Applying the Pythagorean Relationship

3.5

***MathLinks 8*, pages 106–111**

Suggested Timing
80–100 minutes

Blackline Masters
BLM 3–3 Chapter 3 Warm-Up
BLM 3–16 Section 3.5 Extra Practice
BLM 3–17 Section 3.5 Math Link

Mathematical Processes
- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Mathematics and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ☐ Technology (T)
- ✓ Visualization (V)

Specific Outcomes
SS1 Develop and apply the Pythagorean theorem to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, Math Link
Typical	1–3, 5, 7–10, 13, Math Link
Extension/Enrichment	1–3, 8, 11–14

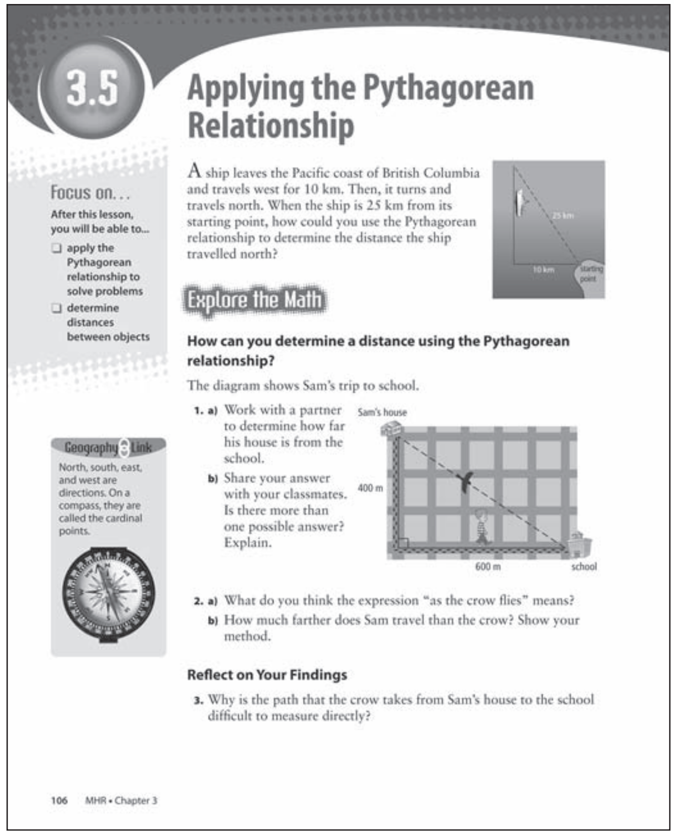
Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce material learned in previous sections.

Discuss the opening paragraph with students. They explored determining the missing leg length of a right triangle in section 3.4, so they may know how to answer the question. You might wish to have students wait to solve the problem, since they will have a chance to do so in the Example 1 Show You Know.

Explore the Math

Depending on the location of your school, there may be interesting shortcuts that cut through parts of the school property to shorten students' walk to the school door. Discuss these with students to set the stage for this Explore the Math. For example, some students


3.5

Applying the Pythagorean Relationship

Focus on...

After this lesson, you will be able to...
- ☐ apply the Pythagorean relationship to solve problems
- ☐ determine distances between objects

A ship leaves the Pacific coast of British Columbia and travels west for 10 km. Then, it turns and travels north. When the ship is 25 km from its starting point, how could you use the Pythagorean relationship to determine the distance the ship travelled north?

Explore the Math

How can you determine a distance using the Pythagorean relationship?

The diagram shows Sam's trip to school.

1. a) Work with a partner to determine how far his house is from the school.

b) Share your answer with your classmates. Is there more than one possible answer? Explain.

2. a) What do you think the expression "as the crow flies" means?

b) How much farther does Sam travel than the crow? Show your method.

Reflect on Your Findings

3. Why is the path that the crow takes from Sam's house to the school difficult to measure directly?

Geography Link

North, south, east, and west are directions. On a compass, they are called the cardinal points.

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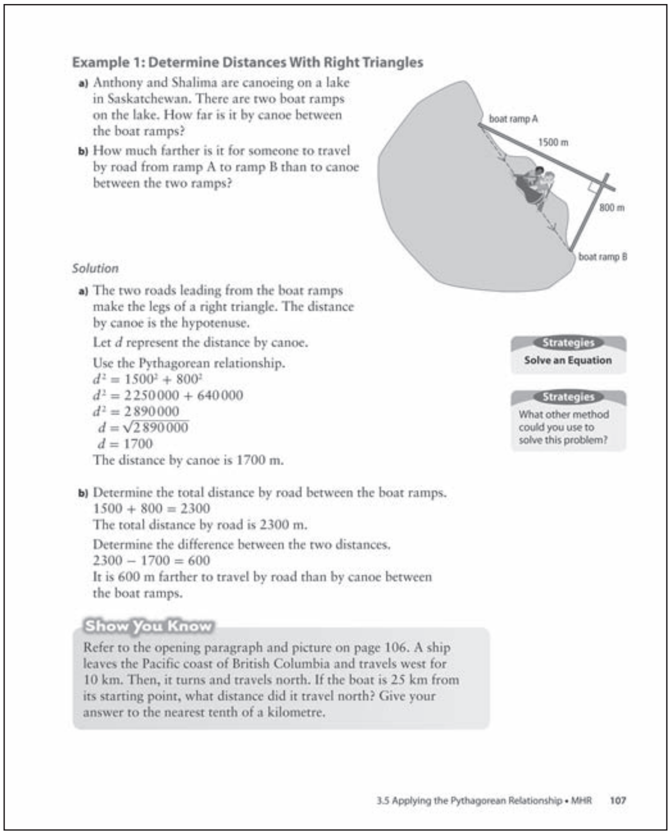
may cut across the playing field to get to the school because they know that route is shorter than walking around the playing field. If such a shortcut exists, sketch the rectangle the roads make and then show the shortcut. Discuss why students take the shortcut. This discussion can lead to Sam's trip to school.

Have students consider the map showing Sam's house and school.
- What would be the shortest way for Sam to get from his house to the school?
- What problems might he run into if he tried to take that route?

Challenge students to do the Explore the Math to find out what distance Sam could save if he could go "as the crow flies." Have them work in pairs. As pairs discuss their ideas, you may wish to provide coaching prompts such as the following to students who are not sure how to proceed:
- What kind of triangle do the dotted lines show on this map?
- What strategies have we developed as a class to solve questions dealing with this type of triangle?



Example 1: Determine Distances With Right Triangles

a) Anthony and Shalima are canoeing on a lake in Saskatchewan. There are two boat ramps on the lake. How far is it by canoe between the boat ramps?

b) How much farther is it for someone to travel by road from ramp A to ramp B than to canoe between the two ramps?

Solution

a) The two roads leading from the boat ramps make the legs of a right triangle. The distance by canoe is the hypotenuse.

Let d represent the distance by canoe.

Use the Pythagorean relationship.

$d^2 = 1500^2 + 800^2$
$d^2 = 2\,250\,000 + 640\,000$
$d^2 = 2\,890\,000$
$d = \sqrt{2\,890\,000}$
$d = 1700$

The distance by canoe is 1700 m.

Strategies
Solve an Equation

Strategies
What other method could you use to solve this problem?

b) Determine the total distance by road between the boat ramps.

$1500 + 800 = 2300$

The total distance by road is 2300 m.

Determine the difference between the two distances.

$2300 - 1700 = 600$

It is 600 m farther to travel by road than by canoe between the boat ramps.

Show You Know

Refer to the opening paragraph and picture on page 106. A ship leaves the Pacific coast of British Columbia and travels west for 10 km. Then, it turns and travels north. If the boat is 25 km from its starting point, what distance did it travel north? Give your answer to the nearest tenth of a kilometre.

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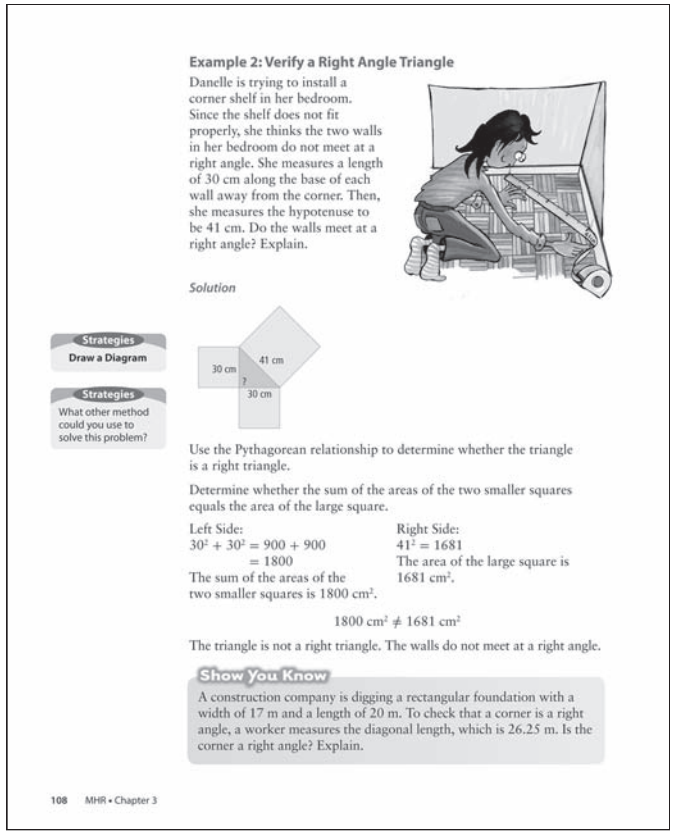

Example 2: Verify a Right Angle Triangle

Danelle is trying to install a corner shelf in her bedroom. Since the shelf does not fit properly, she thinks the two walls in her bedroom do not meet at a right angle. She measures a length of 30 cm along the base of each wall away from the corner. Then, she measures the hypotenuse to be 41 cm. Do the walls meet at a right angle? Explain.

Solution

Strategies
Draw a Diagram

Strategies
What other method could you use to solve this problem?

Use the Pythagorean relationship to determine whether the triangle is a right triangle.

Determine whether the sum of the areas of the two smaller squares equals the area of the large square.

Left Side:
$30^2 + 30^2 = 900 + 900$
$= 1800$
The sum of the areas of the two smaller squares is 1800 cm^2.

Right Side:
$41^2 = 1681$
The area of the large square is 1681 cm^2.

$$1800\text{ cm}^2 \neq 1681\text{ cm}^2$$

The triangle is not a right triangle. The walls do not meet at a right angle.

Show You Know

A construction company is digging a rectangular foundation with a width of 17 m and a length of 20 m. To check that a corner is a right angle, a worker measures the diagonal length, which is 26.25 m. Is the corner a right angle? Explain.

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- What strategy might work here?
- Try it.
- Did that strategy work for you?
- Why or why not?
- Consider how else you might solve this problem.
- How can you show your thinking as you work on a problem such as this one?

Method 1 Have students work in pairs to answer #1 and #2. Have two sets of pairs get together to compare their answers and to discuss #3. Encourage students to review the strategies each pair used and how they might differ from and resemble each other.

Method 2 To work on this type of problem, challenge groups to choose one of the strategies the class developed during section 3.4 (the ones you posted on the wall). Divide the class into groups according to the strategy they chose. Have each group use their specific strategy to solve the Explore the Math. Once the groups have completed #1 to #3, have them compare their solutions and then discuss the advantages, disadvantages, and relative efficiency of each method.

Example 1

Challenge students to consider what solution they would have provided for Example 1 if they had been writing this student resource. Ask them to develop that solution using the strategy they prefer. Have individuals or groups share their alternative suggestions with the class.

Discuss as a class methods students used to record their thinking as they worked. Have them consider why it might be useful to record their thinking.

Encourage students to use the strategy they prefer to solve the Show You Know.

Example 2

Like Example 1, Example 2 provides a context for applying the Pythagorean relationship. Discuss as a class the strategies students might use to prove that a given triangle is a right triangle. Have them try the strategy they prefer to solve this question and decide whether this is a right triangle.

You may wish to discuss with students how, with questions like this one, they need to take into account the fact that the measurements may not have been exact. Therefore, it may be difficult to decide if the sum of the areas of the smaller squares compared to the area of the large square is "close enough" that the angle may be considered a right angle. Explain to students that they have to use their own judgment and that their answer should be reasonable and justified.

The actual angle between the walls is 86.2°. Students will learn how to calculate this angle in future math courses involving trigonometry.

Meeting Student Needs

- For Explore the Math #3, it might help concrete learners to think about their own path to school. Ask them why it is difficult to measure the distance in a straight line between their home and the school.
- Encourage students to solve these problems using the strategies they have developed earlier in the chapter. Ask them to show their thinking so that you can help them identify where calculation or thinking errors may occur.

ELL

- Orally explain the introduction question. Using the picture in the student resource, point to the ship and the route it takes. Review cardinal directions (north, south, east, and west).
- Explain the expression *as the crow flies*. Show a picture of a real crow.
- Ensure that students understand the following terms: *canoe*, *boat ramp*, and *corner shelf*.
- In the Example 2 Show You Know, you might need to explain what the foundation of a house or building is to students who are not familiar with this term.

Answers

Explore the Math

1. a) Answers may be either 1000 m or 721.1 m.

b) Answers may vary. Example: Yes, there is more than one possible answer. You can calculate based on Sam's route or the crow's route.

2. a) Answers may vary. Example: "As the crow flies" means in a straight path.

b) 278.9 m

3. Answers will vary. Example: The crow's path along the ground might be blocked by houses and other obstacles. Also, the distance is too great to measure directly.

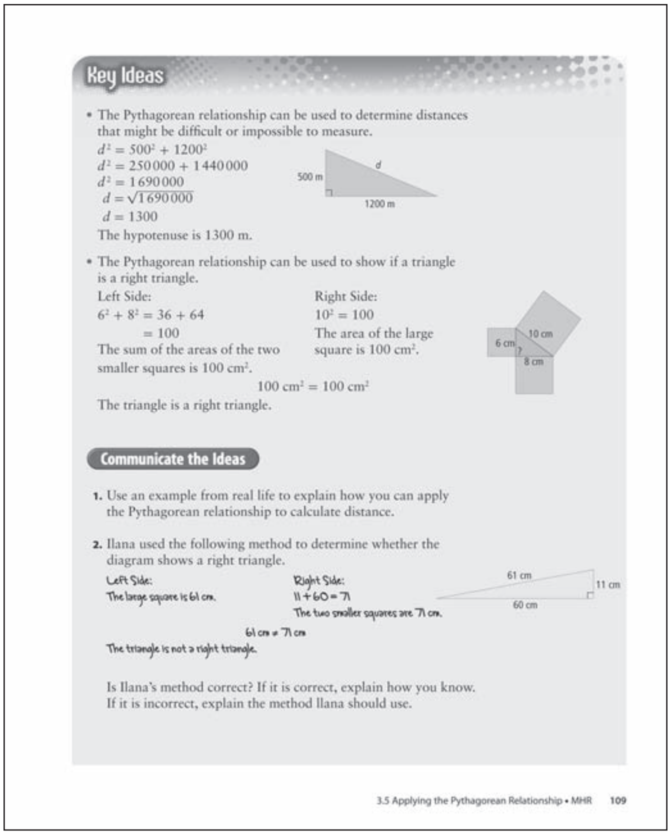
Show You Know: Example 1

22.9 km

Show You Know: Example 2

Yes, the corner is a right angle. $17^2 + 20^2 \approx 26.25^2$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings. They should be able to identify that the crow flies along the hypotenuse whereas Sam walks along both legs of the right triangle.	• Discuss what advantages a bird has when it travels that humans do not (e.g., a bird can fly over houses and other obstacles).
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to draw a diagram and label it. • For the Show You Know, check that students are calculating the leg of the right triangle and not the hypotenuse.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Suggest that students first calculate the areas in order to help them visualize how to approach this problem.



Key Ideas

- The Pythagorean relationship can be used to determine distances that might be difficult or impossible to measure.

$d^2 = 500^2 + 1200^2$
$d^2 = 250\,000 + 1\,440\,000$
$d^2 = 1\,690\,000$
$d = \sqrt{1\,690\,000}$
$d = 1300$

The hypotenuse is 1300 m.

- The Pythagorean relationship can be used to show if a triangle is a right triangle.

Left Side:
$6^2 + 8^2 = 36 + 64$
$= 100$
The sum of the areas of the two smaller squares is 100 cm².

Right Side:
$10^2 = 100$
The area of the large square is 100 cm².

$100 \text{ cm}^2 = 100 \text{ cm}^2$

The triangle is a right triangle.

Communicate the Ideas

1. Use an example from real life to explain how you can apply the Pythagorean relationship to calculate distance.

2. Ilana used the following method to determine whether the diagram shows a right triangle.

Left Side:
The large square is 61 cm.

Right Side:
11 + 60 = 71
The two smaller squares are 71 cm.

61 cm ≠ 71 cm
The triangle is not a right triangle.

Is Ilana's method correct? If it is correct, explain how you know. If it is incorrect, explain the method Ilana should use.

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Key Ideas

The Key Ideas sum up the two main concepts of this final section of the chapter. The first concept is using the Pythagorean relationship to measure distances that are not easily computed by direct measurement. The second concept is using the Pythagorean relationship to check whether or not a triangle has a right angle, given the three side lengths.

Communicate the Ideas

The open-ended nature of #1 may make it a challenging question for some learners. Consider assigning it after students have completed #2. Question 1 might be an effective prompt for the Math Learning Log. Alternatively, brainstorm some ideas with the class, listing them on chart paper. Allow students to use the ideas for their response or as springboards to develop ideas of their own.

Meeting Student Needs

ELL

- Consider allowing students to answer the Communicate the Ideas questions in their own language first. After doing so, students might then find it easier to express their thinking in English.

Common Errors

- Students may not be able to identify the error in #2.

R_x This question involves a common error that students make when they are checking whether a triangle has a right angle, given the three side lengths. To remind students that the Pythagorean relationship involves the squares of the side lengths, not simply the side lengths, have them copy the triangle in #2 and draw the squares attached to each side.

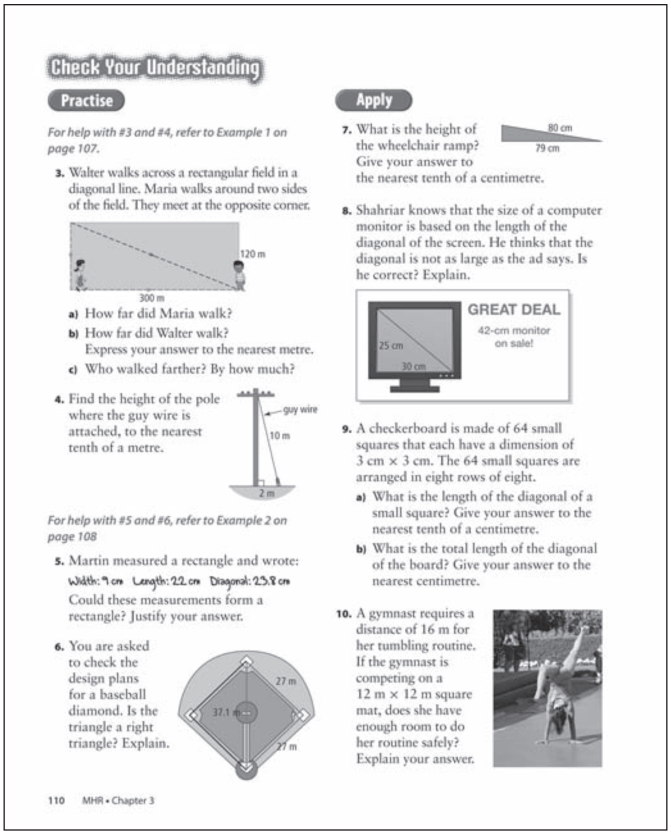
Answers

Communicate the Ideas

1. Answers will vary. Example: It is possible to determine if a piece of plywood is rectangular by measuring the diagonals and the dimensions of the sides. Apply the Pythagorean relationship to determine if the square of the length added to the square of the width equals the square of the diagonal. If this is true, the piece of plywood is rectangular.

2. No, Ilana is not correct. Answers may vary. Example: She compared the lengths of the three sides instead of the squares of the lengths of the three sides. Square the hypotenuse: $61^2 = 3721$. Find the sum of the squares of the two legs: $11^2 + 60^2 = 3721$, which equals the square of the hypotenuse. Therefore, the triangle is a right triangle.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have students complete #1 and #2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may need assistance with creating a real-life situation on their own for #1. Circulate and help students with ideas. • To help them with #2, have students review Example 2. • For #2, encourage students to write the equation, show all of their thinking, and use the form Left Side = Right Side.



Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on page 107.

3. Walter walks across a rectangular field in a diagonal line. Maria walks around two sides of the field. They meet at the opposite corner.

   a) How far did Maria walk?
   b) How far did Walter walk? Express your answer to the nearest metre.
   c) Who walked farther? By how much?

4. Find the height of the pole where the guy wire is attached, to the nearest tenth of a metre.

For help with #5 and #6, refer to Example 2 on page 108

5. Martin measured a rectangle and wrote:
   Width: 9 cm Length: 22 cm Diagonal: 23.8 cm
   Could these measurements form a rectangle? Justify your answer.

6. You are asked to check the design plans for a baseball diamond. Is the triangle a right triangle? Explain.

Apply

7. What is the height of the wheelchair ramp? Give your answer to the nearest tenth of a centimetre.

8. Shahriar knows that the size of a computer monitor is based on the length of the diagonal of the screen. He thinks that the diagonal is not as large as the ad says. Is he correct? Explain.

9. A checkerboard is made of 64 small squares that each have a dimension of 3 cm × 3 cm. The 64 small squares are arranged in eight rows of eight.
   a) What is the length of the diagonal of a small square? Give your answer to the nearest tenth of a centimetre.
   b) What is the total length of the diagonal of the board? Give your answer to the nearest centimetre.

10. A gymnast requires a distance of 16 m for her tumbling routine. If the gymnast is competing on a 12 m × 12 m square mat, does she have enough room to do her routine safely? Explain your answer.

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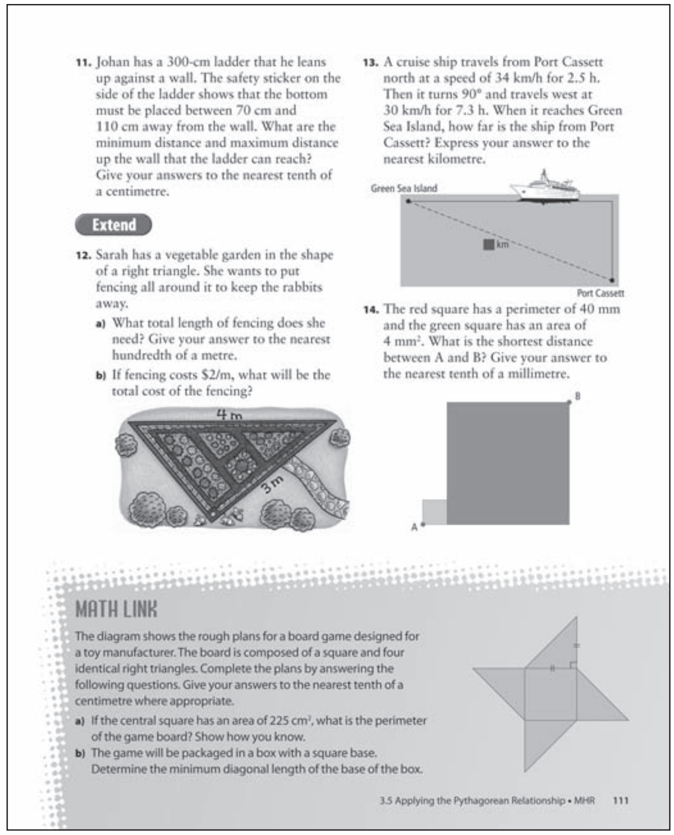

11. Johan has a 300-cm ladder that he leans up against a wall. The safety sticker on the side of the ladder shows that the bottom must be placed between 70 cm and 110 cm away from the wall. What are the minimum distance and maximum distance up the wall that the ladder can reach? Give your answers to the nearest tenth of a centimetre.

Extend

12. Sarah has a vegetable garden in the shape of a right triangle. She wants to put fencing all around it to keep the rabbits away.
    a) What total length of fencing does she need? Give your answer to the nearest hundredth of a metre.
    b) If fencing costs $2/m, what will be the total cost of the fencing?

13. A cruise ship travels from Port Cassett north at a speed of 34 km/h for 2.5 h. Then it turns 90° and travels west at 30 km/h for 7.3 h. When it reaches Green Sea Island, how far is the ship from Port Cassett? Express your answer to the nearest kilometre.

14. The red square has a perimeter of 40 mm and the green square has an area of 4 mm². What is the shortest distance between A and B? Give your answer to the nearest tenth of a millimetre.

MATH LINK

The diagram shows the rough plans for a board game designed for a toy manufacturer. The board is composed of a square and four identical right triangles. Complete the plans by answering the following questions. Give your answers to the nearest tenth of a centimetre where appropriate.

a) If the central square has an area of 225 cm², what is the perimeter of the game board? Show how you know.
b) The game will be packaged in a box with a square base. Determine the minimum diagonal length of the base of the box.

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Check Your Understanding

Practise

These four questions are application problems, due to the nature of this chapter section. The questions involve the same steps as Examples 1 and 2, though the contexts are different.

Apply

The Apply section also includes application questions, though these ones vary more widely from the sample questions in Examples 1 and 2.

Extend

The first two problems are classified as extension questions because they involve additional calculations beyond those covered in the previous questions. In #12, students calculate the perimeter of a fence and then use a cost rate to determine the price of the fencing. In #13, students must use the given speed and time to calculate the distance a cruise ship travels.

In #14, students use their knowledge about squares and their problem solving skills to determine a distance.

Math Link

Remind students that the perimeter of the game is the outer perimeter. Some students may mistakenly add the side lengths of the square, which are not part of the game's perimeter.

Meeting Student Needs

- In the Apply section, have students identify the dimensions found in each question and then draw and label a diagram with the dimensions. Assist them, as necessary, to identify which dimensions relate to each part of the diagram.
- Provide **BLM 3–16 Section 3.5 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure that students understand the following terms: *guy wire*, *wheelchair*, and *computer monitor*.

Common Errors

- Students may become confused by the multiple steps involved in the questions.

R_x Encourage students to record their calculations in an organized fashion vertically down the page.

Answers

Math Link

a) Perimeter ≈ 144.8 cm **b)** Minimum diagonal length ≈ 47.4 cm

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #3, #5, and #7. Students who have no problems with these questions can go on to the remaining Apply questions.	• Some learners will benefit from reviewing Examples 1 and 2 before attempting the Practise questions. • For students who do not know how to begin #3 and #5, have them draw their own simple version of the diagrams. The same approach should be taken with #7; however, guiding students to understand that the ramp is the longest side may also be beneficial. • For those problems that do not include a diagram, such as #9, encourage students to sketch one.
Math Link The Math Link on page 111 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 115.	• It is not essential for students to complete this Math Link, but it is helpful for those students who will create a game board design in the Wrap It Up! Encourage students to show their work. • Students who need help getting started could use **BLM 3–17 Section 3.5 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students respond to the following prompt: • Provide an example from real life in which you would need to calculate an unknown distance using the Pythagorean relationship because it is either impossible or difficult to measure the distance directly.	• Encourage students to use the What I Need to Work On tab of their chapter Foldable to note what they continue to have difficulties with. • For students who need assistance, you may wish to invite a carpenter into the classroom to talk about how to check for right angles when it is impossible to measure the angle itself. Many carpenters use the 3 : 4 : 5 principle in such measurements.