Proportional Reasoning

2.3

***MathLinks 8*, pages 63–69**

Suggested Timing
80–100 minutes

Materials
- ruler
- computer access
- short story
- stopwatch
- sticky notes
- access to recipes in print material and/or on the Internet

Blackline Masters
Master 2 Two Stars and One Wish
Master 17 Frayer Model
Master 19 Multiplication Chart
BLM 2–3 Chapter 2 Warm-Up
BLM 2–10 Section 2.3 Extra Practice
BLM 2–11 Section 2.3 Math Link

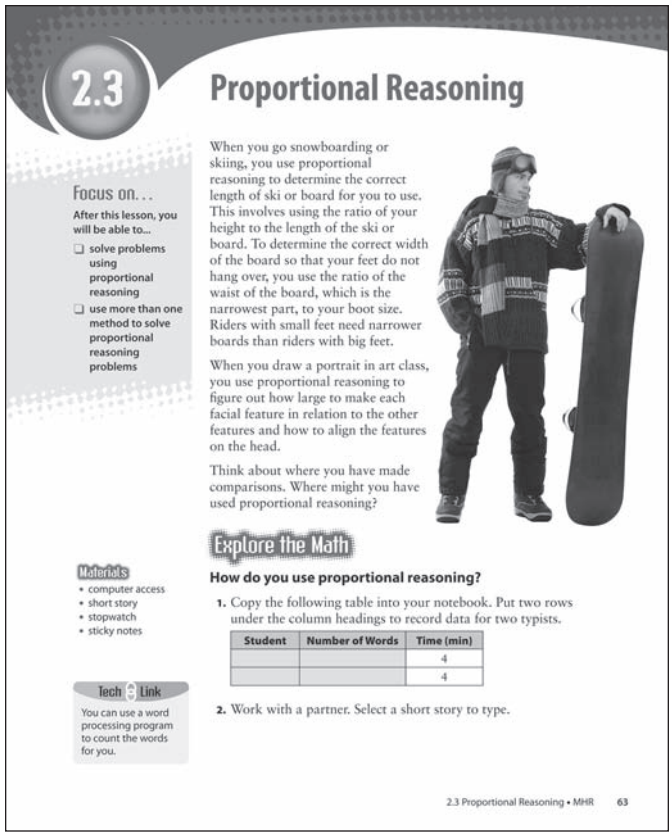
Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☐ Visualization (V)

Specific Outcomes
N5 Solve problems that involve rates, ratios and proportional reasoning.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 3, 4, 6, 8, 10, 11, 14, 15, 18, Math Link
Typical	1, 3, 4, 6, 8, 10, 11, 14, 15–18, 21–23, 25, Math Link
Extension/Enrichment	1, 3, 25–28, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce material learned in previous sections.



2.3 Proportional Reasoning

When you go snowboarding or skiing, you use proportional reasoning to determine the correct length of ski or board for you to use. This involves using the ratio of your height to the length of the ski or board. To determine the correct width of the board so that your feet do not hang over, you use the ratio of the waist of the board, which is the narrowest part, to your boot size. Riders with small feet need narrower boards than riders with big feet.

When you draw a portrait in art class, you use proportional reasoning to figure out how large to make each facial feature in relation to the other features and how to align the features on the head.

Think about where you have made comparisons. Where might you have used proportional reasoning?

Focus on...
After this lesson, you will be able to...
- solve problems using proportional reasoning
- use more than one method to solve proportional reasoning problems

Explore the Math

Materials
- computer access
- short story
- stopwatch
- sticky notes

How do you use proportional reasoning?

1. Copy the following table into your notebook. Put two rows under the column headings to record data for two typists.

Student	Number of Words	Time (min)
		4
		4

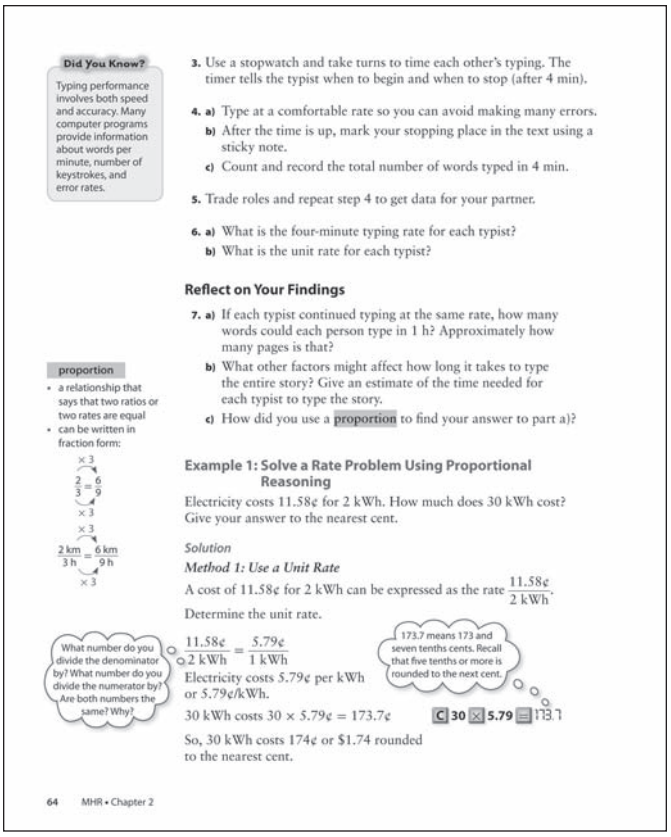
Tech Link
You can use a word processing program to count the words for you.

2. Work with a partner. Select a short story to type.

2.3 Proportional Reasoning • MHR 63

As a class, read and discuss the examples of how people use proportion. Invite volunteers to describe skiing and snowboarding for students who may not be familiar with these activities. Have students share how they use proportional reasoning in daily life. Consider recording their responses on the chalkboard. Note that it is important for students to develop a thorough understanding of proportional reasoning rather than simply applying an algorithm or a specific strategy to solve a problem.

Literacy Link Have students use **Master 17 Frayer Model** to develop a Frayer model showing what they already know about proportions at the beginning of section 2.3. You may wish to have them revisit their Frayer models at the end of the section.



Did You Know?

Typing performance involves both speed and accuracy. Many computer programs provide information about words per minute, number of keystrokes, and error rates.

3. Use a stopwatch and take turns to time each other's typing. The timer tells the typist when to begin and when to stop (after 4 min).

4. a) Type at a comfortable rate so you can avoid making many errors.
   b) After the time is up, mark your stopping place in the text using a sticky note.
   c) Count and record the total number of words typed in 4 min.

5. Trade roles and repeat step 4 to get data for your partner.

6. a) What is the four-minute typing rate for each typist?
   b) What is the unit rate for each typist?

Reflect on Your Findings

7. a) If each typist continued typing at the same rate, how many words could each person type in 1 h? Approximately how many pages is that?
   b) What other factors might affect how long it takes to type the entire story? Give an estimate of the time needed for each typist to type the story.
   c) How did you use a proportion to find your answer to part a)?

proportion
- a relationship that says that two ratios or two rates are equal
- can be written in fraction form:

$\frac{2}{3} = \frac{6}{9}$ (×3)

$\frac{2\text{ km}}{3\text{ h}} = \frac{6\text{ km}}{9\text{ h}}$ (×3)

Example 1: Solve a Rate Problem Using Proportional Reasoning

Electricity costs 11.58¢ for 2 kWh. How much does 30 kWh cost? Give your answer to the nearest cent.

Solution

Method 1: Use a Unit Rate

A cost of 11.58¢ for 2 kWh can be expressed as the rate $\frac{11.58¢}{2\text{ kWh}}$.

Determine the unit rate.

What number do you divide the denominator by? What number do you divide the numerator by? Are both numbers the same? Why?

$\frac{11.58¢}{2\text{ kWh}} = \frac{5.79¢}{1\text{ kWh}}$

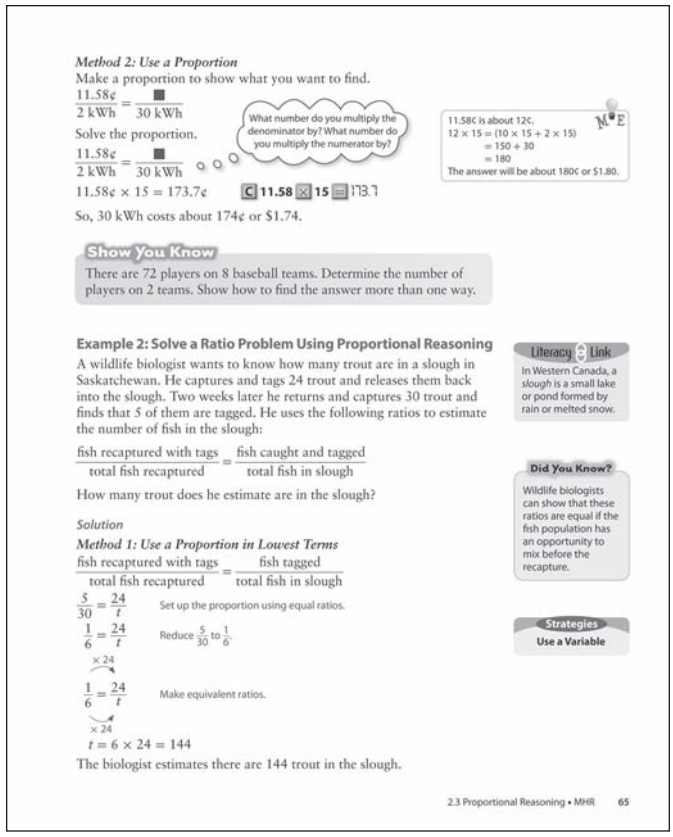
173.7 means 173 and seven tenths cents. Recall that five tenths or more is rounded to the next cent.

Electricity costs 5.79¢ per kWh or 5.79¢/kWh.

30 kWh costs $30 \times 5.79¢ = 173.7¢$ C 30 × 5.79 = 173.7

So, 30 kWh costs 174¢ or $1.74 rounded to the nearest cent.

64 MHR • Chapter 2



Method 2: Use a Proportion

Make a proportion to show what you want to find.

$\frac{11.58¢}{2\text{ kWh}} = \frac{■}{30\text{ kWh}}$

What number do you multiply the denominator by? What number do you multiply the numerator by?

Solve the proportion.

$\frac{11.58¢}{2\text{ kWh}} = \frac{■}{30\text{ kWh}}$

$11.58¢ \times 15 = 173.7¢$ C 11.58 × 15 = 173.7

So, 30 kWh costs about 174¢ or $1.74.

11.58¢ is about 12¢.
$12 \times 15 = (10 \times 15 + 2 \times 15)$
$= 150 + 30$
$= 180$
The answer will be about 180¢ or $1.80.

Show You Know

There are 72 players on 8 baseball teams. Determine the number of players on 2 teams. Show how to find the answer more than one way.

Example 2: Solve a Ratio Problem Using Proportional Reasoning

A wildlife biologist wants to know how many trout are in a slough in Saskatchewan. He captures and tags 24 trout and releases them back into the slough. Two weeks later he returns and captures 30 trout and finds that 5 of them are tagged. He uses the following ratios to estimate the number of fish in the slough:

$\frac{\text{fish recaptured with tags}}{\text{total fish recaptured}} = \frac{\text{fish caught and tagged}}{\text{total fish in slough}}$

How many trout does he estimate are in the slough?

Literacy Link
In Western Canada, a *slough* is a small lake or pond formed by rain or melted snow.

Did You Know?
Wildlife biologists can show that these ratios are equal if the fish population has an opportunity to mix before the recapture.

Solution

Method 1: Use a Proportion in Lowest Terms

$\frac{\text{fish recaptured with tags}}{\text{total fish recaptured}} = \frac{\text{fish tagged}}{\text{total fish in slough}}$

$\frac{5}{30} = \frac{24}{t}$ Set up the proportion using equal ratios.

$\frac{1}{6} = \frac{24}{t}$ Reduce $\frac{5}{30}$ to $\frac{1}{6}$.

(×24)

$\frac{1}{6} = \frac{24}{t}$ Make equivalent ratios.

(×24)

$t = 6 \times 24 = 144$

The biologist estimates there are 144 trout in the slough.

Strategies
Use a Variable

2.3 Proportional Reasoning • MHR 65

Explore the Math

In this exploration, students compare typing rates as an introduction to proportional reasoning.

Method 1 Students need to work in pairs. Gather the materials in advance. Stopwatches may be available from the science or physical education department. You may need to book a lab for computers with access to a word-processing program for the typing activity. If computers are not available, consider adapting the activity by having students compare rates for bouncing a tennis ball or walking a certain distance. Have students discuss their findings as a class.

You might explain how to use a word processor to count words. If students are using Microsoft® Word, they can go to Tools, and select Word Count. If they are using Corel® WordPerfect they can go to File, and look under Document Information.

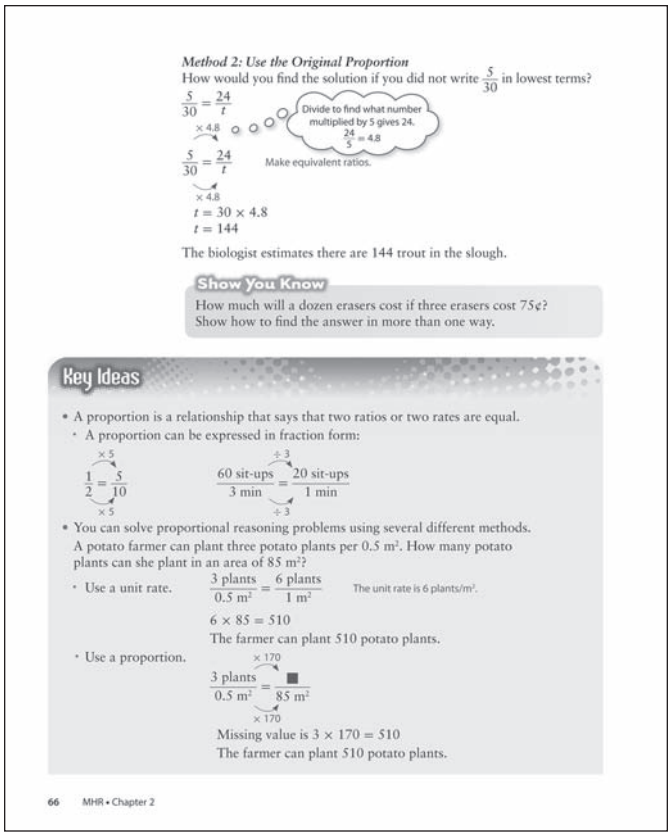
Method 2 If computers are in short supply, consider using two computers and asking for two volunteers to type a short story. Students could use the data collected to complete the activity.

Example 1

Example 1 demonstrates two methods for solving a rate problem using proportional reasoning. Method 1 uses a unit rate that helps students make a connection with the work on rates in section 2.2. Method 2 sets up a proportion and then solves it.

Have students consider the unit rate in Method 1 and discuss how it works. The questions in the thought bubble attached to the rate are intended to assist students in developing some metacognitive thoughts to assist them with using this strategy. Have students consider and discuss these questions. Encourage students to connect solving rates to other types of calculations they have done by having them consider:

- When have you seen this type of operation before?
- What is different about the unit rate you are seeing here?
- What is the same?



Method 2: Use the Original Proportion

How would you find the solution if you did not write $\frac{5}{30}$ in lowest terms?

$\frac{5}{30} = \frac{24}{t}$ (×4.8)

Divide to find what number multiplied by 5 gives 24. $\frac{24}{5} = 4.8$

$\frac{5}{30} = \frac{24}{t}$ (×4.8)

Make equivalent ratios.

$t = 30 \times 4.8$

$t = 144$

The biologist estimates there are 144 trout in the slough.

Show You Know

How much will a dozen erasers cost if three erasers cost 75¢? Show how to find the answer in more than one way.

Key Ideas

- A proportion is a relationship that says that two ratios or two rates are equal.
  - A proportion can be expressed in fraction form:

$\frac{1}{2} = \frac{5}{10}$ (×5, ×5)

$\frac{60 \text{ sit-ups}}{3 \text{ min}} = \frac{20 \text{ sit-ups}}{1 \text{ min}}$ (÷3, ÷3)

- You can solve proportional reasoning problems using several different methods.

A potato farmer can plant three potato plants per 0.5 m². How many potato plants can she plant in an area of 85 m²?

- Use a unit rate. $\frac{3 \text{ plants}}{0.5 \text{ m}^2} = \frac{6 \text{ plants}}{1 \text{ m}^2}$ The unit rate is 6 plants/m².

$6 \times 85 = 510$

The farmer can plant 510 potato plants.

- Use a proportion. $\frac{3 \text{ plants}}{0.5 \text{ m}^2} = \frac{\blacksquare}{85 \text{ m}^2}$ (×170, ×170)

Missing value is $3 \times 170 = 510$

The farmer can plant 510 potato plants.

66 MHR • Chapter 2

Method 2 uses a proportion. Encourage students to connect solving proportions to other types of calculations they have done by having them consider:

- How is this proportion different from the unit rate used in Method 1?
- How is it similar?
- When else have you seen a proportion before?
- What is different about the proportion you are seeing here?
- What is the same?

You might find it helpful to show students variations of the given proportion that would work equally well. For example, they could also solve this question using the following proportion: $\frac{30 \text{ kWh}}{2 \text{ kWh}} = \frac{?}{11.58¢}$.

Show this proportion to students and discuss:

- How does this proportion show the same information as the one in the student resource?
- Why is it different from the one in Method 2?
- How might you solve this proportion? (Some students will use their knowledge of improper fractions, realize that 30 kWh/2 kWh represents 15, and will multiply 11.58¢ by 15. Encourage this type of connection.)
- Did this proportion work as well as the one in Method 2? Justify your response.
- What other proportions could you make to solve this question?
- Which proportion do you find easiest to use? Explain your thinking.

Challenge students to suggest another strategy they could use to solve this Example. You may wish to have students demonstrate other strategies for the class.

Ask students to show two different ways to solve the Show You Know. Challenge them to use a proportion (or a different proportion) to verify their answer. Ask:

- Can you use more than one proportion to answer the Show You Know? Why or why not?
- Which proportion do you prefer? Why?
- What are the advantages and disadvantages of using a unit rate on a question such as this?
- What are the advantages and disadvantages of using a proportion?

Example 2

Example 2 illustrates an application of proportions from wildlife biology. Techniques similar to the capture-tag-recapture of trout highlighted in this example are used in counting wild animals or birds. The example demonstrates two methods for solving the proportion. Method 1 uses a proportion in lowest terms. Method 2 uses the original proportion. You might find it helpful to show students variations of the given proportion that would work equally well.

As in Example 1, have students consider the two methods and how they work.

- What method do individuals prefer? Why?
- What alternative strategy could students use to solve this problem?

Encourage students to use a strategy they feel comfortable with to solve the Show You Know, then to compare their answer with someone who used a different strategy. Students who have different answers could work together to consider what happened.

- Was there a difficulty with the strategy?
- Did they use the strategy incorrectly?
- Was the strategy used correctly but a minor calculation error affected the final answer?

Literacy Link Direct students to the Literacy Link on page 65 that describes the meaning of *slough*.

Meeting Student Needs

- Give students extra time and a ruler to copy the table in the Explore the Math. Alternatively, consider allowing them to use a computer to create the table.
- Consider using simpler number values in Example 1 for students who are having difficulty. For example, electricity costs 10¢ for 2 kWh. How much does 30 kWh cost?
- When discussing Method 1 in Example 1, you may need to coach students who have difficulty with decimals. Ask them to consider what they would divide 2 kWh in the first rate by to get 1 kWh in the second rate. Have them make the connection with equivalent fractions and ask them what they would have to divide 11.58¢ by to get 5.79¢. Have them check their answers with a calculator. Once they recognize that they need to divide by 2, ask them to explain in their own words how a rate works. You may wish to have them write this reminder in their chapter Foldable.
- You may need to repeat the above coaching for Example 1, Method 2, which has an empty box in the numerator of the second rate. Ask students how they can figure out what belongs in the empty box. Have them test their suggestions.
- Some students may need additional reinforcement to process the information about using proportional reasoning to solve problems. Consider completing a second problem using a familiar context. For Example 2, you might have a wildlife biologist count caribou, whales, or polar bears. Biologists tag these animals to monitor their populations.

ELL

- English language learners may have difficulty with the following terms: *narrowest part*, *portrait*, *facial feature*, and *typist*. Have them add new terms to their dictionary.

Common Errors

- Some students may have difficulty setting up a proportion.

R_x Have students identify each ratio or rate provided in a question. Then, have them set up one ratio or rate on one side of a proportion before setting up the other side of the proportion in the same order. One of the quantities will likely be unknown.

Answers

Explore the Math

1.–5. Answers may vary. Example:

Student	Number of Words	Time (min)
Thea	92	4
Jessie	128	4

6. a) Answers may vary. Example: Thea: 92 words per 4 min; Jessie: 128 words per 4 min

b) Thea: 23 words per min; Jessie: 32 words per min

7. a) Answers may vary. Assuming 250 words per page, Thea: 1380 words per h or 5.5 pages per h; Jessie: 1920 words per h or close to 8 pages per h

b) Answers may vary. Look for at least one factor. Examples:
- speeding up or slowing down depending on skill or fatigue
- text containing special characters or numbers that take longer to type
- length of words
- number of errors

Estimates will vary. Example: The story would take 3.6 h for Thea and 2.6 h for Jessie to type, assuming the story was 5000 words in length.

c) Answers may vary. Example: $\frac{23 \text{ words}}{1 \text{ min}} = \frac{x \text{ words}}{60 \text{ min}}$ for Thea; $\frac{32 \text{ words}}{1 \text{ min}} = \frac{y}{60 \text{ min}}$ for Jessie

Show You Know: Example 1

Method 1: Unit Rate: $\frac{72 \text{ players}}{8 \text{ teams}} = \frac{9 \text{ players}}{1 \text{ team}}$, then multiply

$9 \times 2 = 18$ players per 2 teams

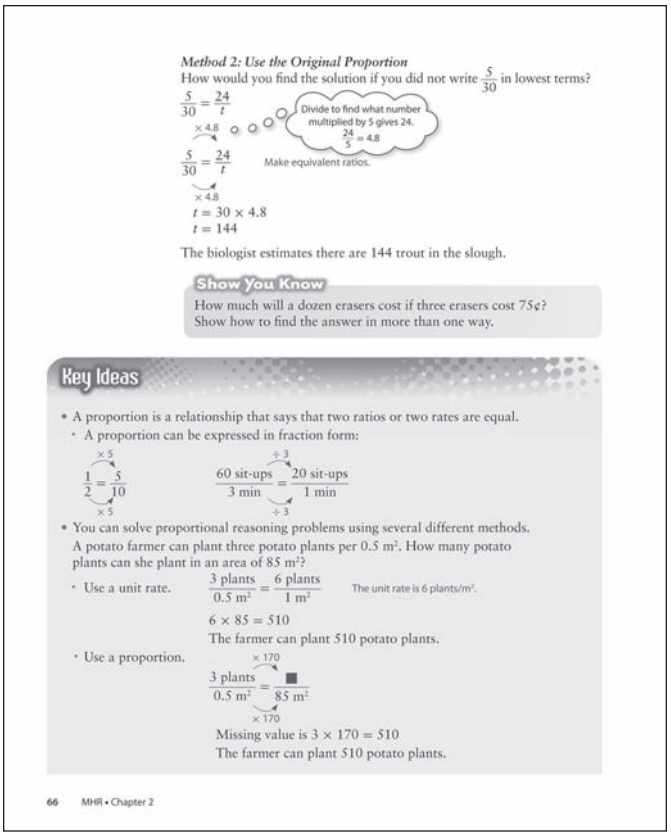
Method 2: Proportion: $\frac{72 \text{ players}}{8 \text{ teams}} = \frac{x \text{ players}}{2 \text{ teams}}$, $x = 18$ players per 2 teams

Show You Know: Example 2

Method 1: Unit Rate: $\frac{75¢}{3 \text{ erasers}} = \frac{25¢}{1 \text{ eraser}}$; 12 erasers cost \$3.00

Method 2: Proportion: $\frac{75¢}{3 \text{ erasers}} = \frac{x¢}{12 \text{ erasers}}$; $x = 300¢$ or \$3.00

Assessment	Supporting Learning
Assessment *as* Learning	
Literacy Link At the beginning of section 2.3, have students work in pairs to develop a Frayer model on proportions.	• Use the students' Frayer models to identify their misconceptions about proportions. Address those misconceptions as you work through the chapter. • At the end of section 2.3, have students revisit their Frayer models and make additions and improvements.
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students share their thinking and generalize how to solve a proportion.	• For #7a), tell students that 250 words per page is considered standard. • As a class, briefly discuss #7b) for a broader range of factors affecting typing rate.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ask students to suggest other ways to solve the problem. For instance, they could multiply by 0.5 to determine the unit rate. • Refer students to Example 1, which models a similar question pattern. • Coach students who need assistance by asking: – What units are provided in the question? – What unit are you looking for in the answer? • Provide **Master 19 Multiplication Chart** to students who need multiplication tables. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage concrete and kinesthetic learners to draw a diagram of three erasers and divide them by 75¢ before setting up a proportion. • Encourage students to think of the given proportion as part of an equation. Have them substitute the given values into the equation. Remind students that one of the four values in the proportion is unknown and that solving for the unknown value will answer the question. • Coach students who need assistance by asking them: – What units are provided in the question? – What unit are you looking for in the answer? • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.



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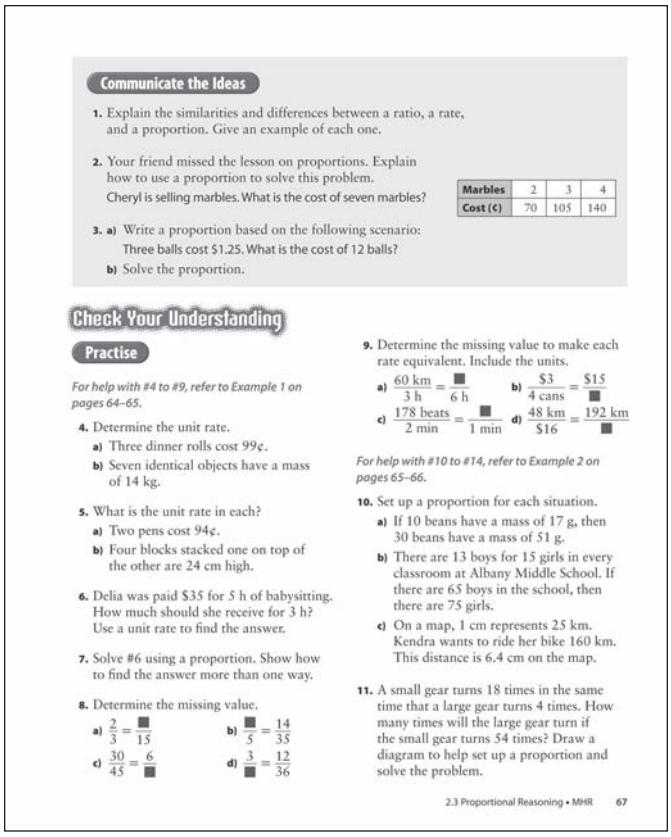
66 MHR • Chapter 2



Communicate the Ideas

1. Explain the similarities and differences between a ratio, a rate, and a proportion. Give an example of each one.
2. Your friend missed the lesson on proportions. Explain how to use a proportion to solve this problem.

Cheryl is selling marbles. What is the cost of seven marbles?

Marbles	2	3	4
Cost (¢)	70	105	140

3. a) Write a proportion based on the following scenario: Three balls cost $1.25. What is the cost of 12 balls?
   b) Solve the proportion.

Check Your Understanding

Practise

For help with #4 to #9, refer to Example 1 on pages 64–65.

4. Determine the unit rate.
   a) Three dinner rolls cost 99¢.
   b) Seven identical objects have a mass of 14 kg.
5. What is the unit rate in each?
   a) Two pens cost 94¢.
   b) Four blocks stacked one on top of the other are 24 cm high.
6. Delia was paid $35 for 5 h of babysitting. How much should she receive for 3 h? Use a unit rate to find the answer.
7. Solve #6 using a proportion. Show how to find the answer more than one way.
8. Determine the missing value.
   a) $\frac{2}{3} = \frac{\blacksquare}{15}$
   b) $\frac{\blacksquare}{5} = \frac{14}{35}$
   c) $\frac{30}{45} = \frac{6}{\blacksquare}$
   d) $\frac{3}{\blacksquare} = \frac{12}{36}$
9. Determine the missing value to make each rate equivalent. Include the units.
   a) $\frac{60 \text{ km}}{3 \text{ h}} = \frac{\blacksquare}{6 \text{ h}}$
   b) $\frac{\$3}{4 \text{ cans}} = \frac{\$15}{\blacksquare}$
   c) $\frac{178 \text{ beats}}{2 \text{ min}} = \frac{\blacksquare}{1 \text{ min}}$
   d) $\frac{48 \text{ km}}{\$16} = \frac{192 \text{ km}}{\blacksquare}$

For help with #10 to #14, refer to Example 2 on pages 65–66.

10. Set up a proportion for each situation.
    a) If 10 beans have a mass of 17 g, then 30 beans have a mass of 51 g.
    b) There are 13 boys for 15 girls in every classroom at Albany Middle School. If there are 65 boys in the school, then there are 75 girls.
    c) On a map, 1 cm represents 25 km. Kendra wants to ride her bike 160 km. This distance is 6.4 cm on the map.
11. A small gear turns 18 times in the same time that a large gear turns 4 times. How many times will the large gear turn if the small gear turns 54 times? Draw a diagram to help set up a proportion and solve the problem.

2.3 Proportional Reasoning • MHR 67

Key Ideas

The Key Ideas summarize information about proportions and the different methods for solving proportional reasoning problems. Have students prepare their own summary of the Key Ideas and record them in the notes on their chapter Foldable.

Communicate the Ideas

These questions allow students to express their understanding of proportional reasoning. Consider having students work in groups or pairs to answer and then discuss the questions. In #1, students differentiate between a ratio, a rate, and a proportion. In #2, students explain how to use a proportion to solve a problem. In #3, students set up and solve a proportion.

Meeting Student Needs

ELL

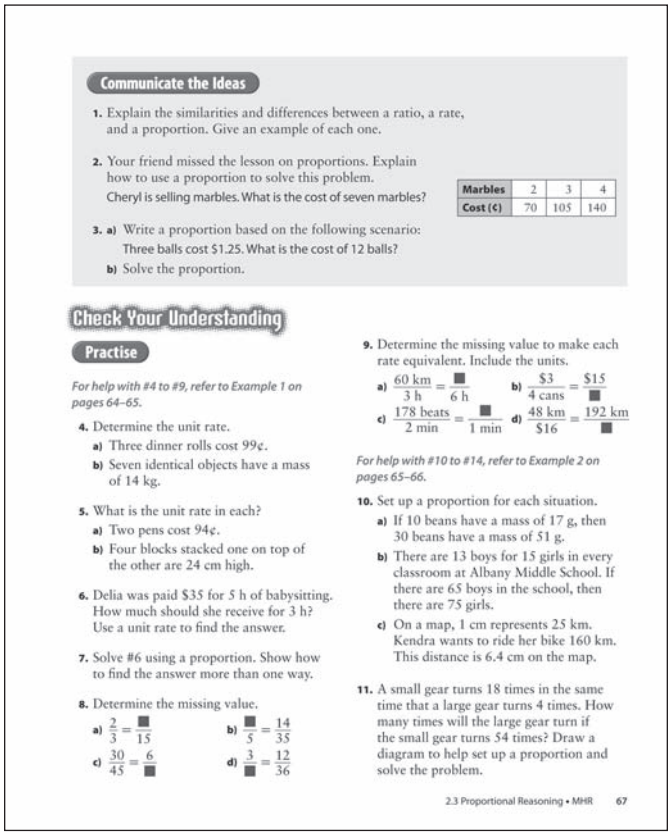
- Encourage students to share their understanding of the Key Ideas during group discussion.
- If several students share a common first language, consider having them discuss or write the answers to #1 and #2 in their first language. This offers students the opportunity to activate their knowledge using familiar language. After doing so, it may be easier for students to ask for the missing vocabulary to express their thinking in English.

Answers

Communicate the Ideas

1. Answers may vary. A ratio is a comparison of two or three quantities in the same units. Example: 20 cm : 25 cm. A rate is a comparison of two quantities measured in different units. Example: 23 m/5 s. A proportion is an equation with two ratios or two rates that are equal. Example: $\frac{\$2.80}{5 \text{ bars}} = \frac{\$x}{12 \text{ bars}}$.
2. Answers may vary. Using a proportion: $\frac{70¢}{2 \text{ marbles}} = \frac{x¢}{7 \text{ marbles}}$; therefore $x = 245¢$ or $2.45
3. a) $\frac{\$1.25}{3 \text{ balls}} = \frac{\$x}{12 \text{ balls}}$ b) $5.00

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and #3. Use student responses to assess their understanding of proportions. Encourage them to share their answer to #2 with a partner and listen to each other's explanation.	• Check each student's answers to #1 and #3. These are key questions; make sure students understand the concepts about proportions before proceeding. • As a class, discuss the response to #1 and have students who need help use the class responses as a springboard for their own answer to #3. • In #2, some students may benefit from using a different strategy such as extending the pattern in the table. • In #3, encourage visual learners to draw arrows and show the factor used to multiply (or divide) each term in the proportion. • Use **Master 2 Two Stars and One Wish** and have students critique other students' writing. This master allows students to write two things they like about a piece and one thing they would like to see improved.



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Check Your Understanding

Practise

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5. What is the unit rate in each?
a) Two pens cost 94¢.
b) Four blocks stacked one on top of the other are 24 cm high.

6. Delia was paid $35 for 5 h of babysitting. How much should she receive for 3 h? Use a unit rate to find the answer.

7. Solve #6 using a proportion. Show how to find the answer more than one way.

8. Determine the missing value.
a) $\frac{2}{3} = \frac{\blacksquare}{15}$ b) $\frac{\blacksquare}{5} = \frac{14}{35}$
c) $\frac{30}{45} = \frac{6}{\blacksquare}$ d) $\frac{3}{\blacksquare} = \frac{12}{36}$

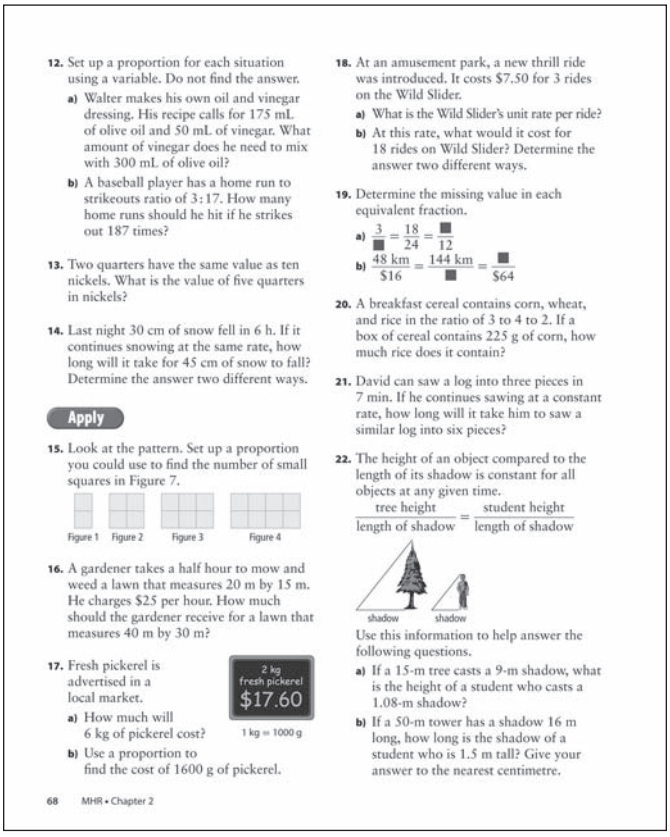
9. Determine the missing value to make each rate equivalent. Include the units.
a) $\frac{60\text{ km}}{3\text{ h}} = \frac{\blacksquare}{6\text{ h}}$ b) $\frac{\$3}{4\text{ cans}} = \frac{\$15}{\blacksquare}$
c) $\frac{178\text{ beats}}{2\text{ min}} = \frac{\blacksquare}{1\text{ min}}$ d) $\frac{48\text{ km}}{\$16} = \frac{192\text{ km}}{\blacksquare}$

For help with #10 to #14, refer to Example 2 on pages 65–66.

10. Set up a proportion for each situation.
a) If 10 beans have a mass of 17 g, then 30 beans have a mass of 51 g.
b) There are 13 boys for 15 girls in every classroom at Albany Middle School. If there are 65 boys in the school, then there are 75 girls.
c) On a map, 1 cm represents 25 km. Kendra wants to ride her bike 160 km. This distance is 6.4 cm on the map.

11. A small gear turns 18 times in the same time that a large gear turns 4 times. How many times will the large gear turn if the small gear turns 54 times? Draw a diagram to help set up a proportion and solve the problem.

2.3 Proportional Reasoning • MHR 67



12. Set up a proportion for each situation using a variable. Do not find the answer.
a) Walter makes his own oil and vinegar dressing. His recipe calls for 175 mL of olive oil and 50 mL of vinegar. What amount of vinegar does he need to mix with 300 mL of olive oil?
b) A baseball player has a home run to strikeouts ratio of 3:17. How many home runs should he hit if he strikes out 187 times?

13. Two quarters have the same value as ten nickels. What is the value of five quarters in nickels?

14. Last night 30 cm of snow fell in 6 h. If it continues snowing at the same rate, how long will it take for 45 cm of snow to fall? Determine the answer two different ways.

Apply

15. Look at the pattern. Set up a proportion you could use to find the number of small squares in Figure 7.

Figure 1 Figure 2 Figure 3 Figure 4

16. A gardener takes a half hour to mow and weed a lawn that measures 20 m by 15 m. He charges $25 per hour. How much should the gardener receive for a lawn that measures 40 m by 30 m?

17. Fresh pickerel is advertised in a local market.
a) How much will 6 kg of pickerel cost?
b) Use a proportion to find the cost of 1600 g of pickerel.

2 kg fresh pickerel $17.60
1 kg = 1000 g

18. At an amusement park, a new thrill ride was introduced. It costs $7.50 for 3 rides on the Wild Slider.
a) What is the Wild Slider's unit rate per ride?
b) At this rate, what would it cost for 18 rides on Wild Slider? Determine the answer two different ways.

19. Determine the missing value in each equivalent fraction.
a) $\frac{3}{\blacksquare} = \frac{18}{24} = \frac{\blacksquare}{12}$
b) $\frac{48\text{ km}}{\$16} = \frac{144\text{ km}}{\blacksquare} = \frac{\blacksquare}{\$64}$

20. A breakfast cereal contains corn, wheat, and rice in the ratio of 3 to 4 to 2. If a box of cereal contains 225 g of corn, how much rice does it contain?

21. David can saw a log into three pieces in 7 min. If he continues sawing at a constant rate, how long will it take him to saw a similar log into six pieces?

22. The height of an object compared to the length of its shadow is constant for all objects at any given time.
$\frac{\text{tree height}}{\text{length of shadow}} = \frac{\text{student height}}{\text{length of shadow}}$

shadow shadow

Use this information to help answer the following questions.
a) If a 15-m tree casts a 9-m shadow, what is the height of a student who casts a 1.08-m shadow?
b) If a 50-m tower has a shadow 16 m long, how long is the shadow of a student who is 1.5 m tall? Give your answer to the nearest centimetre.

68 MHR • Chapter 2

Check Your Understanding

Practise

Note that #4 and #5, #8 and #9, #10 and #12, and #13 and #14 are pairs of similar questions. Consider assigning or allowing students to choose one question from each pair initially. You might assign the second question from each pair to those who would benefit from extra practice.

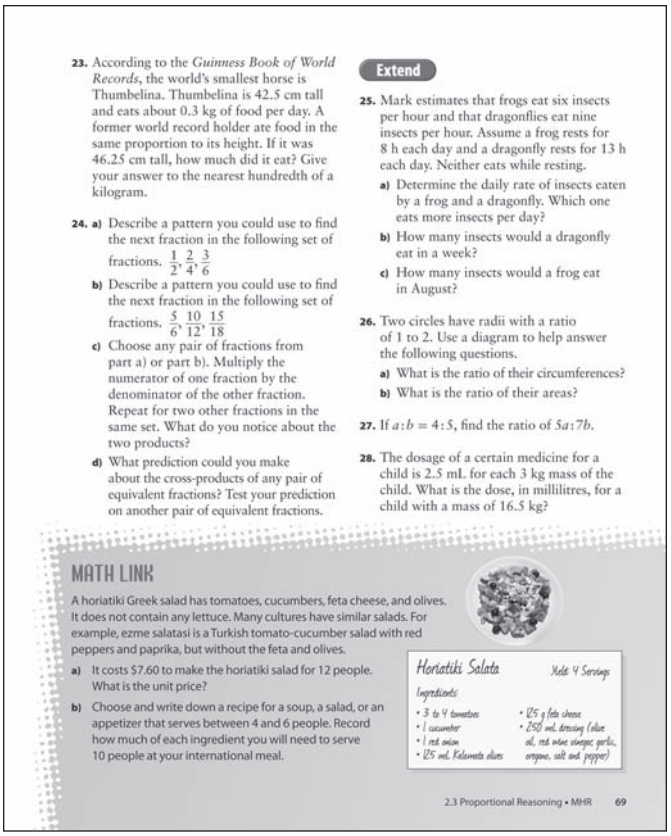
Apply

The Apply questions provide a range of contexts in which proportions can be used. Consider giving students some choice in the questions they do.

Encourage students to draw a diagram to help solve #15. Have them look at the pattern to determine the number of squares and then set up a proportion. In #21, students need to know that it takes five cuts to saw a log into six pieces. In #24, students explore patterns in different sets of fractions and make a prediction about the cross-products for pairs of equivalent fractions.

Extend

In #27, students find the ratio of $5a:7b$, given the proportion $a:b = 4:5$. You may need to remind them that $5a = 5 \times a$. This question allows students to solve the ratio using several methods after they interpret what they are being asked to find.



23. According to the *Guinness Book of World Records*, the world's smallest horse is Thumbelina. Thumbelina is 42.5 cm tall and eats about 0.3 kg of food per day. A former world record holder ate food in the same proportion to its height. If it was 46.25 cm tall, how much did it eat? Give your answer to the nearest hundredth of a kilogram.

24. a) Describe a pattern you could use to find the next fraction in the following set of fractions. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$

b) Describe a pattern you could use to find the next fraction in the following set of fractions. $\frac{5}{6}, \frac{10}{12}, \frac{15}{18}$

c) Choose any pair of fractions from part a) or part b). Multiply the numerator of one fraction by the denominator of the other fraction. Repeat for two other fractions in the same set. What do you notice about the two products?

d) What prediction could you make about the cross-products of any pair of equivalent fractions? Test your prediction on another pair of equivalent fractions.

Extend

25. Mark estimates that frogs eat six insects per hour and that dragonflies eat nine insects per hour. Assume a frog rests for 8 h each day and a dragonfly rests for 13 h each day. Neither eats while resting.

a) Determine the daily rate of insects eaten by a frog and a dragonfly. Which one eats more insects per day?

b) How many insects would a dragonfly eat in a week?

c) How many insects would a frog eat in August?

26. Two circles have radii with a ratio of 1 to 2. Use a diagram to help answer the following questions.

a) What is the ratio of their circumferences?

b) What is the ratio of their areas?

27. If $a:b = 4:5$, find the ratio of $5a:7b$.

28. The dosage of a certain medicine for a child is 2.5 mL for each 3 kg mass of the child. What is the dose, in millilitres, for a child with a mass of 16.5 kg?

MATH LINK

A horiatiki Greek salad has tomatoes, cucumbers, feta cheese, and olives. It does not contain any lettuce. Many cultures have similar salads. For example, ezme salatasi is a Turkish tomato-cucumber salad with red peppers and paprika, but without the feta and olives.

a) It costs $7.60 to make the horiatiki salad for 12 people. What is the unit price?

b) Choose and write down a recipe for a soup, a salad, or an appetizer that serves between 4 and 6 people. Record how much of each ingredient you will need to serve 10 people at your international meal.

Horiatiki Salata — Yield: 4 Servings

Ingredients:

- 3 to 4 tomatoes
- 1 cucumber
- 1 red onion
- 125 mL Kalamata olives
- 125 g feta cheese
- 250 mL dressing (olive oil, red wine vinegar, garlic, oregano, salt and pepper)

2.3 Proportional Reasoning • MHR 69

Math Link

The Math Link provides an opportunity for students to choose a recipe for a soup, a salad, or an appetizer and calculate the quantity of ingredients needed to serve ten people. Students could use the recipe to help them complete the Wrap It Up! at the end of the chapter. Provide or have students bring in recipe books from various cultures. Alternatively, consider allowing students to use the Internet to research recipes. If so, have them key in search terms such as the name of a culture (e.g., Polish) and "recipes."

Meeting Student Needs

- Consider allowing students to work in pairs. They might work on one question together and then work individually on the next one. Ensure that students complete a number of questions individually.
- Consider providing **Master 19 Multiplication Chart** to help students solve equivalent fractions.
- Provide **BLM 2–10 Section 2.3 Extra Practice** to students who would benefit from more practice.

ELL

- Explain *shadow* to English language learners who may not be familiar with the term.
- For #12b), explain the question orally and ensure that students understand the terms.
- For #21, ensure that students understand that saw refers to using a tool to cut wood.
- Encourage English language learners to record their recipe for the Math Link using their first language. After doing so, students might then find it easier to record the recipe in English.

Gifted and Enrichment

- Challenge students to research how proportional reasoning is used to make clothing. For example, students might research how seamstresses design and sew traditional clothing. See *Sinews of Survival: The Living Legacy of Inuit Clothing* by Betty Kobayashi Issenman (UBC Press, 1997).

Common Errors

- Students may set up a proportion incorrectly by inverting a ratio or rate on one side of the proportion.

R_x Encourage students to write each ratio or rate in words, and then substitute numerical values for the words. By writing the proportion in words, it should be apparent if the comparison or the units do not match on both sides of the proportion.

- Students may set up a proportion correctly but have difficulty solving it.

R_x Encourage students to solve it using a unit rate or using a proportion and equivalent fractions. If neither method works, consider showing them how to use cross multiplication.

Answers

Math Link

a) $0.63/person

b) Answers will vary.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise Have students do #4, #6, #8, #10, #11, and #14. Students who have no problems with these questions can go on to the Apply questions.	• Provide additional coaching with Example 1 to students who need help with #4, #6 and #8. Remind them that unit rate is a rate in which the second term is one. Have students explain their thinking on these questions; clarify any misunderstandings. Coach them through these questions. Then have students complete #5, #7, and #9 on their own. Check back with them several times to make sure that they understand the concepts. • Provide additional coaching with Example 2 to students who need help with #10, #11, and #14. Have them explain their thinking on these questions; clarify any misunderstandings. Coach them through #10 and #11. Then have students complete #12 and #13 on their own. Check back with them several times to make sure that they understand the concepts. • Encourage students to set up more than one proportion for #10 and #12. Reinforce the idea that more than one proportion can be recorded for a given question.
Math Link The Math Link on page 69 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 73.	• It is recommended that all students complete the Math Link. • Provide recipe books or have students bring in books from various cultures. Alternatively, consider allowing students to use the Internet to research recipes. • There are unlimited possibilities for recipes. Students might choose a recipe for a heritage food, such as bannock, a heritage food of some First Nations and of Scottish immigrants. • Invite students who are familiar with horiatiki to pronounce its name correctly and describe it. Horiatiki is pronounced *ho-RIA-ti-ki*. • Students who need help getting started could use **BLM 2–11 Section 2.3 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students complete the following statements: • Two methods that can be used to write and solve a proportion are … • What I find most confusing about proportions is …	• To help make the connection between ratios, rates, and proportions, discuss as a class how students use ratios and rates in daily life. • Depending on students' learning style, have them provide oral or written answers. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.