3.3 Estimating Square Roots

***MathLinks 8*, pages 95–100**

Suggested Timing
80–100 minutes

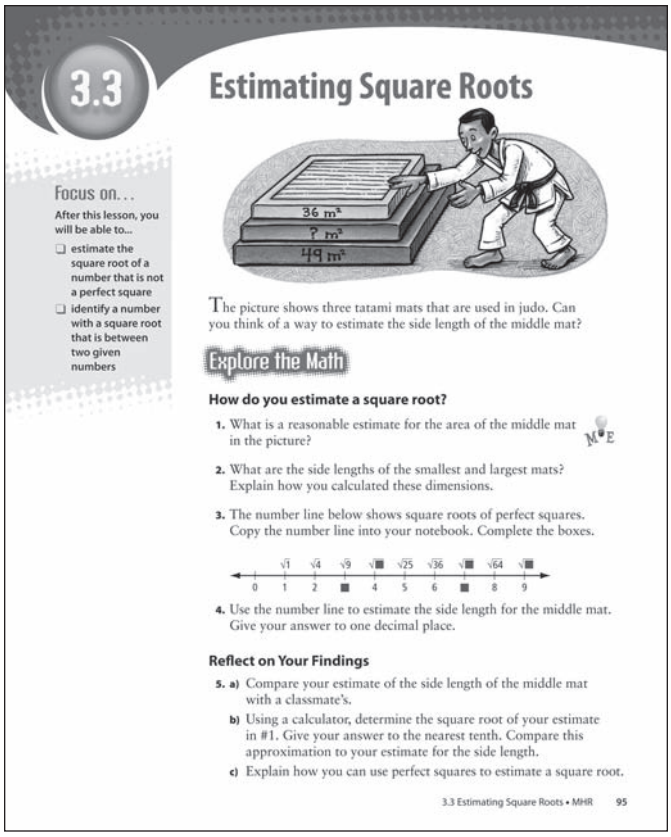
Materials
- ruler
- calculator

Blackline Masters
Master 4 Vertical and Horizontal Number Lines
Master 8 Centimetre Grid Paper
Master 19 Multiplication Chart
BLM 3–3 Chapter 3 Warm-Up
BLM 3–12 Section 3.3 Extra Practice
BLM 3–13 Section 3.3 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
N2 Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 10, Math Link
Typical	1–7, 9–11, 13–16, Math Link
Extension/Enrichment	1–3, 11, 15, 16, 19–21


3.3 Estimating Square Roots

Focus on...
After this lesson, you will be able to...
- estimate the square root of a number that is not a perfect square
- identify a number with a square root that is between two given numbers

The picture shows three tatami mats that are used in judo. Can you think of a way to estimate the side length of the middle mat?

Explore the Math

How do you estimate a square root?

1. What is a reasonable estimate for the area of the middle mat in the picture? ME
2. What are the side lengths of the smallest and largest mats? Explain how you calculated these dimensions.
3. The number line below shows square roots of perfect squares. Copy the number line into your notebook. Complete the boxes.

4. Use the number line to estimate the side length for the middle mat. Give your answer to one decimal place.

Reflect on Your Findings

5. a) Compare your estimate of the side length of the middle mat with a classmate's.
   b) Using a calculator, determine the square root of your estimate in #1. Give your answer to the nearest tenth. Compare this approximation to your estimate for the side length.
   c) Explain how you can use perfect squares to estimate a square root.

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Planning Notes

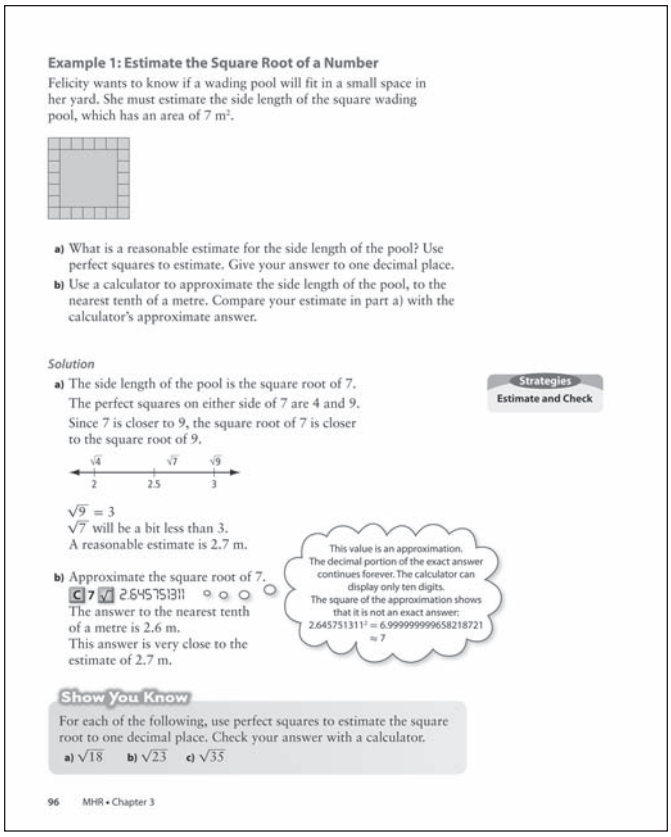
Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce material learned in previous sections.

Read the opening paragraph of section 3.3 with the class. Discuss situations in which people might need to estimate the side length of an object (e.g., a piece of plywood that needs to fit in the trunk of a car, a mattress that must fit through a stairwell). Invite students to brainstorm possible ways to estimate the side length of the middle tatami mat in the picture.

Note that students will need rulers to draw the number lines in this section.

Explore the Math

This investigation provides students with an opportunity to estimate. Answers will vary for the area of the middle mat.



Example 1: Estimate the Square Root of a Number

Felicity wants to know if a wading pool will fit in a small space in her yard. She must estimate the side length of the square wading pool, which has an area of 7 m².

a) What is a reasonable estimate for the side length of the pool? Use perfect squares to estimate. Give your answer to one decimal place.

b) Use a calculator to approximate the side length of the pool, to the nearest tenth of a metre. Compare your estimate in part a) with the calculator's approximate answer.

Solution

a) The side length of the pool is the square root of 7.
The perfect squares on either side of 7 are 4 and 9.
Since 7 is closer to 9, the square root of 7 is closer to the square root of 9.

Strategies
Estimate and Check

$\sqrt{4}$ $\sqrt{7}$ $\sqrt{9}$
2 2.5 3

$\sqrt{9} = 3$
$\sqrt{7}$ will be a bit less than 3.
A reasonable estimate is 2.7 m.

b) Approximate the square root of 7.
C 7 √ 2.645751311
The answer to the nearest tenth of a metre is 2.6 m.
This answer is very close to the estimate of 2.7 m.

This value is an approximation. The decimal portion of the exact answer continues forever. The calculator can display only ten digits.
The square of the approximation shows that it is not an exact answer:
$2.645751311^2 = 6.999999999658218721 \approx 7$

Show You Know

For each of the following, use perfect squares to estimate the square root to one decimal place. Check your answer with a calculator.
a) $\sqrt{18}$ b) $\sqrt{23}$ c) $\sqrt{35}$

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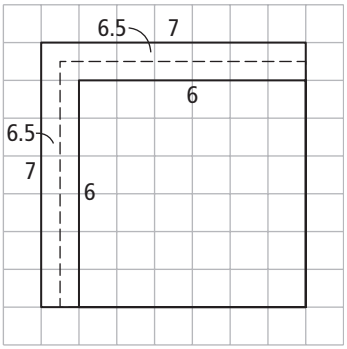
Method 1 Students may need to copy the number line in #3 into their notebooks to help them with their estimate. Alternatively, provide them with **Master 4 Vertical and Horizontal Number Lines**.

Method 2 Have students draw the two known mats on grid paper such as **Master 8 Centimetre Grid Paper**. Ask them to draw one mat on top of the other, clearly showing the length of each side. Have them use their visual to estimate the length of the middle mat as shown in the diagram, then place that number on the number line.



With the class, discuss how you know that the middle mat is not a perfect square.

Example 1

Discuss why some estimates might be more reasonable than others. Explain what a reasonable estimate and an unreasonable estimate might be. For example, the value of 7 is between the perfect squares of 4 and 9. Since 7 is closer to 9 than to 4, an unreasonable estimate is 2.2 since it is closer to the square root of 4. A reasonable estimate is 2.7 since it is closer to the square root of 9.

You may wish to introduce to students another Estimate and Check strategy. After they make their estimate, students square the estimate and see how close it is to 7. Then, they reevaluate it up or down and square it again. They continue until they are satisfied with their estimate.

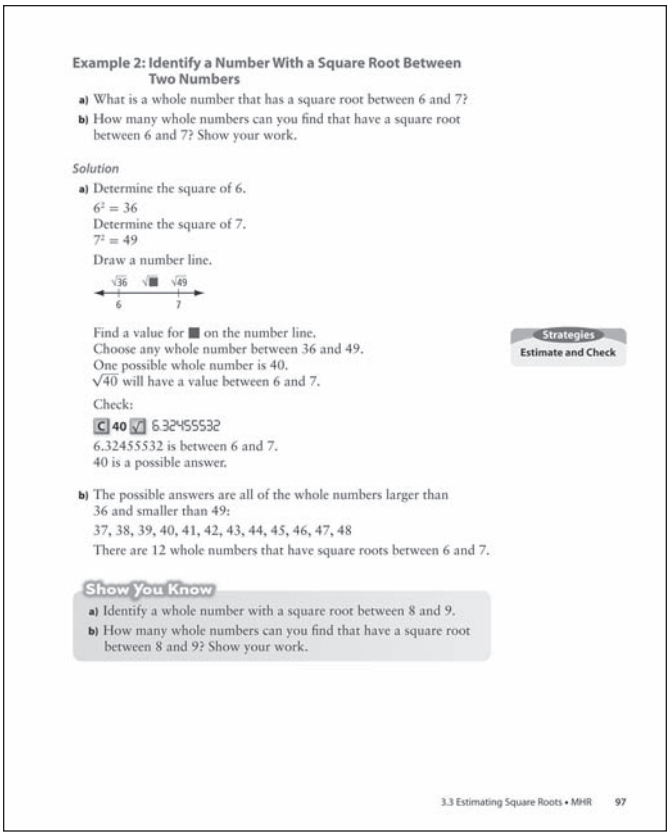
You may wish to use the following strategy to reinforce that what a calculator shows as the square root of a number may be an approximation. Have students copy and fill in the table shown.

Approximation of the Square Root	Square of the Approximation
2.6	6.76
2.65	
2.646	
2.6458	
2.64575	
2.645751	

The table will help students to understand that a calculator cannot display the exact value for $\sqrt{7}$. Explore other numbers so that students see that this is the case for the square root of any non-perfect square number.

Students who have a full understanding of Example 1 will likely be successful with the exercises in this section.

Discuss the thought bubble related to Solution b) with students. Make sure that they understand how the rounding was done and why the symbol ≈ was used.



Example 2: Identify a Number With a Square Root Between Two Numbers

a) What is a whole number that has a square root between 6 and 7?

b) How many whole numbers can you find that have a square root between 6 and 7? Show your work.

Solution

a) Determine the square of 6.

$6^2 = 36$

Determine the square of 7.

$7^2 = 49$

Draw a number line.

$\sqrt{36}$ $\sqrt{\blacksquare}$ $\sqrt{49}$

6 7

Find a value for ■ on the number line.
Choose any whole number between 36 and 49.
One possible whole number is 40.
$\sqrt{40}$ will have a value between 6 and 7.

Strategies
Estimate and Check

Check:

C 40 √ 6.32455532

6.32455532 is between 6 and 7.
40 is a possible answer.

b) The possible answers are all of the whole numbers larger than 36 and smaller than 49:

37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48

There are 12 whole numbers that have square roots between 6 and 7.

Show You Know

a) Identify a whole number with a square root between 8 and 9.

b) How many whole numbers can you find that have a square root between 8 and 9? Show your work.

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Example 2

Make sure students understand that there are a number of correct answers to part a). Explain that the answer can be any whole number between the two perfect squares of 36 and 49. They will determine all of these whole numbers in part b).

Meeting Student Needs

- Some students may require assistance in reactivating their understanding of square root and finding the square root of a perfect square.
- It might be helpful for some students to use **Master 19 Multiplication Chart** to explore perfect squares. Have them colour each perfect square and identify what has been squared. You also might have them explore squares online (see the Web Link on this page).
- Students may benefit from working with a classmate to complete at least two additional problems of the type shown in Examples 1 and 2 before they attempt each Show You Know on their own.

ELL

- Point to the picture in the section 3.3 opener and say the words *tatami mats*, repeating them a couple of times. If any students in the class participate in judo, invite them to explain what this sport involves.
- Ensure that students understand the following terms: *wading pool*, *yard*, *approximate*, *reasonable estimate*, and *whole number*.
- You may wish to show what a reasonable estimate is by giving a simple example and non-example on the board:
  A reasonable estimate: 38 + 10 is close to 50.
  Not a reasonable estimate: 38 +10 is close to 100.

Common Errors

- Students may confuse perfect squares and square roots.

R_x Have students complete a Verbal Visual Chart for these terms, as discussed on TR page 98.

- In Example 2, part b), some students may count 36 and 49 as two of the answers.

R_x Discuss with students that when the word *between* is used, there are cases when the boundary values should be included and there are cases when they should not. Explain that they need to read the question carefully to determine what is being asked. In this case, the word *between* does not mean that 36 and 49 should be included because the objective is to find whole numbers that have a square root between the square roots of 36 and 49. Since the square roots of 36 and 49 (6 and 7) are already given in the question, it would not make sense to include these two numbers as part of the answer.

WWW Web Link

You may wish to have students use a virtual peg board to discover squares of different sizes. Go to www.mathlinks8.ca and follow the links.

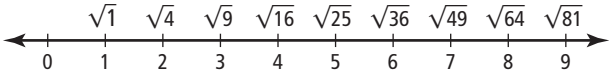
Answers

Explore the Math

1. Answers will vary. Example: The area of the middle mat is 40 m^2.

2. 6 m and 7 m. Answers may vary. Example: $\sqrt{36} = 6$; $\sqrt{49} = 7$

3.



4. Answers may vary. Example: 6.5 m

5. a) Answers will vary. Example: My answer differed from my classmate's by 0.1 m.

b) Answers may vary. Example: 6.3 m. My approximation of 6.3 m differs from my estimate of 6.5 by 0.2 m.

c) Answers may vary. Example: Determine the two perfect squares on either side of the number. Calculate the square root of each perfect square. Estimate the square root of the number between the square roots of the perfect squares. The decimal value of the square root of the number is based on which perfect square the number is closer to.

Show You Know: Example 1

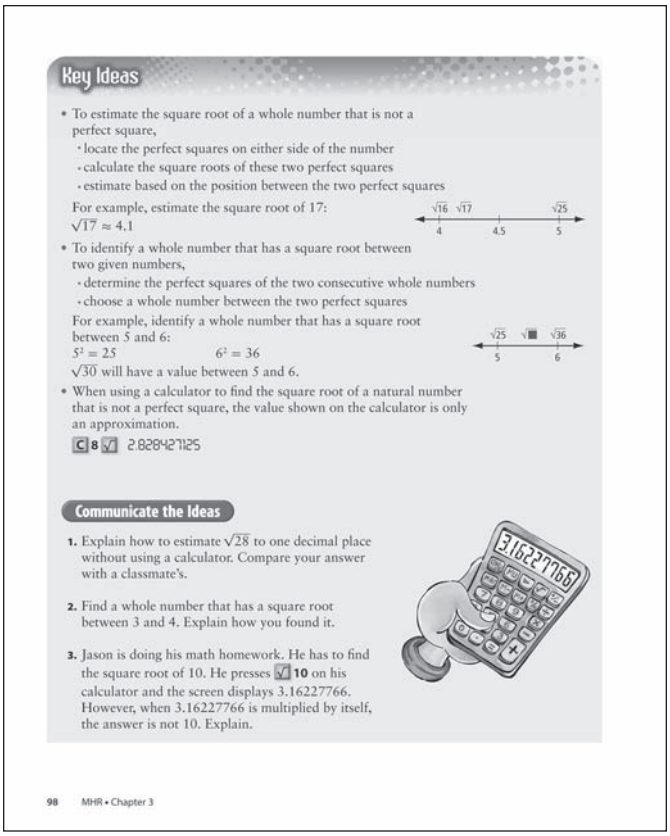
Estimates will vary. Example: **a)** 4.2 **b)** 4.8 **c)** 5.9

Show You Know: Example 2

a) Answers will vary. Example: 70

b) There are 16 numbers that have a square root between 8 and 9: 65 to 80 inclusive.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.	• Check that students can explain how to use perfect squares as benchmarks for estimating square roots in #5c).
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • If students still do not understand how to use perfect squares as benchmarks for estimating square roots, refer them to #3 in the Explore the Math. Then, have students determine the estimated square root of a different value to ensure that they have grasped the concept.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may need clarification regarding the wording of the Show You Know. Explain that the square root of the unknown whole number is between 8 and 9.



Key Ideas

- To estimate the square root of a whole number that is not a perfect square,
  - locate the perfect squares on either side of the number
  - calculate the square roots of these two perfect squares
  - estimate based on the position between the two perfect squares

For example, estimate the square root of 17:
$\sqrt{17} \approx 4.1$

- To identify a whole number that has a square root between two given numbers,
  - determine the perfect squares of the two consecutive whole numbers
  - choose a whole number between the two perfect squares

For example, identify a whole number that has a square root between 5 and 6:
$5^2 = 25$ $6^2 = 36$
$\sqrt{30}$ will have a value between 5 and 6.

- When using a calculator to find the square root of a natural number that is not a perfect square, the value shown on the calculator is only an approximation.

C 8 √ 2.828427125

Communicate the Ideas

1. Explain how to estimate $\sqrt{28}$ to one decimal place without using a calculator. Compare your answer with a classmate's.
2. Find a whole number that has a square root between 3 and 4. Explain how you found it.
3. Jason is doing his math homework. He has to find the square root of 10. He presses √ 10 on his calculator and the screen displays 3.16227766. However, when 3.16227766 is multiplied by itself, the answer is not 10. Explain.

Key Ideas

This section reinforces how to estimate the square root of a natural number that is not a perfect square. It also reiterates how to identify a natural number that has a square root between two consecutive whole numbers.

Communicate the Ideas

In #1, students explain how to estimate a square root. In #2, students explain how to determine a number with a square root between 3 and 4. In #3, students are reminded that the values that calculators compute for square roots of non-perfect squares are approximations.

Meeting Student Needs

ELL

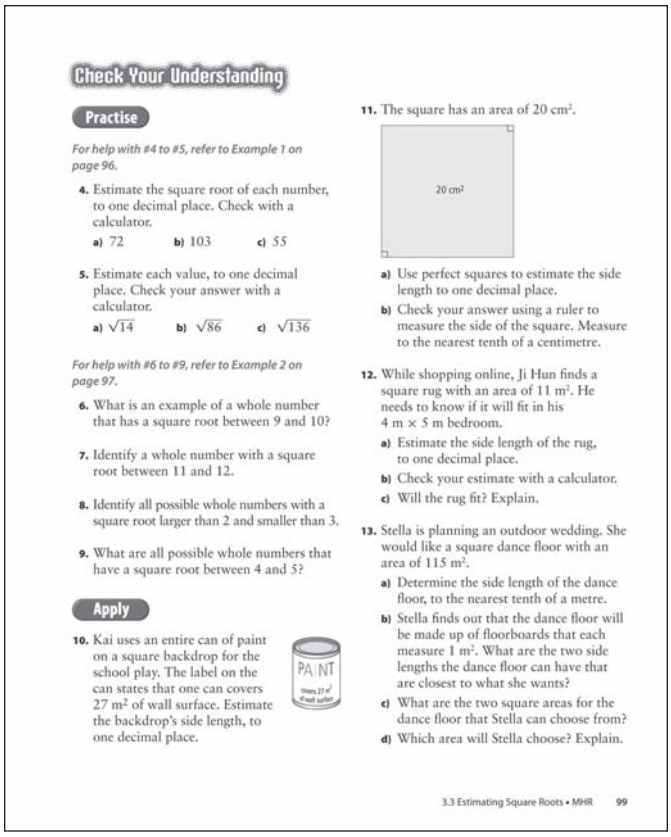
- Ensure that students understand the word *consecutive*.

Answers

Communicate the Ideas

1. Answers may vary. Example: The number 28 is between the two perfect squares 25 and 36. The square root of 25 is 5 and the square root of 36 is 6. Therefore, a reasonable estimate is 5.2, since 28 is closer to 25 than to 36.
2. Answers may vary. Example: A whole number that has a square root between 3 and 4 is 11. The square of 3 is 9 and the square of 4 is 16. A number between 9 and 16 is 11.
3. The number 3.16227766 is an approximation for the square root of 10. The decimal portion of the answer continues but the calculator can only show nine digits.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to #3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students who do not understand how to respond to #1 should be referred back to the number line in #3 of the Explore the Math. The use of a multiplication chart may also be beneficial. Provide these students with **Master 19 Multiplication Chart**. • Some students may find a multiplication chart or calculator useful as they apply the Estimate and Check strategy to answer #2. • The response to #3 should be discussed as a class so that all learners have the concept clarified before moving on.



Check Your Understanding

Practise

For help with #4 to #5, refer to Example 1 on page 96.

4. Estimate the square root of each number, to one decimal place. Check with a calculator.
a) 72 **b)** 103 **c)** 55

5. Estimate each value, to one decimal place. Check your answer with a calculator.
a) $\sqrt{14}$ **b)** $\sqrt{86}$ **c)** $\sqrt{136}$

For help with #6 to #9, refer to Example 2 on page 97.

6. What is an example of a whole number that has a square root between 9 and 10?

7. Identify a whole number with a square root between 11 and 12.

8. Identify all possible whole numbers with a square root larger than 2 and smaller than 3.

9. What are all possible whole numbers that have a square root between 4 and 5?

Apply

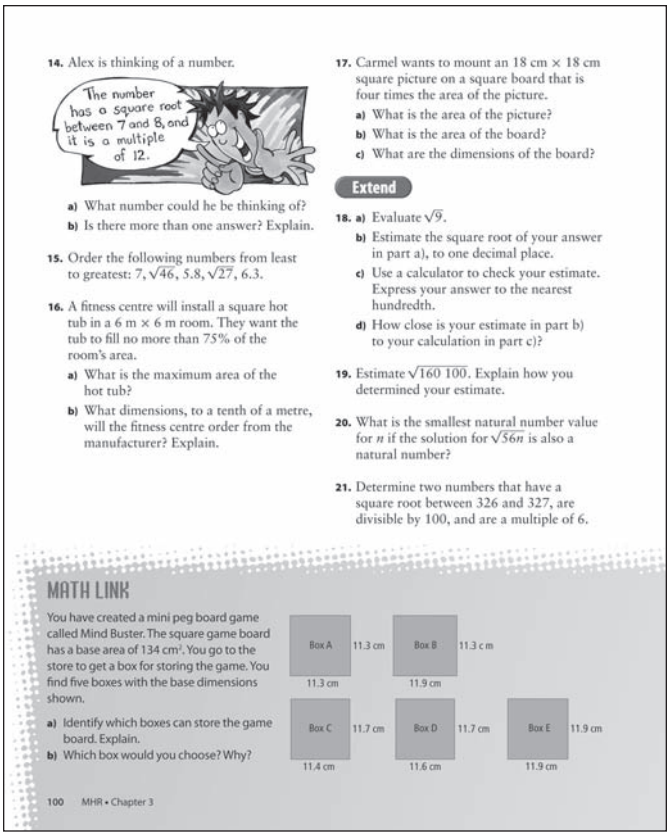
10. Kai uses an entire can of paint on a square backdrop for the school play. The label on the can states that one can covers 27 m² of wall surface. Estimate the backdrop's side length, to one decimal place.

11. The square has an area of 20 cm².

a) Use perfect squares to estimate the side length to one decimal place.
b) Check your answer using a ruler to measure the side of the square. Measure to the nearest tenth of a centimetre.

12. While shopping online, Ji Hun finds a square rug with an area of 11 m². He needs to know if it will fit in his 4 m × 5 m bedroom.
a) Estimate the side length of the rug, to one decimal place.
b) Check your estimate with a calculator.
c) Will the rug fit? Explain.

13. Stella is planning an outdoor wedding. She would like a square dance floor with an area of 115 m².
a) Determine the side length of the dance floor, to the nearest tenth of a metre.
b) Stella finds out that the dance floor will be made up of floorboards that each measure 1 m². What are the two side lengths the dance floor can have that are closest to what she wants?
c) What are the two square areas for the dance floor that Stella can choose from?
d) Which area will Stella choose? Explain.



14. Alex is thinking of a number.

a) What number could he be thinking of?
b) Is there more than one answer? Explain.

15. Order the following numbers from least to greatest: 7, $\sqrt{46}$, 5.8, $\sqrt{27}$, 6.3.

16. A fitness centre will install a square hot tub in a 6 m × 6 m room. They want the tub to fill no more than 75% of the room's area.
a) What is the maximum area of the hot tub?
b) What dimensions, to a tenth of a metre, will the fitness centre order from the manufacturer? Explain.

17. Carmel wants to mount an 18 cm × 18 cm square picture on a square board that is four times the area of the picture.
a) What is the area of the picture?
b) What is the area of the board?
c) What are the dimensions of the board?

Extend

18. **a)** Evaluate $\sqrt{9}$.
b) Estimate the square root of your answer in part a), to one decimal place.
c) Use a calculator to check your estimate. Express your answer to the nearest hundredth.
d) How close is your estimate in part b) to your calculation in part c)?

19. Estimate $\sqrt{160\ 100}$. Explain how you determined your estimate.

20. What is the smallest natural number value for n if the solution for $\sqrt{56n}$ is also a natural number?

21. Determine two numbers that have a square root between 326 and 327, are divisible by 100, and are a multiple of 6.

MATH LINK

You have created a mini peg board game called Mind Buster. The square game board has a base area of 134 cm². You go to the store to get a box for storing the game. You find five boxes with the base dimensions shown.

a) Identify which boxes can store the game board. Explain.
b) Which box would you choose? Why?

Check Your Understanding

Practise

There are three sets of paired questions in this section. Some learners may need to do only one question from each of the following pairs: #4 and #5, #6 and #7, and #8 and #9.

Apply

Students will need a ruler to answer #11. Some students may need assistance with how to approach #16. Encourage students to try to complete this question without a calculator.

Extend

For #19, you may wish to point out to students that the work they did on #26 and #27 in section 3.1 may help them with this question. Note that in section 3.1, Extend #27, students found a quick method to determine that $\sqrt{640\ 000} = 800$. For #19 in this section, they can use the same method to determine that an estimate for $\sqrt{160\ 100}$ is $\sqrt{160\ 000} = 400$.

For #20, you might wish to suggest that students express 56 as a product of prime factors.

For #21, students will need to recall how to determine divisibility by a given number and what a multiple is.

Math Link

Since the game and box are both three dimensional, some students may ask about the height of the boxes. Tell students that the height is slightly greater than the height of the game but that it is not necessary to know this information to respond to the Math Link.

Meeting Student Needs

- The context of #14 offers the opportunity for students to discuss wedding ceremonies in various cultures. Have students share cultural traditions familiar to them.
- Provide **BLM 3–12 Section 3.3 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure that students understand the following terms: *backdrop*, *mount* (i.e., to mount a picture on poster board), *wedding*, *floorboards*, and *hot tub*.

Common Errors

- In #13, instead of multiplying the area of the picture by 4, some students may multiply the dimensions of the picture by 4 to determine the area of the poster board.

R_x Encourage students to read the question carefully. You may wish to have them determine the area of the poster board both ways: first, by multiplying the dimensions of the picture by 4, and then, by multiplying the area of the picture by 4, so they can see that the two resulting values are not the same.

- For the Math Link, some students may multiply the dimensions of the possible boxes and then compare base areas.

R_x Make sure students understand that they must determine the square root of the base area of the game and then compare the dimensions.

Answers

Math Link

a) Answers will vary. Example: The dimensions of the square game board to the nearest hundredth of a centimetre are 11.58 cm × 11.58 cm. The game board can be stored in Boxes D and E, since they each have dimensions large enough to fit the game board.

b) Answers may vary. Example: Box D is smaller so it might be less expensive.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #4, #6, and #10. Students who have no problems with these questions can go on to the remaining Apply questions.	• To complete #4 and #6, some students may benefit from referring back to the number line in #3 of the Explore the Math. Have students verbally identify what they are looking for. They should verbalize where they believe the numbers would be located on the number line and why. Have students try selected parts of #5 and #7 before continuing. • For #10, it may help some students to draw a diagram and label the information. Referring back to Example 1 may also assist them. Have them try #11 on their own before continuing.
Math Link The Math Link on page 100 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 115.	• The Math Link reinforces students' understanding of squares, square roots, and area. • Students who need help getting started could use **BLM 3–13 Section 3.3 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students complete the following statements: • To estimate the square root of 45 to one decimal place, I would … • The part I find the easiest about estimating square roots is … • The part I find most difficult is …	• Encourage students to use the What I Need to Work On tab of their chapter Foldable to note what they continue to have difficulties with.