconds, and then multiplying the number of beats by four. Next, have students do some exercise (e.g., walking up and down stairs) and measure their heart rate after a few minutes. Have them take their heart rate once more two to ten minutes after exercise. Ask volunteers to compare their heart rates.

- Have students compare fuel efficiency rates for different models of snowmobiles, motorbikes, cars, or trucks.
- Work with students who have organizational problems. Have them consider different strategies for organizing data, choose one, and then explain how they could use that strategy. If they use a table, ask them what headings they might need across the top and down the side.
- Assist students in connecting their understanding of monetary conversions to how a conversion rate might work with measurement. For example, using money manipulatives and their prior knowledge, ask students to record the number of quarters in a dollar as a rate. ( $\frac{4 \ quarters}{1 \ dollar}$  or  $4 \ quarters : 1 \ dollar$ ) How could they use the same idea to show the number of standard clips to jumbo clips it takes to measure something? ( $\frac{\# \ standard \ clips}{\# \ jumbo \ clips}$  or  $\# \ standard$ clips :  $\# \ jumbo \ clips$ ) Encourage students to use this idea to develop a conversion rate for the measurements they have done.

# ELL

- English language learners may have difficulty with terms such as *trainers*, *accurately*, *reliably*, *monitor*, *heart rate*, *equine competitor*, *evaluate*, *at rest*, *during exercise*, *during recovery*, *beats per minute*, *growth rates*, and *fuel efficiency rates*. Have students add new terms to their dictionary.
- In the Explore the Math, point out the rows, columns, and headings in the table. Have students record these terms on their copy of the table.
  - For #4, make sure that students understand the term *convert*. Use a visual showing that  $\frac{5}{10}$  is 0.5.
  - For #6a), help students recall the terms denominator and numerator. Consider using the following mnemonic to help students remember the term. In Korea, the character for denominator means parent and the character for numerator means child. The parent holds up the child.
- For Example 2, explain that 0.408¢/ mL means that each millilitre of orange juice costs 0.408¢.

#### **Gifted and Enrichment**

- Challenge students to suggest what objects they might use to measure the length and width of the room. For example, they could use the length of their student resource for one unit and the width of their student resource as a second unit. Have them develop conversion rates using these new objects. You might also challenge them to develop conversion rates between the paper clips and these objects. Discuss why they would not want to use paper clips for such a large measurement.
- You may wish to challenge students to research conversion rates used in other sources. Have students share their findings with the class.
- Provide practice determining unit rates by having students research a local or national long-distance running event, and compare the performance of winners from different years. Have them compare the distance travelled to the time it takes to complete the race.
- Challenge students to determine the fuel economy for a family vehicle. This activity may require some planning at the beginning of the chapter. After obtaining parental permission, have students prepare and use a log such as the one that follows to track fuel purchases, kilometres driven, and fuel economy over several weeks.

Amount of Gas Purchased (L)	Beginning Odometer Reading (km)	Ending Odometer Reading (km)	Total Km Travelled	Fuel Efficiency

# **Common Errors**

- Students may reverse the order of the units in a rate by dividing the numerator into the denominator.
- $\mathbf{R}_{\mathbf{x}}$  Use examples to reinforce that the rate for buying an item compares cost to the amount of an item (e.g., 1.69/100 g). Similarly, speed is a rate that compares the distance travelled to the time it takes (e.g., 50 km/h).
- · Some students may have difficulty rounding answers when using a calculator.
- $\mathbf{R}_{\mathbf{x}}$  Show students how to use a place 2743.819 value chart labelled from thousands to thousandths. Have them practise rounding answers to the nearest tenth, hundredth, and thousandth.

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- Students may not round answers to the correct decimal place.
- $\mathbf{R}_{\mathbf{x}}$  Post a place value chart in the classroom. Reinforce that giving an answer to the nearest tenth means rounding to one decimal place, while giving an answer to the nearest hundredth means rounding to two decimal places.

#### **Answers**

#### **Explore the Math**

- 1. Answers will vary. See #3 for sample measurements.
- 2. jumbo paper clips, standard paper clips
- 3. Answers will vary. Example:
  - Use ratios to compare the number of standard clips and jumbo clips for each object.
  - · Convert each fraction to a decimal.
  - Compare the decimal values.
  - The conversion rate is about 1.5.

Students may develop a table to organize results. Example:

Object	Length in Standard Paper Clips	Length in Jumbo Paper Clips	Standard Clips Jumbo Clips	Decimal to the Nearest Tenth
Calculator	5.5	3.5	$\frac{5.5}{3.5}$	1.6
Notebook	9	6	$\frac{9}{6}$	1.5
Pen	4.5	3	$\frac{4.5}{3}$	1.5
Disk case	4	3	$\frac{4}{3}$	1.3
Tissue box	6.5	4.5	$\frac{6.5}{4.5}$	1.4
Book	8.5	5.5	<u>8.5</u> 5.5	1.5

- 4. a) A conversion rate is sometimes called a unit rate because it tells what factor to multiply by to get one unit in another measure.
  - b) The conversion rate would be less than one, because the conversion would be a reduction. The conversion rate for a reduction is less than one.
  - c) Yes, because the clips are standard lengths

#### Show You Know: Example 1

a) 6 m/s b) \$11/h c) \$1.80/can

#### Show You Know: Example 2

Small size: 3.44/425 mL = 0.81 c/mLMedium size: 6.29/642 mL = 0.98 ¢/mLLarge size: 15.49/1700 mL = 0.91 c/mLThe small size costs the least per millilitre. Therefore, it is the best buy.

Assessment	Supporting Learning
Assessment <i>as</i> Learning	
<b>Literacy Link</b> At the beginning of section 2.2, have students work in pairs to develop a Frayer model on rates.	<ul> <li>Use students' Frayer models to identify their misconceptions about rates. Address those misconceptions as you work through the chapter.</li> <li>At the end of section 2.2, have students revisit their Frayer model and make additions and improvements.</li> </ul>
<b>Reflect on Your Findings</b> Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings. Check their responses for understanding of unit rate. You might ask the following question: Would the conversion rate for the number of jumbo clips for one very small clip be greater or less than the rate in #4b)?	<ul> <li>Encourage students to model the situation using actual paper clips. Show that it takes about eight standard clips to equal the length of five jumbo clips.</li> <li>Clarify the terms <i>greater than</i> and <i>less than</i>.</li> <li>Have students use the class responses as springboards to prepare their own answers.</li> </ul>
Assessment for Learning	
<b>Example 1</b> Have students do the Show You Know related to Example 1.	<ul> <li>Encourage students to verbalize their thinking.</li> <li>You may wish to have students work with a partner.</li> <li>Even though students are not asked to estimate, it is excellent practice to encourage them to estimate before calculating each unit rate. Have them check the estimate against the calculated answer for reasonableness.</li> <li>Have students who are using a calculator show their keying sequences.</li> <li>Refer students to Example 1, which provides a similar question pattern.</li> <li>Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.</li> </ul>
Example 2 Have students do the Show You Know related to Example 2.	<ul> <li>Encourage students to verbalize their thinking.</li> <li>You may wish to have students work with a partner.</li> <li>Remind students that the numbers must be in the same units in order to compare prices. Ask what the common units might be for the containers of salsa.</li> <li>Encourage students to show their thinking.</li> <li>Have students who are using a calculator show their keying sequences for at least one of the calculations.</li> <li>Show samples or a flyer of various similar products with unit pricing information. Or, have students bring in samples of unit pricing information and use the information to provide additional questions.</li> <li>Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.</li> </ul>



# Key Ideas

The Key Ideas summarize rates with emphasis on unit rates and unit prices. Emphasize the difference between ratios and rates; namely, rates are measured in different units and ratios are measured in the same units. Have students prepare their own summary of the Key Ideas and record them in the notes on their chapter Foldable.

# **Communicate the Ideas**

These questions allow students to apply their understanding of ratios and rates. Have students work individually to answer the questions. In #1, students identify examples of ratios and rates from the data in a table. In #2, students explain how to find the unit price for different brands of dog food. In #3, students give examples of rates used in daily life.

**Literacy Link** Identifying similarities and differences is an effective strategy for learning. Developing a comparison using a graphic organizer, such as a double bubble organizer, provides students with a visual and memory aid. You may wish to complete an overhead copy of **BLM 2–7 Compare a Ratio and a Rate** as a class.

# **Meeting Student Needs**

• Double bubble organizers are useful for visual learners. For #1, have students fill out **BLM 2–7 Compare a Ratio and a Rate** on their own.

#### **Answers**

#### Communicate the Ideas

- **1.** a) Answers may vary. Example: The birth mass of black bears to polar bears is 0.3:0.7.
  - **b)** Answers may vary. A rate is a comparison of two quantities measured in different units. Example: The growth rate of black bears is 6.5 kg/60 days.
  - c) Answers may vary. Example: The unit rate for black bears is 0.108 kg/day.
- 2. For Brand A, divide \$13.60 by 8 = \$1.70/can. For brand B, divide \$8.75 by 5 = \$1.75/can. Assuming both products are the same quality and quantity, the can with the least price per can is the best buy. In this case, it would be Brand A.
- **3.** a) Answers may vary. Look for two rates. Examples:
  speed limit of 60 km/h
  temperature change of +2 °C/h
  - b) Answers may vary. Example: The units are km, h, and °C.
  - c) A rate cannot be expressed as a percent because a percent is a ratio that compares the same units and a rate compares two quantities expressed in two different units.

Assessment	Supporting Learning
Assessment as Learning	
<b>Communicate the Ideas</b> Have all students complete #1 and #2. As a class, have students share their responses to #1. Have students share their responses to #2 and #3 with a partner and listen to each other's explanations. Use student responses to assess their understanding of rates.	<ul> <li>Consider having students work in groups or pairs.</li> <li>Check each student's answers to #1 and #2. These are key questions; make sure students understand the concepts related to rates before proceeding.</li> <li>Remind students who need help with #1 that ratios compare quantities measured in the same units. For example, the mass of black bears and of polar bears can be compared at birth or after 60 days. Rates compare quantities measured in different units. For example, technically, the growth rate of the black bear is 6.5 kg/60 days. For more practice, have students record a different example of a ratio and rate using the same data.</li> </ul>

- a) Give two examples of rates that are common in every day life Share your examples with a classmate.
- b) What units measure each of the rates in part a)?
- c) Explain why a rate cannot be expressed as a percent.

#### Check Your Understanding

#### Practise

- For help with #4 to #6, refer to Example 1 on pages 56–57.
- 4. Determine the unit rate in each situation.a) An orca swims 110 km in 2 h.
- b) A Canada goose flies 800 km in 12.5 h.c) Cathy plants 45 daffodils in 30 min.
- c) Cathy plants 45 datfodils in 30 min.
- 5. What is the unit rate in each?a) A blue whale eats 8 t of krill in 2 days
- b) The cruising speed of a blue whale allows it to travel 193 km in 10 h.
- c) A bull moose bellows 15 times in  $2\frac{1}{2}$  h.
- 6. Gina earns \$78.00 for working 6 h. Asad makes \$192.50 after working 14 h. Determine each person's unit rate of pay. Who has a greater hourly rate of pay?
- For help with #7 to #9, refer to Example 2 on pages 57–58.
- The table shows the price of different-sized packages of mixed nuts.

Nut Package	Mass	Price
1	300 g	\$2.19
2	500 g	\$3.09
3	700 g	\$4.83

each package?

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What is the unit price per 100 mL for the l-L carton?
What is the unit price per 100 mL for the l-L carton?
What is the unit price per 100 mL for the l-L carton?
Which carton of milk is the best buy? Explain why.
Mala is shopping for honey. Her favourite brand is available in two sizes.

b) Which package is the best buy?

8. Fraser is shopping for milk. It is available

Explain your choice.

52.79 52.59 a) Estimate which is the better buy. Show your thinking.

b) Determine the better buy. Show your work

# Check Your Understanding

# **Practise**

All of these questions involve determining unit rates or unit prices. Note that #4 and #5 are similar questions. Consider assigning one question initially. Assign the second question to students who would benefit from extra practice.

# Apply

These questions provide a range of contexts in which rates are used. Since each question involves a different context, consider giving students some choice in the questions they do. Consider assigning two questions and then allowing students to choose two additional questions. This allows students to take responsibility and select problems that are of personal interest.

# Extend

In #17, students are introduced to rotation rates for planets. In #18, they use conversion rates for exchanging currency. Although many students may solve #18 on their own, you might consider having them work with a partner on this question. In #19, they convert speed from km/h to metres per second.



# **Math Link**

The Math Link allows students to apply their understanding of ratios and rates by calculating the quantity of ingredients needed to serve ten people using a recipe that serves four people.

# **Meeting Student Needs**

- Have students who need practice with rates and ratios complete the Math Link.
- Provide **BLM 2–8 Section 2.2 Extra Practice** to students who would benefit from more practice.

#### ELL

- For #12, use the picture of the glacier and the following terms to help students understand the problem: *melt, receded, annual rate.*
- Consider assigning fewer Apply questions to English language learners as they may struggle with the vocabulary in these questions.



Assessment	Supporting Learning
Assessment for Learning	
<b>Practise</b> Have students do #4, #6, #7, and #8. Students who have no problems with these questions can go on to the Apply questions.	<ul> <li>Provide additional coaching with Example 1 to students who need help with #4 and #6. For #4, have them explain the meaning of unit rate and identify the units for each part of the question. Coach students through corrections to their answers, and then have them complete #5 on their own.</li> <li>Provide additional coaching with Example 2 to students who need help with #7 and #8. Coach them through corrections to their answers, and then have them complete #9 on their own.</li> <li>Check back with students several times to make sure that they understand the concepts.</li> </ul>
Math Link The Math Link on page 62 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 73.	<ul> <li>Ask students how much rice would be needed to make kheer for 8 people, 16 people, and 24 people, then have them use the numbers in the pattern to determine the amount of rice needed for 20 people (i.e., halfway between 16 people and 24 people).</li> <li>Students who need help getting started can use BLM 2–9 Section 2.2 Math Link, which provides scaffolding.</li> </ul>
Assessment as Learning	
<ul> <li>Math Learning Log</li> <li>Have students complete the following statements:</li> <li>The similarities and differences between a ratio and a rate are</li> <li>What I find difficult about ratios and rates is</li> </ul>	<ul> <li>Consider having students work in groups and to present their ideas using a format of their choice (e.g., oral report, written response, poster, role play).</li> <li>Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.</li> </ul>