# Using the Pythagorean Relationship

3.4

***MathLinks 8*, pages 101–105**

**Suggested Timing**
80–100 minutes

**Materials**
- centimetre grid paper
- ruler

**Blackline Masters**
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
BLM 3–3 Chapter 3 Warm-Up
BLM 3–14 Section 3.4 Extra Practice
BLM 3–15 Section 3.4 Math Link

**Mathematical Processes**
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

**Specific Outcomes**
**SS1** Develop and apply the Pythagorean theorem to solve problems.

| Category | Question Numbers |
|---|---|
| Essential (minimum questions to cover the outcomes) | 1–3, 5, 6, 9, 10, Math Link |
| Typical | 1–3, 5, 6, 9–14, Math Link |
| Extension/Enrichment | 1–3, 10, 11, 13–16 |

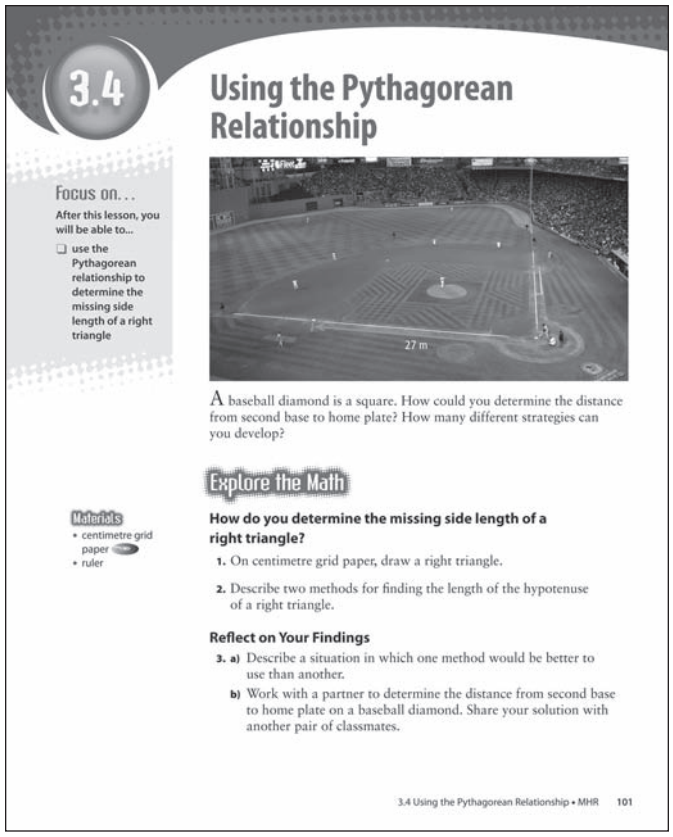
## Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce material learned in previous sections.

### Explore the Math

Students will need centimetre grid paper to draw the diagram. You may wish to provide them with **Master 8 Centimetre Grid Paper** or **Master 9 0.5 Centimetre Grid Paper**.

Have students work together to brainstorm as many strategies as they can to solve this problem. If they do not develop at least one strategy that uses the Pythagorean relationship, challenge them to consider how they could use what they learned about the Pythagorean relationship to solve this problem.


3.4

Using the Pythagorean Relationship

Focus on...

After this lesson, you will be able to...

- use the Pythagorean relationship to determine the missing side length of a right triangle




A baseball diamond is a square. How could you determine the distance from second base to home plate? How many different strategies can you develop?

Explore the Math

Materials
- centimetre grid paper
- ruler

**How do you determine the missing side length of a right triangle?**

1. On centimetre grid paper, draw a right triangle.
2. Describe two methods for finding the length of the hypotenuse of a right triangle.

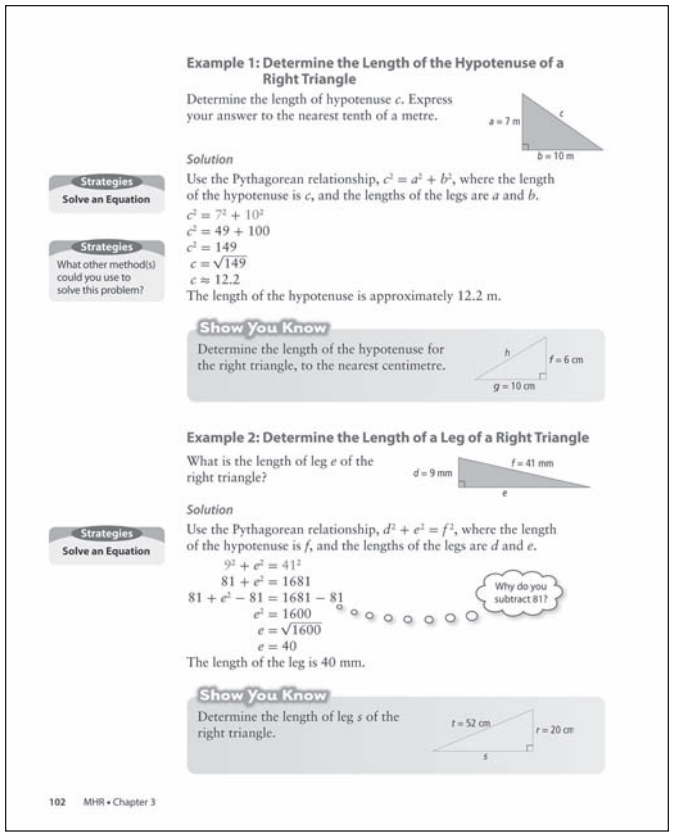
**Reflect on Your Findings**

3. a) Describe a situation in which one method would be better to use than another.
   b) Work with a partner to determine the distance from second base to home plate on a baseball diamond. Share your solution with another pair of classmates.

3.4 Using the Pythagorean Relationship • MHR 101

As students work, observe what they are doing and provide coaching questions such as
- What method could you use to solve this problem?
- Verify that your idea works.
- What other method could you use to solve this problem?
- Verify that this idea works.
- What does the Pythagorean relationship suggest about the relationship between the two legs of a right triangle and the hypotenuse?
- How can you use this information to solve this problem?

When students start to consider how they can use what they are learning to solve the question about the baseball diamond, ask someone who knows a lot about baseball to sketch a picture of a diamond on the board and label home plate, first base, second base, and third base. Students may wish to



**Example 1: Determine the Length of the Hypotenuse of a Right Triangle**

Determine the length of hypotenuse $c$. Express your answer to the nearest tenth of a metre.

$a = 7$ m, $b = 10$ m, $c$

*Solution*

**Strategies** Solve an Equation

Use the Pythagorean relationship, $c^2 = a^2 + b^2$, where the length of the hypotenuse is $c$, and the lengths of the legs are $a$ and $b$.

$c^2 = 7^2 + 10^2$
$c^2 = 49 + 100$
$c^2 = 149$
$c = \sqrt{149}$
$c \approx 12.2$

**Strategies** What other method(s) could you use to solve this problem?

The length of the hypotenuse is approximately 12.2 m.

**Show You Know**

Determine the length of the hypotenuse for the right triangle, to the nearest centimetre.

$h$, $f = 6$ cm, $g = 10$ cm

**Example 2: Determine the Length of a Leg of a Right Triangle**

What is the length of leg $e$ of the right triangle?

$d = 9$ mm, $f = 41$ mm, $e$

*Solution*

**Strategies** Solve an Equation

Use the Pythagorean relationship, $d^2 + e^2 = f^2$, where the length of the hypotenuse is $f$, and the lengths of the legs are $d$ and $e$.

$9^2 + e^2 = 41^2$
$81 + e^2 = 1681$
$81 + e^2 - 81 = 1681 - 81$
$e^2 = 1600$
$e = \sqrt{1600}$
$e = 40$

Why do you subtract 81?

The length of the leg is 40 mm.

**Show You Know**

Determine the length of leg $s$ of the right triangle.

$t = 52$ cm, $r = 20$ cm, $s$

102 MHR • Chapter 3

refer to that diagram as they discuss the Reflect on Your Findings. You may wish to provide individual coaching such as

- What are the side measurements of the square used for a baseball diamond?
- How can you use what you are learning to determine the distance from home plate to second base on a baseball diamond using these measurements?
- How can you draw a diagram to help you?

Once student pairs and groups have developed some strategies for solving this problem, have each group present their favourite strategy and describe how it works. You may wish to post examples of each strategy developed by students.

## Example 1

Example 1 provides one strategy for solving for the length of a hypotenuse if you know the length of two legs of a right triangle. If students did not develop this strategy during the Explore the Math, present this as an alternative strategy to the ones they developed. Ask them to discuss the strategy in groups and then to have one representative from each group report to the class on how the strategy works.

You may wish to provide individual coaching as students read the solution in the student resource. Questions such as the following will help them develop metacognition as they work on strategies such as this one:

- Why do you substitute the values of 7 for $a$ and 10 for $b$?
- Would it have made a difference if you had substituted 10 for $a$ and 7 for $b$?
- Why do you square 7 and 10 before adding them?
- How would the answer change if you added and then squared?
- What does the number 149 represent? How is this important?

Then, challenge students to solve the problem in the Show You Know using other strategies or using the Pythagorean relationship in a different way. Have individuals or groups share their alternative strategies with the class and verify that they work.

## Example 2

Introduce Example 2 as a class challenge: In the Explore the Math and Example 1, students worked with a right triangle in which they knew the length of the two legs. What if they know the length of the hypotenuse and one leg? Before having students read Example 2, ask them to consider how they could solve such a problem. In small groups, have them brainstorm and post ideas.

Discuss their ideas as a class, and then have pairs or small groups of students use a strategy of their choice to solve the problem in Example 2. As a class, discuss the following:

- How does your strategy differ from the one shown here?
- How is it similar?
- What do you prefer about your own strategy?
- What do you prefer about the one shown in the student resource?

Again, you may wish to provide individual coaching as students read the solution in the student resource using questions such as the following:

- Why do you substitute the values of 9 for $d$ and 41 for $f$?
- Would it have made a difference if you had substituted 41 for $d$ and 9 for $f$?
- How does the order of operations help you complete the second line?
- How can you use mental math to help you find the square root of 1600?

After discussing the differing strategies developed by students and the one presented in the student resource, ask students to solve the Show You Know using a strategy of their choice and compare their answer with someone who has used a different strategy.

## Meeting Student Needs

- Students may benefit from assistance in recalling the order of operations. Make sure they understand that determining the square root is the last step.

### ELL

- Ensure that students understand the following term in the opening paragraph for the chapter: *baseball diamond*.
- Explain how baseball is played and what the different parts of the field are called.
- Orally discuss the questions in the Reflect on Your Findings section. Simplify the language and use visuals to ensure that English language learners understand what they are being asked.

### Gifted and Enrichment

- When doing the Explore the Math, challenge students to determine the length of the hypotenuse using another shape besides a square. For example, how might they use their knowledge of circle area if they drew a circle or half circle on each side of the triangle? What other shapes might they try?

## Common Errors

- For Example 2, some students may find the sum of the square of the hypotenuse length and the square of the given leg length.

**$R_x$** Make sure they understand why they need to subtract the square of the given leg length from the square of the hypotenuse length before calculating the missing leg. You may need to guide students to think about the relative size of the missing leg length in comparison to the hypotenuse length. This check may help some students to catch their computational error.

## Answers

**Explore the Math**

**1.** Answers may vary.

**2.** Answers may vary. Example: You can measure the hypotenuse of the triangle directly, or you can use the Pythagorean relationship.

**3. a)** Answers may vary. Example: If the triangle is very large, it might not be possible to measure the hypotenuse directly. If the length of the legs are known, you could use the Pythagorean relationship to find the length of the hypotenuse.

**b)** 38.2 m

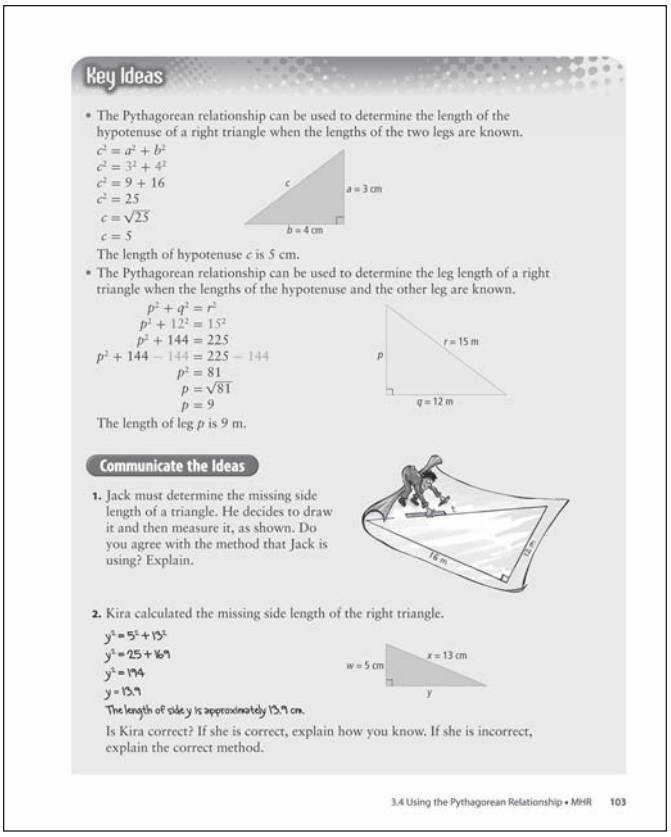
**Show You Know: Example 1**

12 cm

**Show You Know: Example 2**

48 cm

| Assessment | Supporting Learning |
|---|---|
| **Assessment *as* Learning** | |
| **Reflect on Your Findings**<br>Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings. | • Have students use a strategy with which they feel comfortable. If students make errors, you may need to discuss the strategy being used and assess whether it is effective for that type of question. |
| **Assessment *for* Learning** | |
| **Example 1**<br>Have students do the Show You Know related to Example 1. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Students may benefit from seeing the square root of both sides of the equation so that it is clear how $c \approx 12.2$ was obtained:<br>$c^2 = 149$<br>$\sqrt{c^2} = \sqrt{149}$<br>$c \approx 12.2$<br>• Discuss with students the concept of assigning variables to the sides. Make sure they understand that it does not matter which leg is, for example, $a$ or $b$, since it will not affect the answer as long as they do not confuse the lengths of the legs with the length of the hypotenuse in the equation. |
| **Example 2**<br>Have students do the Show You Know related to Example 2. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Have students draw a diagram and label it with the lengths of the hypotenuse and the leg to assist them in substituting correctly into the Pythagorean relationship. |



Key Ideas

- The Pythagorean relationship can be used to determine the length of the hypotenuse of a right triangle when the lengths of the two legs are known.

$c^2 = a^2 + b^2$
$c^2 = 3^2 + 4^2$
$c^2 = 9 + 16$
$c^2 = 25$
$c = \sqrt{25}$
$c = 5$

The length of hypotenuse $c$ is 5 cm.

- The Pythagorean relationship can be used to determine the leg length of a right triangle when the lengths of the hypotenuse and the other leg are known.

$p^2 + q^2 = r^2$
$p^2 + 12^2 = 15^2$
$p^2 + 144 = 225$
$p^2 + 144 - 144 = 225 - 144$
$p^2 = 81$
$p = \sqrt{81}$
$p = 9$

The length of leg $p$ is 9 m.

Communicate the Ideas

1. Jack must determine the missing side length of a triangle. He decides to draw it and then measure it, as shown. Do you agree with the method that Jack is using? Explain.

2. Kira calculated the missing side length of the right triangle.

$y^2 = 5^2 + 13^2$
$y^2 = 25 + 169$
$y^2 = 194$
$y = 13.9$

The length of side $y$ is approximately 13.9 cm.

Is Kira correct? If she is correct, explain how you know. If she is incorrect, explain the correct method.

3.4 Using the Pythagorean Relationship • MHR 103

## Communicate the Ideas

In #1, students must understand that when a right triangle is very large, measurement is not the best method for determining an unknown side length. Students may not answer this question correctly if they fail to note the significance of the side lengths being in metres.

In #2, students complete some error analysis by looking at one of the most common mistakes made when using the Pythagorean relationship: substituting the values into the formula incorrectly.

## Answers

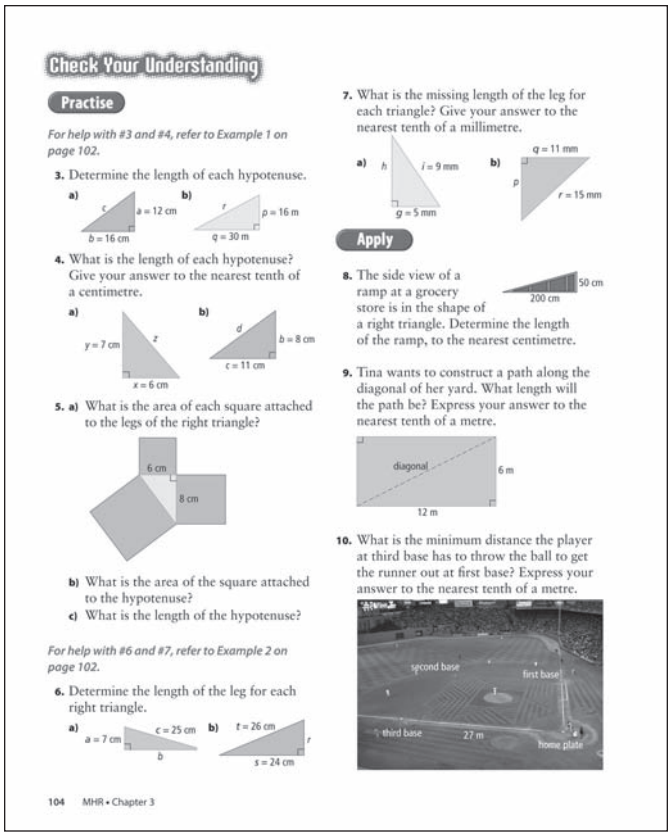
**Communicate the Ideas**

1. No, this method is not effective because the triangle is too large. Jack should use the Pythagorean relationship since the lengths of the two legs are known.
2. No, Kira is not correct. She should have written the equation as $13^2 = 5^2 + y^2$ instead of $y^2 = 5^2 + 13^2$.

Key Ideas

This section reinforces how to use the Pythagorean relationship to calculate the length of the hypotenuse or a leg, given the other two sides. As students write Key Ideas notes in their Foldable or notebook, encourage them to include their own examples in addition to, or instead of, the ones in the student resource.

| Assessment | Supporting Learning |
|---|---|
| **Assessment *as* Learning** | |
| **Communicate the Ideas**<br>Have all students complete #1 and #2. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Students who need assistance with #1 may benefit from referring back to Reflect on Your Findings #4b) to look for similarities.<br>• It may help students who are unsure of how to answer #2 to sketch their own copy of the diagram and label it. Then, coach them through determining the missing side length, without looking at Kira's calculations. Have them compare their calculations with Kira's. It may also help to have them refer back to the steps in Example 2. |



Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on page 102.

3. Determine the length of each hypotenuse.

a) $a = 12$ cm, $b = 16$ cm

b) $p = 16$ m, $q = 30$ m

4. What is the length of each hypotenuse? Give your answer to the nearest tenth of a centimetre.

a) $y = 7$ cm, $x = 6$ cm

b) $b = 8$ cm, $c = 11$ cm

5. a) What is the area of each square attached to the legs of the right triangle?

6 cm, 8 cm

b) What is the area of the square attached to the hypotenuse?

c) What is the length of the hypotenuse?

For help with #6 and #7, refer to Example 2 on page 102.

6. Determine the length of the leg for each right triangle.

a) $a = 7$ cm, $c = 25$ cm

b) $t = 26$ cm, $s = 24$ cm

7. What is the missing length of the leg for each triangle? Give your answer to the nearest tenth of a millimetre.

a) $i = 9$ mm, $g = 5$ mm

b) $q = 11$ mm, $r = 15$ mm

Apply

8. The side view of a ramp at a grocery store is in the shape of a right triangle. Determine the length of the ramp, to the nearest centimetre.

50 cm, 200 cm

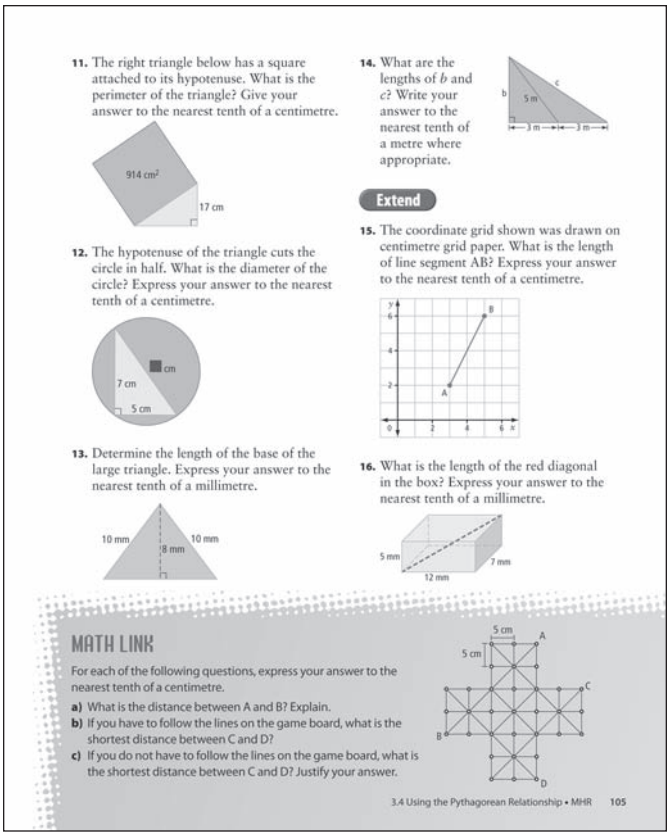
9. Tina wants to construct a path along the diagonal of her yard. What length will the path be? Express your answer to the nearest tenth of a metre.

diagonal, 6 m, 12 m

10. What is the minimum distance the player at third base has to throw the ball to get the runner out at first base? Express your answer to the nearest tenth of a metre.

second base, first base, third base, 27 m, home plate

104 MHR • Chapter 3



11. The right triangle below has a square attached to its hypotenuse. What is the perimeter of the triangle? Give your answer to the nearest tenth of a centimetre.

914 cm², 17 cm

12. The hypotenuse of the triangle cuts the circle in half. What is the diameter of the circle? Express your answer to the nearest tenth of a centimetre.

7 cm, 5 cm

13. Determine the length of the base of the large triangle. Express your answer to the nearest tenth of a millimetre.

10 mm, 10 mm, 8 mm

14. What are the lengths of $b$ and $c$? Write your answer to the nearest tenth of a metre where appropriate.

5 m, 3 m, 3 m

Extend

15. The coordinate grid shown was drawn on centimetre grid paper. What is the length of line segment AB? Express your answer to the nearest tenth of a centimetre.

A, B

16. What is the length of the red diagonal in the box? Express your answer to the nearest tenth of a millimetre.

5 mm, 7 mm, 12 mm

MATH LINK

For each of the following questions, express your answer to the nearest tenth of a centimetre.

a) What is the distance between A and B? Explain.

b) If you have to follow the lines on the game board, what is the shortest distance between C and D?

c) If you do not have to follow the lines on the game board, what is the shortest distance between C and D? Justify your answer.

5 cm, 5 cm, A, B, C, D

3.4 Using the Pythagorean Relationship • MHR 105

# Check Your Understanding

## Practise

In #3 to #5, students are given opportunities to calculate the length of the hypotenuse when given the length of both legs of a right triangle. In #6 and #7, students are provided with the chance to calculate the length of the missing leg of a right triangle.

## Apply

Question 10 on page 104 is very similar to the final question in the Reflect on Your Findings on page 101. You may wish to assign this question to all students and encourage them to use a different strategy from the one they used in the Explore the Math. Note that the same measurements are used. This time, students are asked to find the shortest distance from first base to third base.

Once student have completed #12, you may want to have them explore inscribing various circles with triangles that have the diameter as their hypotenuse, so students can see that the result is always a right triangle. For #13, students may need to be instructed that the height of the isosceles triangle bisects the base.

## Extend

In #15, students will need to view the line segment as the hypotenuse of a right triangle. This question introduces students to an important concept of coordinate geometry that they will use in later years. This concept involves finding the distance between any two points on a coordinate grid.

### Math Link

In part c), students need to recognize that the shortest distance means going beyond the boundaries of the board. Emphasize that it is acceptable to do this in order to answer the question.

## *Meeting Student Needs*

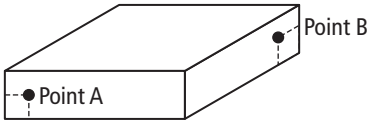
- Provide **BLM 3–14 Section 3.4 Extra Practice** to students who would benefit from more practice.

### ELL

- Ensure that students understand the following terms: *ramp*, *grocery store*, and *minimum distance*.

### Gifted and Enrichment

- To add a challenge to #16, have students determine the length from the top corner of the classroom to the opposite bottom corner. As an additional challenge, choose Point A on the classroom wall and then choose Point B on a different wall (see the diagram). Ask students what the minimum distance is that a bug must travel to get from Point A to Point B if the bug can walk along the floor, walls, and ceiling. Make sure that students work with whole number values only so they are not going beyond the grade 8 curriculum.




## Answers

**Math Link**

**a)** 28.3 cm **b)** 24.1 cm **c)** 22.4 cm

| Assessment | Supporting Learning |
|---|---|
| **Assessment *for* Learning** | |
| **Practise and Apply**<br>Have students do #3, #5, #6, #9, and #10. Students who have no problems with these questions can go on to the remaining Apply questions. | • Students who are unsure of how to answer #3 should be encouraged to use both a diagram and the equation. Coach students through #3b) and have them try #4a) before continuing with other questions.<br>• For #5, students may benefit from reviewing Example 1. Have them label the sides of their diagrams with *hypotenuse*, *leg*, and *leg*, as well as the values for the areas. Encourage them to show all of their thinking.<br>• Students who need assistance with #6 should also be encouraged to label their triangles and to review Example 2. Coach them through #6b) and then have them try #7a) on their own.<br>• For #9, students may find it helpful to identify the hypotenuse and to review Example 1. Coach them through #9 and then have them try #10 on their own.<br>• For #10, students may find it beneficial to work with a partner who used a different strategy from the one they used in the Explore the Math. |
| **Math Link**<br>The Math Link on page 105 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 115. | • Students who need help getting started could use **BLM 3–15 Section 3.4 Math Link**, which provides scaffolding.<br>• Students may need to be reminded that the shortest distance between any two points is a straight line.<br>• Have students compare their answers with a classmate's. |
| **Assessment *as* Learning** | |
| **Math Learning Log**<br>Give students a piece of centimetre grid paper and the following instructions: Carefully measure and draw a right triangle with legs of 7 cm and 10 cm. Determine the length of the hypotenuse, using a ruler and using the Pythagorean relationship. Have students compare the two answers and complete the following statements:<br>• The method I prefer is … because …<br>• I get confused when … | • This activity encompasses the important points of this section. As a second or alternative activity, you may wish to have students measure and draw a right triangle with a leg of 7 cm and a hypotenuse of 10 cm and then complete the activity as described.<br>• You may wish to provide students with **Master 8 Centimetre Grid Paper**.<br>• Encourage students to use the What I Need to Work On tab of their chapter Foldable to note what they continue to have difficulties with. |