

Squares and Square Roots

3.1

MathLinks 8, pages 80–87

Suggested Timing

80–100 minutes

Materials

- square tiles
- grid paper
- ruler

Blackline Masters

- Master 2 Two Stars and One Wish
- Master 8 Centimetre Grid Paper
- Master 9 0.5 Centimetre Grid Paper
- Master 19 Multiplication Chart
- BLM 3–3 Chapter 3 Warm-Up
- BLM 3–6 Section 3.1 Extra Practice
- BLM 3–7 Section 3.1 Math Link

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- Reasoning (R)
- Technology (T)
- Visualization (V)

Specific Outcomes

N1 Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers).


Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–5, 7, 9, 11, 15, 17, Math Link
Typical	1–5, 7, 9, 11, 15, 17, 18, 23, 24, Math Link
Extension/Enrichment	1–4, 19, 21, 22, 24–27

3.1

Squares and Square Roots

FOCUS ON...
After this lesson, you will be able to...

- determine the square of a whole number
- determine the square root of a perfect square



The Pythagoreans were members of an academy of study that existed 2500 years ago. They created square numbers by arranging pebbles in equal numbers of rows and columns. Nine pebbles could be arranged in three rows and three columns. Nine is a square number because $3 \times 3 = 9$. The picture shows the first four square numbers that the Pythagoreans found: 1, 4, 9, and 16. How can you determine the next square number?

Literacy Link
A square number is the product of the same two numbers. $3 \times 3 = 9$, so 9 is a square number. A square number is also known as a perfect square. A number that is not a perfect square is called a non-perfect square.

Did You Know?
Pythagoras (about 580–500 B.C.E.) was the leader of a group of academics called the Pythagoreans. They believed that patterns in whole numbers could help explain the universe.

Explore the Math

How can you identify a perfect square?

1. Use square tiles to make five rectangles with the dimensions shown. What is the area of each rectangle?

Length (cm)	Width (cm)
5	3
8	2
9	1
4	3
9	4

Materials
• square tiles

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Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce material learned in previous sections.

You may wish to have students learn more about Pythagoras (see the Web Link on TR page 104). Clarify for students where and when he lived by indicating the location of Greece on a map and by marking on a timeline when he lived. Discuss with students how long ago 2500 years is.

Literacy Link Review the Literacy Link on page 80. Have students list the first five square numbers (1, 4, 9, 16, 25).

2. Try to rearrange the tiles in each rectangle to make a square.


- Which rectangles can you make into squares?
- What is the side length of each square?
- How is the area of each square related to its side length?

3. a) Choose three perfect squares and three non-perfect squares.
 b) Express each number as a product of prime factors.
 c) For each number, how many times does each prime factor appear? Compare your results with a partner's results.

4. a) What do all of the perfect squares have in common?
 b) What do all of the non-perfect squares have in common?

Reflect on Your Findings

- How can square tiles help you to determine if a number is a perfect square?
- How can prime factors help you to determine if a number is a perfect square?



Literacy Link

Prime Numbers and Prime Factors

A prime number is a whole number greater than 1 that has only two factors: 1 and itself.

Prime factors are factors that are prime numbers.

For example, the prime factors of 10 are 2 and 5.

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Example 1: Identify Perfect Squares

prime factorization

- a number written as the product of its prime factors
- the prime factorization of 6 is 2×3

perfect square

- a number that is the product of the same two factors
- has only an even number of prime factors
- $5 \times 5 = 25$, so 25 is a perfect square



Solution

a) $24 = 2 \times 2 \times 2 \times 3$ $36 = 2 \times 2 \times 3 \times 3$ $81 = 3 \times 3 \times 3 \times 3$

b) To be a perfect square, each prime factor in the prime factorization must occur an even number of times. 36 and 81 are perfect squares because each prime factor occurs an even number of times.

$36 = 2 \times 2 \times 3 \times 3$ two factors of 2, two factors of 3
 $81 = 3 \times 3 \times 3 \times 3$ four factors of 3

24 is not a perfect square because at least one of the prime factors occurs an odd number of times.
 $24 = 2 \times 2 \times 2 \times 3$ three factors of 2, one factor of 3

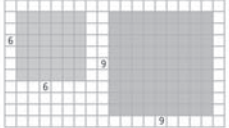
c) To determine the side length of the squares, look at the product of prime factors for the area.

$36 = 2 \times 2 \times 3 \times 3$ $81 = 3 \times 3 \times 3 \times 3$

Rearrange the prime factors into two equal groups.

$36 = (2 \times 3) \times (2 \times 3)$
 $36 = 6 \times 6$

$81 = (3 \times 3) \times (3 \times 3)$
 $81 = 9 \times 9$



Show You Know

Write the prime factorization of each number. Which number is not a perfect square? Explain how you know.

a) 45 b) 100

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Explore the Math

In this exploration, students use square tiles and prime factorization to identify perfect squares.

Method 1 Have students work in groups of two or three. Provide each group with 40 centimetre cubes. If you do not have this many cubes, give each group 20 cubes and instruct students to join one other group to build the last rectangle. Circulate and monitor progress.

Observe how students are organizing their results. If they are keeping random track of data, you may wish to work with individuals to develop a useful recording method.

Encourage students to use divisibility rules to develop the prime factors of each number they are working with. Some students may start to develop the prime factors, but leave in some that are not prime. Encourage them to consider whether a number besides 1 divides into a number they have listed as a prime factor.

Once students have completed the activity, have each small group join with another group or two to discuss how they might identify a perfect square. Ask this larger group to prepare a brief report for the class outlining their answer to #5.

Discuss #5 with the entire class. You may also wish to discuss the following:

- How did your group record data? Why was this method useful?
- How else might you have recorded data?
- What data recording methods seemed to be the most useful? Why?
- What strategies did you use to solve the problem in the Explore the Math?
- What other strategies might you use?
- How can you test your strategy to verify that it works?

Method 2 In the absence of centimetre cubes, have students draw the rectangles and squares on centimetre grid paper or 0.5 centimetre grid paper. You may wish to provide them with **Master 8 Centimetre Grid Paper** or **Master 9 0.5 Centimetre Grid Paper**. To solve #5, use the same teaching strategies as those in Method 1.

Literacy Link Review the Literacy Link on page 81. Have students list the prime numbers between 0 and 20 (2, 3, 5, 7, 11, 13, 17, 19).

Example 2: Determine the Square of a Number

Determine the area of a square picture with a side length of 13 cm.

Solution

$$A = s^2$$

$$A = 13^2$$

$$A = 13 \times 13$$

$$A = 169$$

The area is 169 cm².


Strategies
 Draw a Diagram

Literacy Link

You can write a repeated multiplication like 13×13 as a square: $13 \times 13 = 13^2$. 13^2 is read as thirteen squared.

Show You Know

Determine the area of a square with a side length of 16 mm.

Example 3: Determine the Square Root of a Perfect SquareEdgar knows that the square case for his computer game has an area of 144 cm². What is the side length of the case?

Solution

Method 1: Use Inspection

To find the side length, determine what positive number when multiplied by itself equals 144.

$12 \times 12 = 144$

The square root of 144 is 12, or $\sqrt{144} = 12$.

The side length is 12 cm.

Method 2: Use Guess and Check

Find the positive value for the blank boxes.

$\blacksquare \times \blacksquare = 144$

$10 \times 10 = 100$ Too low

$13 \times 13 = 169$ Too high

$12 \times 12 = 144$ Correct!

$12 = \sqrt{144}$

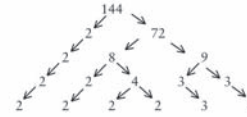
The side length is 12 cm.

square root

- a number that when multiplied by itself equals a given value
- 6 is the square root of 36 because $6 \times 6 = 36$

Literacy Link
Reading Square RootsThe symbol for square root is $\sqrt{\quad}$.Read $\sqrt{9}$ as the square root of 9,

square root 9, or root 9.

Method 3: Use Prime FactorizationThe prime factorization of 144 is $2 \times 2 \times 2 \times 2 \times 3 \times 3$.

Rearrange the prime factors into two equal groups.

$144 = (2 \times 2 \times 3) \times (2 \times 2 \times 3)$

$144 = 12 \times 12$

$\sqrt{144} = 12$

The side length is 12 cm.

Tech Link

You can use a calculator to find the square root of a number. Try the following key sequences on your calculator. Then, record the one that works on your calculator.

$\text{C } 144 \sqrt{\quad} =$

or

$\text{C } \sqrt{\quad} 144 =$

Show You KnowDetermine the side length of a square with an area of 196 cm².**Key Ideas**

- The square of a number is the number multiplied by itself.

$5 \times 5 = 25$, or $5^2 = 25$

- The square of a whole number is a perfect square. $2^2 = 4$

So, 4 is a perfect square.

- The square of a number can be thought of as the area of a square.

$4^2 = 16$

The area is 16 cm².

- The square root of a number can be thought of as the side length of a square.

$\sqrt{16} = 4$

The side length is 4 cm.

- The square root of a value is a number that when multiplied by itself equals the value.

$6 \times 6 = 36$, so $\sqrt{36} = 6$

- In the prime factorization of a perfect square, there is an even number of each prime factor.

$36 = 2 \times 2 \times 3 \times 3$ two factors of 2, two factors of 3

**Example 1**

You may wish to assist students in reactivating their understanding of prime numbers and prime factorization, along with their skills in drawing a factor tree to determine prime factors.

For part c), students will need grid paper. You may wish to provide them with **Master 9 0.5 Centimetre Grid Paper**.

Example 2

Discuss with students the problem solving strategy of drawing a diagram. Have them explore two possible ways of drawing the square: on plain paper, as in the student resource, or on grid paper. Encourage students to think about which method they prefer for solving the problem, or if they prefer not to draw a diagram at all.

Literacy Link Review the Literacy Link at the top of page 83. Have students write a number such as 6^2 using multiplication, and then show 11×11 as 11^2 .

Example 3

This activity provides an opportunity for students to explore using a calculator to calculate the square root of a number. Students with calculators that have a graphical display of the square root sign will need to press the square root button prior to inputting the number. For other calculators that do not display the square root symbol in the calculation window, students will need to input the number before pressing the square root button. Refer students to the Tech Link on page 84.

Literacy Link For Example 3, students will need to understand the term *square root*. Go over the definition and then direct students' attention to the Literacy Link on page 83, called Reading Square Roots. You may wish to explain to students that *square root* is most commonly used as a noun, e.g., "The square root of 9 is 3."

Meeting Student Needs

- Some students may benefit from having section 3.1 covered over two lessons. The first lesson might focus on perfect squares—Examples 1 and 2—and the second on finding the square roots of perfect squares—Example 3. Choose questions from the Practise section to support each lesson, rather than assigning all of the Practise questions at once at the end.
- As they work on Example 2 and the related Show You Know, it might be useful for visual and kinesthetic learners to draw the squares on grid paper and count the interior boxes to assist them in determining the area.

ELL

- Ensure that students understand the following terms: *academy of study*, *existed*, *arranging*, *pebbles*, *square numbers*, *record*, *area*, *rearrange*, *appears*, and *in common*.

Gifted and Enrichment

- Students may wish to do some research on the Pythagoreans. There are interesting connections with geometry, number patterns, music, and astronomy. Have them type *Pythagoreans* into a search engine to learn more about Pythagoras' followers.

Common Errors

- Some students may think that 1 is a prime number.
- R_x** Emphasize to students that each prime number has two distinct factors (1 and itself). In the case of 1, these two factors are not distinct.
- Some students may include 1 as a branch in a factor tree. For example, they will have the branches 1 and 3 coming down from 3.
- R_x** Remind students that 1 is never used in a factor tree. The bottom branch is always a prime number.
- Students may be confused when they see different factor trees for the same given number.
- R_x** Discuss that factor trees for a given number may not be the same. Have students explore choosing different pairs of factors at each branching point for a number such as 12 or 30.



Web Link

For information about and pictures of Pythagoras, go to www.mathlinks8.ca and follow the links.

You may wish to have students interested in Pythagoras explore the Golden Rectangle through a virtual manipulative. Go to www.mathlinks8.ca and follow the links.

Students may benefit from further practice with factor trees online. Go to www.mathlinks8.ca and follow the links.

Answers

Explore the Math

1., 2. a), b)

Length (cm)	Width (cm)	Area (cm ²)	Square? (yes/no)	Side Length of Square (cm)
5	3	15	no	
8	2	16	yes	4
9	1	9	yes	3
4	3	12	no	
9	4	36	yes	6

2. c) Answers may vary. Example: The area of the square is found by multiplying the side length of the square by itself.
3. Answers will vary. Example:
- Perfect squares: 4, 25, 64. Non-perfect squares: 6, 28, 40
 - 4: 2×2 ; 25: 5×5 ; 64: $2 \times 2 \times 2 \times 2 \times 2 \times 2$; 6: 2×3 ; 28: $2 \times 2 \times 7$; 40: $2 \times 2 \times 2 \times 5$
 - 4: two factors of 2; 25: two factors of 5; 64: six factors of 2; 6: one factor of 2 and one factor of 3; 28: two factors of 2 and one factor of 7; 40: three factors of 2 and one factor of 5

- Each perfect square has an even number of prime factors.
 - Each non-perfect square has at least one prime factor that occurs an odd number of times in its prime factorization.
- Answers will vary. Example:
 - If you can make a square with centimetre cubes that has an area equal to the number, the number is a perfect square.
 - If each prime factor of a number occurs an even number of times in its prime factorization, the number is a perfect square.

Show You Know: Example 1

a) $45 = 3 \times 3 \times 5$ b) $100 = 2 \times 2 \times 5 \times 5$

The number 45 is not a perfect square because there is an odd number of 5s in the prime factorization.

Show You Know: Example 2

256 mm²

Show You Know: Example 3

14 cm

Assessment	Supporting Learning
Assessment as Learning	
<p>Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.</p>	<ul style="list-style-type: none"> • Review examples of perfect squares. Some students will not make the connection that each perfect square has even numbers of prime factors. • Some students may benefit from examples of non-perfect squares. This “negative definition” may assist students to understand what a perfect square is. Sometimes, it helps to see what something is not, rather than what it is. • You may wish to show students some prime factorizations and ask them what prime numbers could be added or removed to create a perfect square. For example, write the following prime factorization on the board: $3 \times 3 \times 3 \times 5 \times 5$. Ask students to explain why it is not a perfect square. Then, ask how this number could be changed to make a perfect square by removing or adding a number. (Add or remove one 3.) • Have students share their answers with a classmate. You may wish to distribute Master 2 Two Stars and One Wish, which students can use for peer evaluation. They record two things they like about their classmate’s work and one thing that they would like to see improved.
Assessment for Learning	
<p>Example 1 Have students do the Show You Know related to Example 1.</p>	<ul style="list-style-type: none"> • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Circulate among students in order to see which students require one-on-one help with factor trees. Explore the first five prime numbers with these students: 2, 3, 5, 7, and 11. • Reinforce that each branch will end with a prime number. It may be necessary to assist students in recalling the divisibility rules for 2, 3, and 5.
<p>Example 2 Have students do the Show You Know related to Example 2.</p>	<ul style="list-style-type: none"> • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may find it helpful to draw the picture first. Reinforce that what they are finding is the area of the square. Provide grid paper or hand out Master 9 0.5 Centimetre Grid Paper. • Some students may benefit from using a multiplication chart. Point out where the diagonal line of perfect squares is and how they can use this line to determine the square root. You may wish to hand out Master 19 Multiplication Chart, which students can tape to the inside of their notebook or binder.
<p>Example 3 Have students do the Show You Know related to Example 3.</p>	<ul style="list-style-type: none"> • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • It may be helpful to point out to students that the side of a square is the square root and the area is the square. The area is always larger than the square root. • Reinforce that the square root can be obtained by rearranging the prime factors into two equal groups. • Encourage students to show all of their work.

Method 3: Use Prime Factorization

The prime factorization of 144 is $2 \times 2 \times 2 \times 2 \times 3 \times 3$.

Rearrange the prime factors into two equal groups.

$$144 = \underbrace{2 \times 2 \times 3}_{12} \times \underbrace{2 \times 2 \times 3}_{12}$$

$$144 = 12 \times 12$$

$$\sqrt{144} = 12$$

The side length is 12 cm.

Tech Link
You can use a calculator to find the square root of a number. Try the following key sequences on your calculator. Then, record the one that works on your calculator.
 $\square 144 \sqrt{\square} =$
 or
 $\square \sqrt{\square} 144 =$

Show You Know
Determine the side length of a square with an area of 196 cm^2 .

Key Ideas

- The square of a number is the number multiplied by itself.
 $5 \times 5 = 25$, or $5^2 = 25$
- The square of a whole number is a perfect square. $2^2 = 4$
 So, 4 is a perfect square.
- The square of a number can be thought of as the area of a square.
 $4^2 = 16$
 The area is 16 cm^2 .
- The square root of a number can be thought of as the side length of a square.
 $\sqrt{16} = 4$
 The side length is 4 cm.
- The square root of a value is a number that when multiplied by itself equals the value.
 $6 \times 6 = 36$, so $\sqrt{36} = 6$
- In the prime factorization of a perfect square, there is an even number of each prime factor.
 $36 = 2 \times 2 \times 3 \times 3$ two factors of 2, two factors of 3

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Communicate the Ideas

- Explain how to square the number 7.
- How would you use prime factorization to determine the square root of 225? Compare your answer with a classmate's.
- The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Use words and/or diagrams to explain how you know which factor is the square root of 36.
- Explain how squaring a number is the reverse of finding the square root of a number. Include an example with your explanation.

Check Your Understanding

Practise

For help with #5 to #8, refer to Example 1 on page 82.

- Determine the prime factorization of 4.
 - Is 4 a perfect square? Explain.
 - Draw the square and label its side length.
- A rectangle has an area of 64 m^2 .
 - Determine the prime factorization of 64.
 - Is 64 a perfect square? Explain.
 - Draw a square with that area and label its side length.
- Write the prime factorization of each number. Identify the perfect squares.
 - 42
 - 169
 - 256
- Determine the prime factorization of each number. Which numbers are perfect squares?
 - 144
 - 60
 - 40

For help with #9 to #12, refer to Example 2 on page 83.

- What is the area of a square with each side length?
 - 10
 - 16
- Determine the area of a square with each side length.
 - 20
 - 17
- What is the square of each number?
 - 9
 - 11
- Determine the square of each number.
 - 3
 - 18

For help with #13 to #16, refer to Example 3 on pages 83–84.

- What is the side length of the square shown?

3.1 Squares and Square Roots • MHR 85

Key Ideas

This section reinforces the concept of squares, particularly perfect squares and the relationship between the square of a number and the area of a square. The section also focuses on square roots, particularly square roots of perfect squares in the form of positive integers. The connection is made between square root and the side length of a square.

Communicate the Ideas

In #1, students describe the process of squaring a number. In #2, they review the process of using prime factorization to find the square root of a perfect square. In #3, students make the connection between perfect squares and their square root, using both words and diagrams. In #4, students are reminded that the processes of squaring a number and taking the square root of the squared number are inverse processes (for non-negative numbers).

Meeting Student Needs

- Some students may need more practice with prime factorization. It may help to assist them in recalling their skills with prime numbers and the divisibility rules for 2, 3, and 5.

- Concrete learners may benefit from seeing a correct solution to #3 that includes diagrams.

Common Errors

- Some students may struggle with creating factor trees and then recording the number as a product of prime factors.
- R_x** Ensure that students completely factor the number. Also, make sure that students record every number at the end of every branch in their prime factorization.

Answers

Communicate the Ideas

- Multiply 7 by itself: $7 \times 7 = 49$.
- The prime factorization of 225 is $225 = 3 \times 3 \times 5 \times 5$.
 Rearrange the prime factors into two equal groups.
 $225 = 3 \times 5 \times 3 \times 5$
 $225 = 15 \times 15$
 $\sqrt{225} = 15$
 The square root of 225 is 15.
- Answers may vary. Example: Square each of the factors. The square of 6 is 36, so it is the square root of 36.
- Answers may vary. Example: If a number is squared, it is multiplied by itself. If you take this product and find its square root, you are determining which number when multiplied by itself results in the product. Therefore, these two operations can be thought of as reverse operations. For example, $4 \times 4 = 16$ and $\sqrt{16} = 4$.

Assessment as Learning

Communicate the Ideas

Have all students complete #1 to #3.

- Encourage students to verbalize their thinking.
- You may wish to have students work with a partner.
- Students may benefit from drawing a diagram in #1 to show the square of 7.
- For #2, encourage students to draw a picture, use factor trees, or group prime numbers. They can also use a multiplication chart, such as **Master 19 Multiplication Chart**, on which the diagonal shows the perfect squares.
- Some students may need help determining two factors in order to begin #2. Assist them in recalling the divisibility rules for 3.
- Encourage students to solve #3 with a diagram. Share some successful responses to #3.

Communicate the Ideas

1. Explain how to square the number 7.
2. How would you use prime factorization to determine the square root of 225? Compare your answer with a classmate's.
3. The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Use words and/or diagrams to explain how you know which factor is the square root of 36.
4. Explain how squaring a number is the reverse of finding the square root of a number. Include an example with your explanation.



Check Your Understanding

Practise

For help with #5 to #8, refer to Example 1 on page 82.

5. a) Determine the prime factorization of 4.
b) Is 4 a perfect square? Explain.
c) Draw the square and label its side length.
6. A rectangle has an area of 64 m^2 .
a) Determine the prime factorization of 64.
b) Is 64 a perfect square? Explain.
c) Draw a square with that area and label its side length.
7. Write the prime factorization of each number. Identify the perfect squares.
a) 42 b) 169 c) 256
8. Determine the prime factorization of each number. Which numbers are perfect squares?
a) 144 b) 60 c) 40

For help with #9 to #12, refer to Example 2 on page 83.

9. What is the area of a square with each side length?
a) 10 b) 16
10. Determine the area of a square with each side length.
a) 20 b) 17
11. What is the square of each number?
a) 9 b) 11
12. Determine the square of each number.
a) 3 b) 18

For help with #13 to #16, refer to Example 3 on pages 83–84.

13. What is the side length of the square shown?



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14. Determine the side length of a square with an area of 900 cm^2 .

15. Evaluate.
a) $\sqrt{49}$ b) $\sqrt{64}$ c) $\sqrt{625}$

16. Determine the value.
a) $\sqrt{9}$ b) $\sqrt{25}$ c) $\sqrt{1600}$

Apply

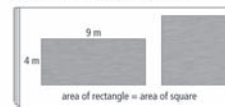
17. A fridge magnet has an area of 54 mm^2 . Is 54 a perfect square? Use prime factorization to find the answer.

18. A floor mat for gymnastics is a square with a side length of 14 m. What is the area of the floor mat in square metres?



19. The gym teacher told the students to run twice around the perimeter of the school field. The area of the square field is $28\,900 \text{ m}^2$. What distance did the students run?

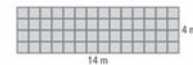
20. Adam's uncle has instructions for building a shed. One page of the instructions, shown below, is not very clear.



- a) What is the area of the rectangle?
- b) What is the side length of the square?

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21. Kate is going to put a patio in her backyard. The patio stones she is using each have an area of 1 m^2 . She has created the rectangular design shown.



- a) What is the area of the patio?
 - b) What are the dimensions of another rectangular patio she could build with the same area?
 - c) Kate decides to make a patio with the same area but she wants it to be a square with whole number side lengths. Is this possible? Explain your reasoning.
22. The world's largest city square is Tiananmen Square in Beijing, China. It has an area of $396\,900 \text{ m}^2$.



- a) What are the dimensions of the square?
- b) If the square had dimensions of 629 m by 629 m, what would be the area?
- c) If the square had an area less than $394\,000 \text{ m}^2$ and greater than $386\,000 \text{ m}^2$, what are all of the possible whole number dimensions that it could have?

23. A helicopter landing pad has a square shape. The area is 400 m^2 . Use prime factorization to find the side length of the pad.

Check Your Understanding

Practise

The following pairs of questions are very similar. Some students may need to do only one in each set: #5 and #6, #7 and #8, #9 and #10, and #11 and #12.

Apply

For #19, you may need to assist students in recalling what *perimeter* is. For #21b), some students may benefit from writing the area of the patio as a product of prime factors.

Extend


Consider having students work in pairs to complete the Extend questions.

Math Link

This Math Link is not crucial for completing the Wrap It Up! at the end of the chapter. However, the question is an interesting challenge as it requires students to find more than one answer.

Extend


24. The first three triangular numbers are



a) What are the next three triangular numbers?
 b) Add together any two consecutive triangular numbers. What do you notice about the sums?

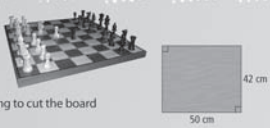
25. A square digital photo on the computer has an area of 144 cm^2 .
 a) What is the side length of the photo?
 b) The photo is enlarged so that the side length is now 36 cm . What is the area of the enlarged photo?
 c) How many times as large as the original area is the enlarged area?

Imagine your dog is 80 cm tall and your cat is 40 cm tall. How many times as tall as your cat is your dog? What operation did you perform?



MATH LINK
 Chess is played on a square board. The board is made up of 32 white squares and 32 dark squares.

You decide to make your own chessboard. You are going to cut the board out of the $42 \text{ cm} \times 50 \text{ cm}$ piece of wood shown.



Each square on the board will have whole number side lengths. The chess pieces fit on squares that are no smaller than 9 cm^2 . What are all of the possible dimensions that your board could have?

3.1 Squares and Square Roots • MHR 87

d) How many times as large as the original side length is the enlarged side length?
 e) Use what you know about the square root of a perfect square to identify the relationship between the numbers in parts c) and d).

26. a) Determine which of the following numbers are perfect squares: 10 , 100 , 1000 , 10000 , 100000 .
 b) State the square root of each perfect square.
 c) Choose one of the numbers that is not a perfect square. Explain how you know that it is not a perfect square.
 d) Describe a quick method for determining mentally if the numbers are perfect squares.
 e) Use your method in part d) to decide if 1000000000 is a perfect square.

27. a) Determine the square root of each number: 6400 , 640000 , 64000000 .
 b) Describe a quick method for determining mentally the square root of each number in part a).
 c) Explain why this method does not work for evaluating $\sqrt{640}$.
 d) Use your method in part b) to evaluate $\sqrt{640000000000}$. Explain how you determined the answer.

Meeting Student Needs

- You may wish to allow students to use square tiles to assist them as they complete the questions in Check Your Understanding.
- Provide **BLM 3–6 Section 3.1 Extra Practice** to students who would benefit from more practice.
- Visual, concrete, and kinesthetic learners may complete the Math Link by constructing the chessboard with paper and labelling the dimensions.

ELL

- Ensure that students understand the following words: *fridge magnet*, *gymnastics*, *helicopter*, *landing pad*, and *shed*.

Common Errors

- For the Math Link, some students may stop after they check that $3 \text{ cm} \times 3 \text{ cm}$ squares will work.

R_x Encourage students to try $4 \text{ cm} \times 4 \text{ cm}$ and $5 \text{ cm} \times 5 \text{ cm}$ squares.

Answers

Math Link

$40 \text{ cm} \times 40 \text{ cm}$; $32 \text{ cm} \times 32 \text{ cm}$; $24 \text{ cm} \times 24 \text{ cm}$

Assessment	Supporting Learning
Assessment for Learning	
<p>Practise and Apply Have students do #5, #7, #9, #11, #15, and #17. Students who have no problems with these questions can go on to the remaining Apply questions.</p>	<ul style="list-style-type: none"> For #5, some students may benefit from drawing diagrams, such as factor trees. Have them refer back to Example 1. Reactivating their understanding of divisibility rules may also help. Then, have them complete #6 and all or part of #7 and #8. For #9 and #11, you may wish to encourage students to draw squares and shade in the area that represents the square of the side. They should then try one or both of #10 and #12. Students may benefit from drawing squares for #15. It may also be useful for them to use a multiplication chart or a calculator. For #17, it may be beneficial for some students to complete a factor tree and/or a diagram of a square. This may help them to visualize their thinking.
<p>Math Link The Math Link on page 87 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 115.</p>	<ul style="list-style-type: none"> Emphasize that it may be possible to build the chessboard with squares that are larger than 9 cm^2. Students who need help getting started could use BLM 3-7 Section 3.1 Math Link, which provides scaffolding. Grid paper may also be helpful to some students. You may wish to hand out Master 9 0.5 Centimetre Grid Paper.
Assessment as Learning	
<p>Math Learning Log Have students complete the following statements:</p> <ul style="list-style-type: none"> I can use a factor tree to find ... The difference between a square and a square root is ... The part I find most confusing is 	<ul style="list-style-type: none"> Encourage students to use the What I Need to Work On tab of their chapter Foldable to note what they continue to have difficulties with.