3 Pythagorean Relationship

General Outcomes

- Develop number sense.
- Use direct or indirect measurement to solve problems.

Specific Outcomes

N1 Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers).
N2 Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).
SS1 Develop and apply the Pythagorean theorem to solve problems.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
3.1	✓ identify a perfect square using square tiles and prime factorization
	✓ determine the square of a whole number
	✓ determine the factors of a perfect square
	✓ determine the square root of a perfect square
3.2	✓ model the Pythagorean relationship
	✓ describe how the Pythagorean relationship applies to right triangles
	✓ determine whether or not a triangle is a right triangle
3.3	✓ estimate the square root of a number that is not a perfect square
	✓ identify a number with a square root that is between two given numbers
	✓ explain that a square root on a calculator may be an approximation
3.4	✓ use the Pythagorean relationship to determine the missing side length of a right triangle
3.5	✓ apply the Pythagorean relationship to solve problems
	✓ use the Pythagorean relationship to determine distances between objects
	✓ verify that a triangle is a right triangle

Assessment	Supporting Learning
Assessment *for* Learning	
Method 1: Use the Math Link introduction on page 79 in *MathLinks 8* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about factors, perimeter and area of rectangles and triangles, comparing and ordering decimals, and divisibility rules.	• **BLM 3–1 Chapter 3 Math Link Introduction** provides scaffolding for the Math Link introduction. • Have students use the What I Need to Work On tab of their chapter Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete the Get Ready materials available on **BLM 3–2 Chapter 3 Get Ready**, in the *MathLinks 8 Practice and Homework Book*, and at the www.mathlinks8.ca book site.
Assessment *as* Learning	
Literacy Link (page 77) Have students develop a Verbal Visual Chart (VVC) for each Key Word listed on page 77. This can be done at the beginning of the chapter or as each term is introduced throughout the chapter.	• Encourage students to use the glossary starting on page 517 to help them. • Students who computerize their VVCs may wish to access the *MathLinks 8* online glossary by going to www.mathlinks8.ca and following the links. • Brainstorm ideas for the How I Will Remember It box for each term. Record the ideas on an overhead or chart paper. Encourage students to use these ideas as springboards for developing ideas of their own.
Chapter 3 Foldable As students work on each section in Chapter 3, have them keep track of any problems they are having on the What I Need to Work On tab.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled.
Assessment *for* Learning	
BLM 3–3 Chapter 3 Warm-Up This BLM includes five warm-ups, one to be used at the beginning of each section. Each warm-up provides cumulative review questions for the entire student resource to that point, as well as mental math practice.	• As students complete questions from previous chapters, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing mental math calculations.

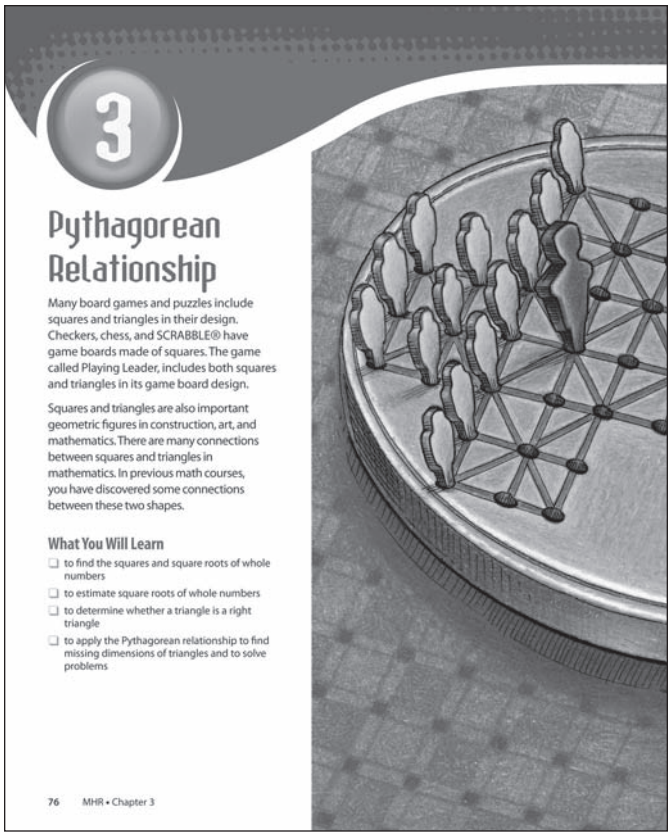
Problems of the Week

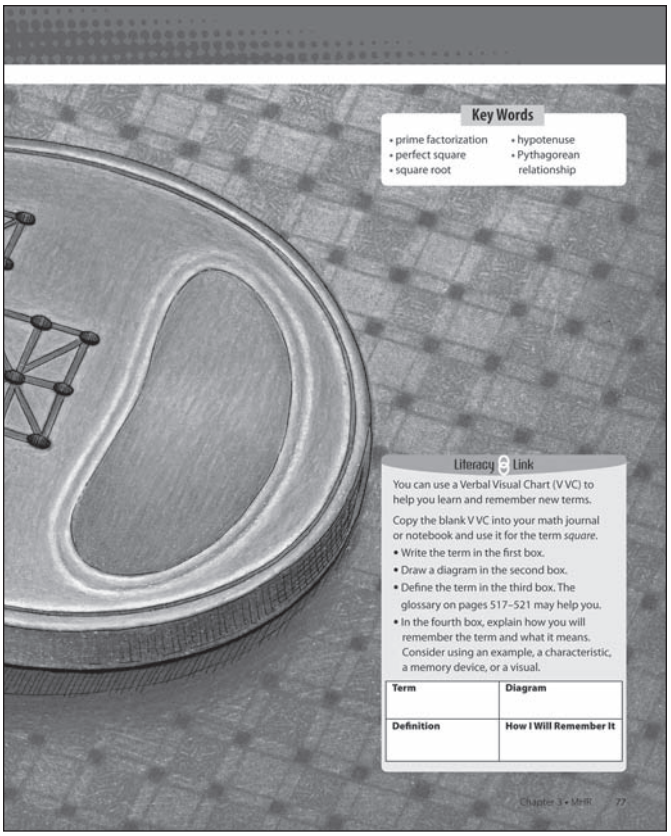
Have all students try at least one of the problems on **BLM 3–4 Chapter 3 Problems of the Week**. Many of these problems require students to think outside the box and experiment with a variety of approaches. Some have definitive answers; others can be answered in more than one way.

Students can take the problems home and consult with parents or guardians, work with other students when their work is completed, or try them on their own. The questions take a varying amount of time to solve, depending on the particular student and the problem itself. You may wish to give out these problems at the beginning of the chapter and discuss the solutions at appropriate times throughout your work on the chapter.

Chapter 3 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Extra Support	Assessment		
						Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter 3 Opener • 40–50 minutes (TR page 97)	Students should be familiar with • types of triangles • horizontal and vertical distances • calculating the area of a square, rectangle, and triangle	• eight sheets of grid paper • stapler • ruler • scissors • coloured counters	Master 9 0.5 Centimetre Grid Paper BLM 3–1 Chapter 3 Math Link Introduction BLM 3–2 Chapter 3 Get Ready BLM 3–4 Chapter 3 Problems of the Week BLM 3–5 Playing Leader Board		Online Learning Centre	TR page 96 Chapter 3 Foldable, TR page 96	TR page 96	
3.1 Squares and Square Roots • 80–100 minutes (TR page 101)	Students should be familiar with • factors, including prime numbers • applying divisibility rules • calculating the area of a square and a rectangle	• square tiles • grid paper • ruler	Master 2 Two Stars and One Wish Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper Master 19 Multiplication Chart BLM 3–3 Chapter 3 Warm-Up BLM 3–6 Section 3.1 Extra Practice BLM 3–7 Section 3.1 Math Link	**Essential:** 1–5, 7, 9, 11, 15, 17, Math Link **Typical:** 1–5, 7, 9, 11, 15, 17, 18, 23, 24, Math Link **Extension/Enrichment:** 1–4, 19, 21, 22, 24–27	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 105, 107 Math Learning Log, TR page 108 Chapter 3 Foldable, TR page 108 Master 2 Two Stars and One Wish	TR pages 105, 108	
3.2 Exploring the Pythagorean Relationship • 80–100 minutes (TR page 109)	Students should be familiar with • measuring angles • constructing triangles • rounding decimal numbers	• centimetre grid paper • scissors • transparent tape • ruler • protractor	Master 5 Tangram Master 8 Centimetre Grid Paper BLM 3–3 Chapter 3 Warm-Up BLM 3–8 Table for Explore the Math BLM 3–9 Triangles for Explore the Math BLM 3–10 Section 3.2 Extra Practice BLM 3–11 Section 3.2 Math Link	**Essential:** 1–3, 5, 6, 8, 10, 12a), c), Math Link **Typical:** 1–3, 5, 6, 8, 10, 12–17, Math Link **Extension/Enrichment:** 1–3, 14, 17, 18–21	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 112, 113 Math Learning Log, TR page 115 Chapter 3 Foldable, TR page 115	TR pages 112, 115	
3.3 Estimating Square Roots • 80–100 minutes (TR page 116)	Students should be familiar with • comparing and ordering decimal numbers • multiples	• ruler • calculator	Master 4 Vertical and Horizontal Number Lines Master 8 Centimetre Grid Paper Master 19 Multiplication Chart BLM 3–3 Chapter 3 Warm-Up BLM 3–12 Section 3.3 Extra Practice BLM 3–13 Section 3.3 Math Link	**Essential:** 1–4, 6, 10, Math Link **Typical:** 1–7, 9–11, 13–16, Math Link **Extension/Enrichment:** 1–3, 11, 15, 16, 19–21	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 119, 120 Math Learning Log, TR page 122 Chapter 3 Foldable, TR page 122	TR pages 119, 122	
3.4 Using the Pythagorean Relationship • 80–100 minutes (TR page 123)	Students should be familiar with • solving an equation • calculating the perimeter and area of triangles	• centimetre grid paper • ruler	Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper BLM 3–3 Chapter 3 Warm-Up BLM 3–14 Section 3.4 Extra Practice BLM 3–15 Section 3.4 Math Link	**Essential:** 1–3, 5, 6, 9, 10, Math Link **Typical:** 1–3, 5, 6, 9–14, Math Link **Extension/Enrichment:** 1–3, 10, 11, 13–16	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 125, 126 Math Learning Log, TR page 128 Chapter 3 Foldable, TR page 128	TR pages 125, 128	
3.5 Applying the Pythagorean Relationship • 80–100 minutes (TR page 129)	Students should be familiar with • adding and subtracting decimal numbers		BLM 3–3 Chapter 3 Warm-Up BLM 3–16 Section 3.5 Extra Practice BLM 3–17 Section 3.5 Math Link	**Essential:** 1–3, 5, 7, Math Link **Typical:** 1–3, 5, 7–10, 13, Math Link **Extension/Enrichment:** 1–3, 8, 11–14	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 131, 132 Math Learning Log, TR page 134 Chapter 3 Foldable, TR page 134	TR pages 131, 134	
Chapter 3 Review • 40–50 minutes (TR page 135)			BLM 3–7 Section 3.1 Extra Practice BLM 3–10 Section 3.2 Extra Practice BLM 3–12 Section 3.3 Extra Practice BLM 3–13 Section 3.4 Extra Practice BLM 3–14 Section 3.5 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	*MathLinks 8 Practice and Homework Book* *MathLinks 8 CAB*	Chapter 3 Foldable, TR page 136	TR page 136	
Chapter 3 Practice Test • 40–50 minutes (TR page 137)			BLM 3–18 Chapter 3 Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 3–5, 7, 8, 10a), 11, 13, 15	*MathLinks 8 CAB*	TR page 138		TR page 138 BLM 3–19 Chapter 3 Test
Chapter 3 Wrap It Up! • 80–100 minutes (TR page 139)		• poster board, cardboard • coloured pencils, coloured markers • scissors • glue • game pieces • ruler • protractor • grid paper • other materials for creating a board game	Master 1 Project Rubric Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper BLM 3–1 Chapter 3 Math Link Introduction BLM 3–8 Section 3.1 Math Link BLM 3–11 Section 3.2 Math Link BLM 3–13 Section 3.3 Math Link BLM 3–15 Section 3.4 Math Link BLM 3–17 Section 3.5 Math Link BLM 3–19 Chapter 3 Wrap It Up!		Online Learning Centre			TR page 140 Master 1 Project Rubric
Chapter 3 Math Games • 30–40 minutes (TR page 142)		• two dice per pair of students					TR page 142	
Chapter 3 Challenge in Real Life • 40–50 minutes (TR page 143)		• grid paper • ruler	Master 1 Project Rubric Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper BLM 3–20 Chapter 3 BLM Answers		Online Learning Centre		TR page 144	TR page 144 Master 1 Project Rubric





***MathLinks 8*, pages 76–79**

Suggested Timing
40–50 minutes

Materials
- eight sheets of grid paper
- stapler
- ruler
- scissors
- coloured counters

Blackline Masters
Master 9 0.5 Centimetre Grid Paper
BLM 3–1 Chapter 3 Math Link Introduction
BLM 3–2 Chapter 3 Get Ready
BLM 3–4 Chapter 3 Problems of the Week
BLM 3–5 Playing Leader Board

Key Words
prime factorization
perfect square
square root

What's the Math?

In this chapter, students learn how to identify and determine perfect squares. They determine the factors of a perfect square and also find the square root of a perfect square. They then explore estimating square roots, using technology and perfect square benchmarks. Next, they learn how to identify a number with a square root that is between two given numbers. Students also develop the Pythagorean relationship and explore how it applies to right triangles. Then, they apply the Pythagorean relationship to determine the missing side lengths of right triangles. Finally, they apply their knowledge of the Pythagorean relationship to real-life situations.

Planning Notes

Before starting Chapter 3, explain that students will be working with squares and triangles to determine important relationships between their dimensions. Ask students,

- Can you make a triangle from a square?
- Can you make a square from a triangle?
- What kinds of triangles can you create?
- What other types of triangles are there?

Literacy Link The Verbal Visual Chart (VVC) strategy is helpful for vocabulary development and may be used throughout this chapter as new vocabulary is introduced or old vocabulary reinforced. By creating a graphic organizer that contains each term and its definition, along with a visual and a personal association, students deepen their understanding of the essential characteristics of a concept.

At the beginning of Chapter 3, students use the term *square* to refer to a shape. In section 3.1, they learn a new meaning of the term *square*. Have students create a VVC for the term *square* at the beginning of the chapter. Allow time for students to share their ideas about this 2-D shape with the class. You may wish to brainstorm different associations for the term.

Once students have done the Explore the Math in section 3.1, have them develop a second VVC for *square* using the new definition they have learned. Identify the two words as synonyms. Also, have students develop a VVC for the term *square root*. Post examples of the three VVCs side by side, since these are words that some students may confuse.

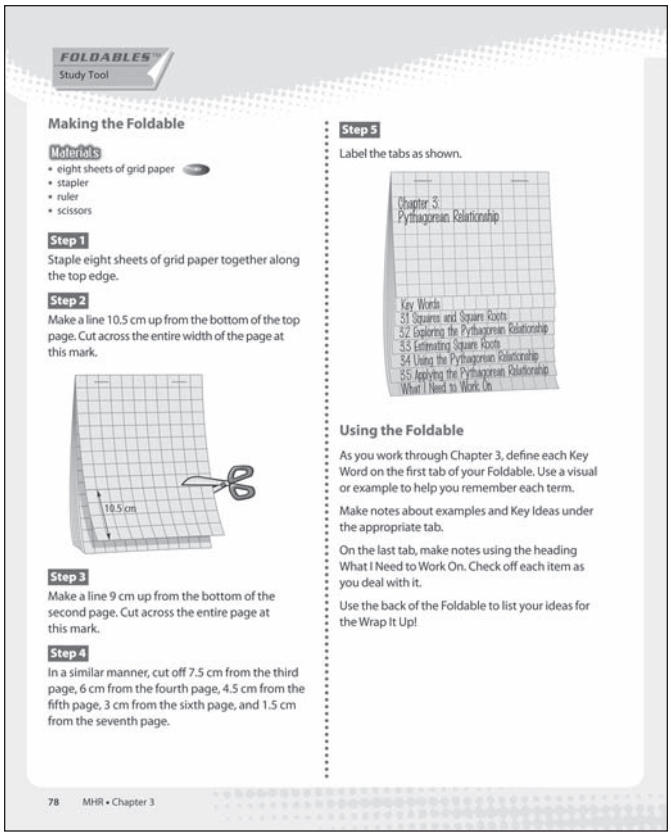
Work as a class to develop a memory device for remembering the difference between the terms *square* and *square root*.

Meeting Student Needs

- Have students use small cards for their VVCs. By making a VVC for each important word in the chapter, they can develop a small deck of cards they can use for review. Students might wish to make a duplicate set of cards that they cut into four pieces, shuffle with other cards from the chapter, and then reassemble to show that they remember the meaning, visual, and association for each term.
- Consider having students complete the questions on **BLM 3–2 Chapter 3 Get Ready** to activate the prerequisite skills for this chapter.

Gifted and Enrichment

- Introduce the term *mnemonic* as a synonym for *memory device*.



FOLDABLES™
Study Tool

Making the Foldable

Materials
- eight sheets of grid paper
- stapler
- ruler
- scissors

Step 1
Staple eight sheets of grid paper together along the top edge.

Step 2
Make a line 10.5 cm up from the bottom of the top page. Cut across the entire width of the page at this mark.

10.5 cm

Step 3
Make a line 9 cm up from the bottom of the second page. Cut across the entire page at this mark.

Step 4
In a similar manner, cut off 7.5 cm from the third page, 6 cm from the fourth page, 4.5 cm from the fifth page, 3 cm from the sixth page, and 1.5 cm from the seventh page.

Step 5
Label the tabs as shown.

Chapter 3: Pythagorean Relationship
Key Words
3.1 Squares and Square Roots
3.2 Exploring the Pythagorean Relationship
3.3 Estimating Square Roots
3.4 Using the Pythagorean Relationship
3.5 Applying the Pythagorean Relationship
What I Need to Work On

Using the Foldable

As you work through Chapter 3, define each Key Word on the first tab of your Foldable. Use a visual or example to help you remember each term.

Make notes about examples and Key Ideas under the appropriate tab.

On the last tab, make notes using the heading What I Need to Work On. Check off each item as you deal with it.

Use the back of the Foldable to list your ideas for the Wrap It Up!

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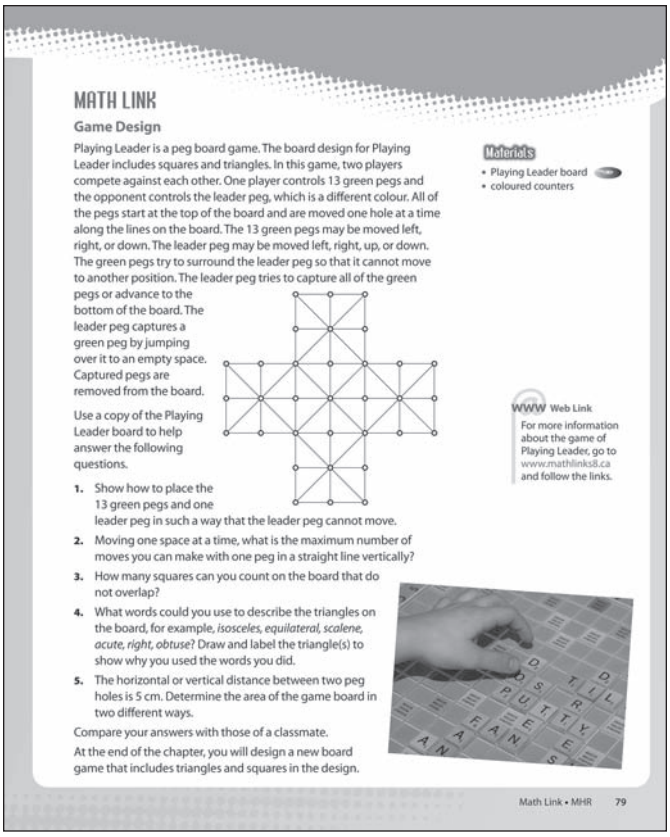


MATH LINK

Game Design

Playing Leader is a peg board game. The board design for Playing Leader includes squares and triangles. In this game, two players compete against each other. One player controls 13 green pegs and the opponent controls the leader peg, which is a different colour. All of the pegs start at the top of the board and are moved one hole at a time along the lines on the board. The 13 green pegs may be moved left, right, or down. The leader peg may be moved left, right, up, or down. The green pegs try to surround the leader peg so that it cannot move to another position. The leader peg tries to capture all of the green pegs or advance to the bottom of the board. The leader peg captures a green peg by jumping over it to an empty space. Captured pegs are removed from the board.

Materials
- Playing Leader board
- coloured counters

Use a copy of the Playing Leader board to help answer the following questions.

1. Show how to place the 13 green pegs and one leader peg in such a way that the leader peg cannot move.
2. Moving one space at a time, what is the maximum number of moves you can make with one peg in a straight line vertically?
3. How many squares can you count on the board that do not overlap?
4. What words could you use to describe the triangles on the board, for example, *isosceles, equilateral, scalene, acute, right, obtuse*? Draw and label the triangle(s) to show why you used the words you did.
5. The horizontal or vertical distance between two peg holes is 5 cm. Determine the area of the game board in two different ways.

Compare your answers with those of a classmate.

At the end of the chapter, you will design a new board game that includes triangles and squares in the design.

WWW Web Link
For more information about the game of Playing Leader, go to www.mathlinks8.ca and follow the links.

Math Link • MHR 79

Foldables Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter. Filling in the What I Need to Work On tab as they progress through the chapter will assist them in identifying and solving any difficulties with concepts, skills, and processes.

You may wish to give students **Master 9 0.5 Centimetre Grid Paper** to create the Foldable, but full grid paper will likely work better.

This Foldable does not allow students to keep track of their progress on the chapter problem as they work on the Math Link introduction on page 79 and the section Math Links on pages 87, 94, 100, 105, and 111. You may wish to have students store these items in their math portfolio for this chapter. Students can list their ideas for the Wrap It Up! on the back of the Foldable.

Math Link

Refer students to the design of the board for Playing Leader in the chapter opener. Have students discuss why the board is part of this chapter. Brainstorm other games that involve triangles and squares. Consider having students play a game of Playing Leader. Hand out **BLM 3–5 Playing Leader Board**, on which students can play the game. They could use 13 counters of one colour to represent the green pegs, and a counter of another colour to represent the leader peg.

Tell students that they will design a board game at the end of the chapter. Have students read the Wrap It Up! on page 115 to give them a sense of where the Math Link is heading. The Wrap It Up! problem is a summative assessment.

Meeting Student Needs

- Some students may benefit from using **BLM 3–1 Chapter 3 Math Link Introduction**, which provides scaffolding for this activity.
- You may wish to mention to students that the Aboriginal name for Playing Leader is *musinaykahwhan metowaywin* and that this game is very similar to the Scandinavian game called Fox and Geese. Ask if any students are familiar with either of these games. Have students research the game Playing Leader, Fox and Geese, or other board games from various cultures.
- It may be helpful to some students for the class to discuss the Key Words before beginning the chapter. You may wish to familiarize students with these additional terms: *prime number*, *right triangle*, and *product*. A suggested activity might be to construct a crossword of all of these vocabulary terms.

Gifted and Enrichment

- Once they have completed #5 of the Math Link, have students think of a way to determine the total area of the board using triangles. Tell them to list the dimensions of each triangle, tell the number of triangles they are using, find the area of the triangles they are using, and show all of the steps and computations they use to get the total area.

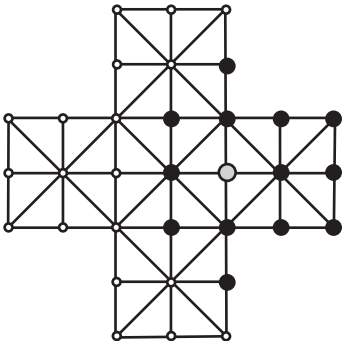
Web Link

For information about the game *musinaykahwhan metowaywin*, or Playing Leader, and other board games of Aboriginal peoples, go to www.mathlinks8.ca and follow the links.

Answers

Math Link

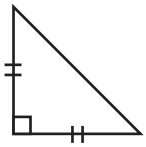
1. Answers will vary. Example:



2. 6

3. There are twenty 5×5 squares and five 10×10 squares.
$20 + 5 = 25$ squares

4. isosceles, right



5. **Method 1:**
Area of 5×5 square $= 25$ cm^2
$20 \times 25 = 500$ cm^2

Method 2:
Area of 10×10 square $= 100$ cm^2
$5 \times 100 = 500$ cm^2