Fractions, Decimals, and Percents

4.2

***MathLinks 8*, pages 130–137**

Suggested Timing
80–100 minutes

Materials
- hundred grids
- calculator (optional)

Blackline Masters
Master 10 Hundred Grids
BLM 4–3 Chapter 4 Warm-Up
BLM 4–7 Section 4.2 Explore the Math
BLM 4–8 Section 4.2 Extra Practice
BLM 4–9 Section 4.2 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

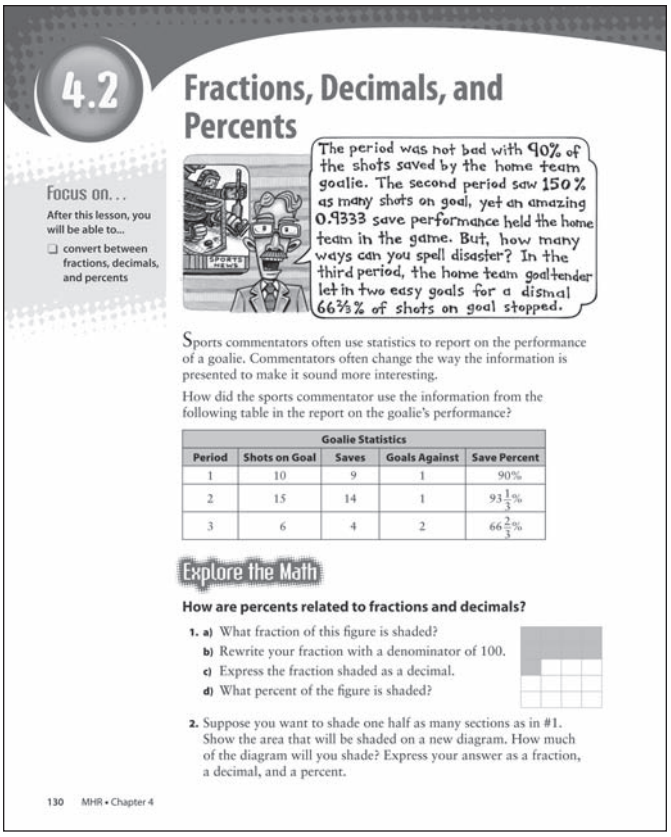
Specific Outcomes
N3 Demonstrate an understanding of percents greater than or equal to 0%.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 4, 6, 8, 13, Math Link
Typical	1, 2, 4, 6, 8, 13–18, Math Link
Extension/Enrichment	1, 2, 10, 17, 21

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce material learned in previous sections.

As a class, read and discuss the cartoon. Have students share how the sports commentator used the statistics in the table to report on the goalie's performance. Use other sports statistics to begin a discussion about how fractions, decimals, and percents are used interchangeably in sports reports. To do this, collect sports statistics of interest to your students, or record and play an excerpt from a sports commentator's report. See the Web Link on TR page 170 for NHL statistics you might use.



4.2 Fractions, Decimals, and Percents

Focus on...
After this lesson, you will be able to...
- convert between fractions, decimals, and percents

The period was not bad with 90% of the shots saved by the home team goalie. The second period saw 150% as many shots on goal, yet an amazing 0.9333 save performance held the home team in the game. But, how many ways can you spell disaster? In the third period, the home team goaltender let in two easy goals for a dismal 66⅔% of shots on goal stopped.

Sports commentators often use statistics to report on the performance of a goalie. Commentators often change the way the information is presented to make it sound more interesting.

How did the sports commentator use the information from the following table in the report on the goalie's performance?

Goalie Statistics

Period	Shots on Goal	Saves	Goals Against	Save Percent
1	10	9	1	90%
2	15	14	1	$93\frac{1}{3}\%$
3	6	4	2	$66\frac{2}{3}\%$

Explore the Math

How are percents related to fractions and decimals?

1. a) What fraction of this figure is shaded?
   b) Rewrite your fraction with a denominator of 100.
   c) Express the fraction shaded as a decimal.
   d) What percent of the figure is shaded?

2. Suppose you want to shade one half as many sections as in #1. Show the area that will be shaded on a new diagram. How much of the diagram will you shade? Express your answer as a fraction, a decimal, and a percent.

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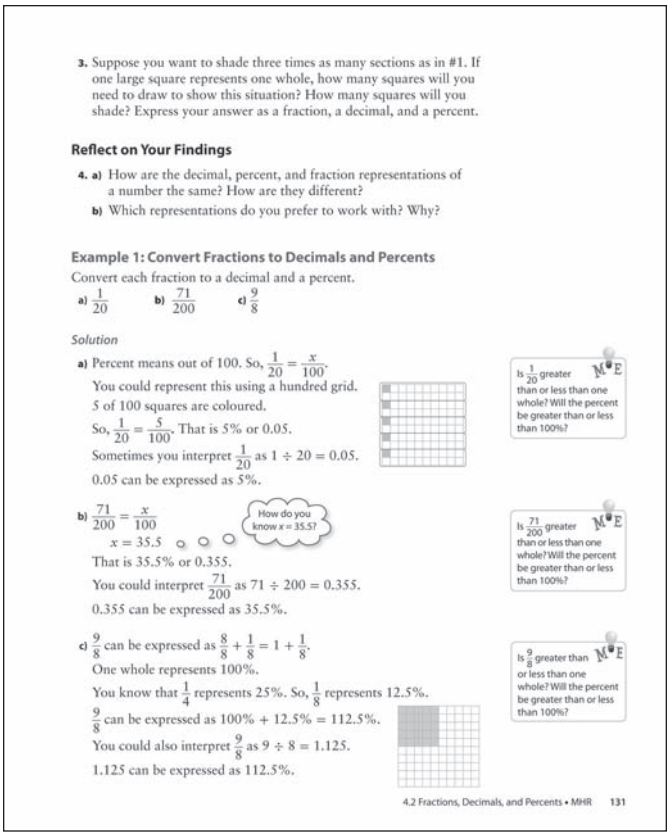
Explore the Math

In this exploration, students compare fraction, decimal, and percent representations of a number.

Method 1 Have students work in small groups. Encourage them to compare the new percent values they are exploring to what they know already about percents between 1% and 100%. Have the groups share and discuss their findings with the class. Encourage students to share what they have learned.

Method 2 Have students start by working with **BLM 4–7 Section 4.2 Explore the Math** and **Master 10 Hundred Grids**. Have them shade the twenty grid as in #1, write the shaded part as a fraction, and then shade one of the hundred grids using the same fraction. Have students record the number they worked with as equivalent fractions, a decimal, and a percent ($\frac{9}{20} = \frac{45}{100} = 0.45 = 45\%$). For #2, have students shade a second twenty grid with half the squares they did in #1. Discuss how they can do this. Next, have them shade the same



3. Suppose you want to shade three times as many sections as in #1. If one large square represents one whole, how many squares will you need to draw to show this situation? How many squares will you shade? Express your answer as a fraction, a decimal, and a percent.

Reflect on Your Findings

4. a) How are the decimal, percent, and fraction representations of a number the same? How are they different?
b) Which representations do you prefer to work with? Why?

Example 1: Convert Fractions to Decimals and Percents

Convert each fraction to a decimal and a percent.

a) $\frac{1}{20}$ b) $\frac{71}{200}$ c) $\frac{9}{8}$

Solution

a) Percent means out of 100. So, $\frac{1}{20} = \frac{x}{100}$.
You could represent this using a hundred grid.
5 of 100 squares are coloured.
So, $\frac{1}{20} = \frac{5}{100}$. That is 5% or 0.05.
Sometimes you interpret $\frac{1}{20}$ as $1 \div 20 = 0.05$.
0.05 can be expressed as 5%.

Is $\frac{1}{20}$ greater than or less than one whole? Will the percent be greater than or less than 100%?

b) $\frac{71}{200} = \frac{x}{100}$
$x = 35.5$
How do you know $x = 35.5$?
That is 35.5% or 0.355.
You could interpret $\frac{71}{200}$ as $71 \div 200 = 0.355$.
0.355 can be expressed as 35.5%.

Is $\frac{71}{200}$ greater than or less than one whole? Will the percent be greater than or less than 100%?

c) $\frac{9}{8}$ can be expressed as $\frac{8}{8} + \frac{1}{8} = 1 + \frac{1}{8}$.
One whole represents 100%.
You know that $\frac{1}{4}$ represents 25%. So, $\frac{1}{8}$ represents 12.5%.
$\frac{9}{8}$ can be expressed as 100% + 12.5% = 112.5%.
You could also interpret $\frac{9}{8}$ as $9 \div 8 = 1.125$.
1.125 can be expressed as 112.5%.

Is $\frac{9}{8}$ greater than or less than one whole? Will the percent be greater than or less than 100%?

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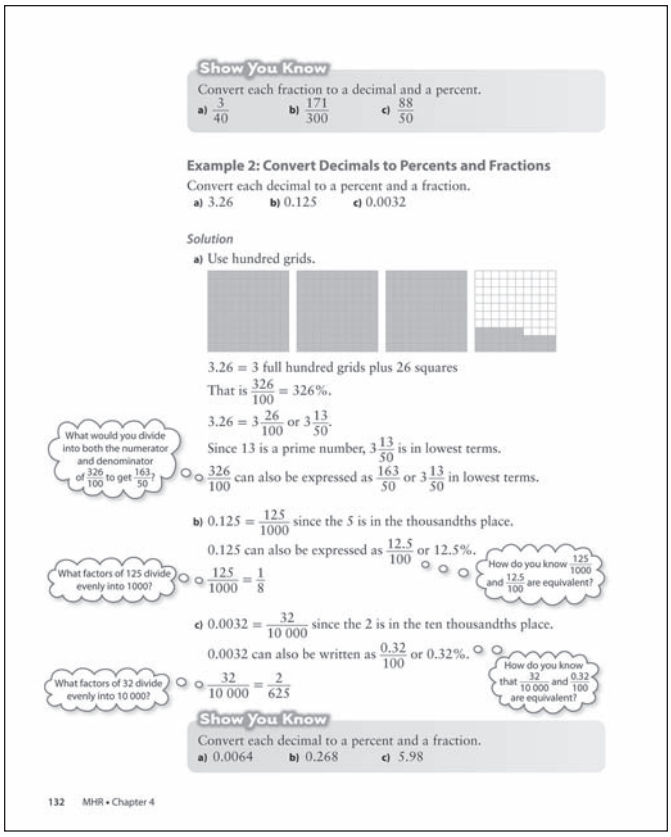

Show You Know

Convert each fraction to a decimal and a percent.

a) $\frac{3}{40}$ b) $\frac{171}{300}$ c) $\frac{88}{50}$

Example 2: Convert Decimals to Percents and Fractions

Convert each decimal to a percent and a fraction.

a) 3.26 b) 0.125 c) 0.0032

Solution

a) Use hundred grids.

3.26 = 3 full hundred grids plus 26 squares
That is $\frac{326}{100}$ = 326%.
$3.26 = 3\frac{26}{100}$ or $3\frac{13}{50}$.
Since 13 is a prime number, $3\frac{13}{50}$ is in lowest terms.
What would you divide into both the numerator and denominator of $\frac{326}{100}$ to get $\frac{163}{50}$?
$\frac{326}{100}$ can also be expressed as $\frac{163}{50}$ or $3\frac{13}{50}$ in lowest terms.

b) $0.125 = \frac{125}{1000}$ since the 5 is in the thousandths place.
0.125 can also be expressed as $\frac{12.5}{100}$ or 12.5%.
What factors of 125 divide evenly into 1000?
$\frac{125}{1000} = \frac{1}{8}$
How do you know $\frac{125}{1000}$ and $\frac{12.5}{100}$ are equivalent?

c) $0.0032 = \frac{32}{10\,000}$ since the 2 is in the ten thousandths place.
0.0032 can also be written as $\frac{0.32}{100}$ or 0.32%.
What factors of 32 divide evenly into 10 000?
$\frac{32}{10\,000} = \frac{2}{625}$
How do you know that $\frac{32}{10\,000}$ and $\frac{0.32}{100}$ are equivalent?

Show You Know

Convert each decimal to a percent and a fraction.

a) 0.0064 b) 0.268 c) 5.98

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fraction on a hundred grid and record the number as equivalent fractions, a decimal, and a percent ($\frac{4.5}{20} = \frac{22\frac{1}{2}}{100} = 0.225 = 22.5\%$). Have students confirm how they know that their 0.5% is correct. For #3, have students shade twenty grids with three times the number of squares they did in #1. Discuss how many twenty grids they need to do this. Have them shade the same fraction on a pair of hundred grids and record the number as equivalent fractions, a decimal, and a percent ($\frac{27}{20} = \frac{135}{100} = 1.35 = 135\%$). As a class, discuss the advantages and disadvantages of using fractions, decimals, and percents to represent various numbers. Some students will find it helpful to think of percents greater than 100% as mixed numbers.

Example 1

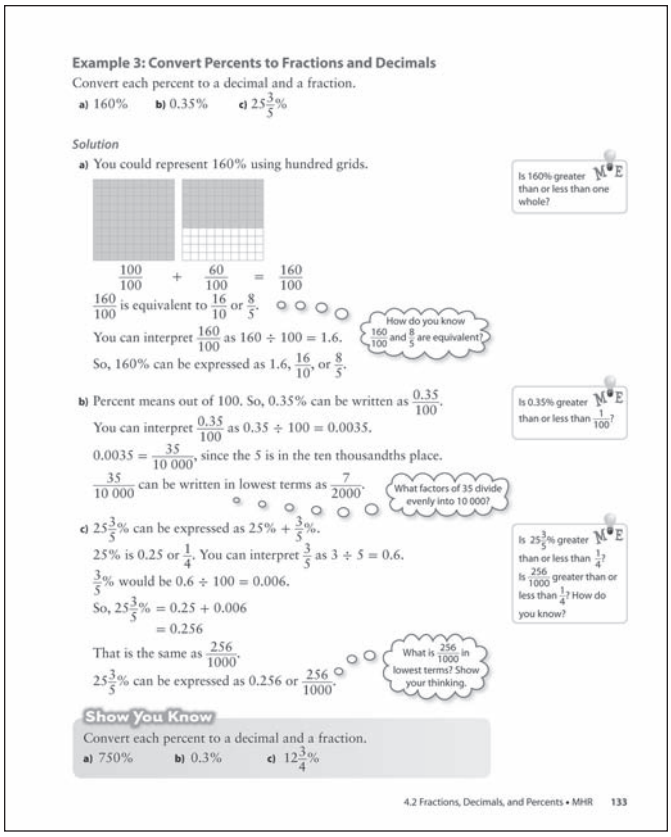
Before students consider any of the methods shown in Example 1, have them use what they know about fractions and percents to develop referents for fractions that are greater than 100% and less than 1%. For example, you might ask the following:

- What number represents 100%? (1)
- Is this fraction (referring to one of the fractions in Example 1) more or less than 100%?
- Is this fraction more or less than 50%?
- Is this fraction more or less than 10%?
- Is this fraction more or less than 1%?

Then have students assess why and how the methods shown in Example 1 work, and what other parts of Example 1 they might be useful in solving. Ask probing questions such as:

- Why is a hundred grid used to solve part a)?
- What are the benefits of a hundred grid?
- What other fraction(s) could you use a hundred grid to solve?
- How would you solve it using that method?
- What method is used to verify the answer to parts a), b), and c)?
- How is this useful?
- How are proportions used in the sample answers to this Example?
- How is each proportion solved?
- How is part c) solved?
- What other method could you use to solve ____ (a, b, or c)?

Have students estimate the percents that represent each of the fractions in the Show You Know before starting to do any conversions. Then, have them work in teams. Challenge each member of the team to use a different method to solve the Show You Know.



Example 3: Convert Percents to Fractions and Decimals

Convert each percent to a decimal and a fraction.

a) 160% b) 0.35% c) $25\frac{3}{5}\%$

Solution

a) You could represent 160% using hundred grids.

Is 160% greater than or less than one whole?

$\frac{100}{100} + \frac{60}{100} = \frac{160}{100}$

$\frac{160}{100}$ is equivalent to $\frac{16}{10}$ or $\frac{8}{5}$.

How do you know $\frac{160}{100}$ and $\frac{8}{5}$ are equivalent?

You can interpret $\frac{160}{100}$ as $160 \div 100 = 1.6$.

So, 160% can be expressed as 1.6, $\frac{16}{10}$, or $\frac{8}{5}$.

b) Percent means out of 100. So, 0.35% can be written as $\frac{0.35}{100}$.

Is 0.35% greater than or less than $\frac{1}{100}$?

You can interpret $\frac{0.35}{100}$ as $0.35 \div 100 = 0.0035$.

$0.0035 = \frac{35}{10\,000}$, since the 5 is in the ten thousandths place.

$\frac{35}{10\,000}$ can be written in lowest terms as $\frac{7}{2000}$.

What factors of 35 divide evenly into 10 000?

c) $25\frac{3}{5}\%$ can be expressed as $25\% + \frac{3}{5}\%$.

Is $25\frac{3}{5}\%$ greater than or less than $\frac{1}{4}$? Is $\frac{256}{1000}$ greater than or less than $\frac{1}{4}$? How do you know?

25% is 0.25 or $\frac{1}{4}$. You can interpret $\frac{3}{5}$ as $3 \div 5 = 0.6$.

$\frac{3}{5}\%$ would be $0.6 \div 100 = 0.006$.

So, $25\frac{3}{5}\% = 0.25 + 0.006$

$= 0.256$

That is the same as $\frac{256}{1000}$.

What is $\frac{256}{1000}$ in lowest terms? Show your thinking.

$25\frac{3}{5}\%$ can be expressed as 0.256 or $\frac{256}{1000}$.

Show You Know

Convert each percent to a decimal and a fraction.

a) 750% b) 0.3% c) $12\frac{3}{4}\%$

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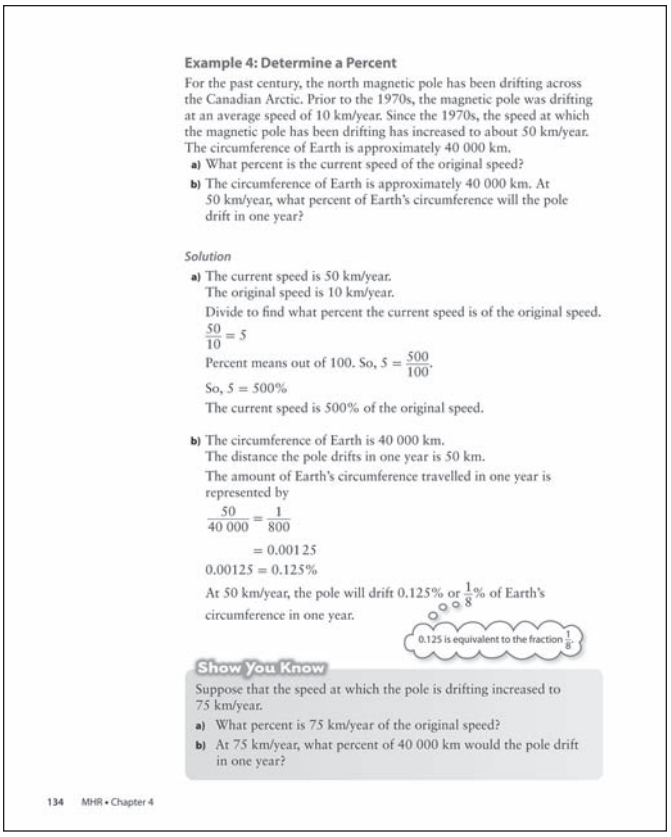

Example 4: Determine a Percent

For the past century, the north magnetic pole has been drifting across the Canadian Arctic. Prior to the 1970s, the magnetic pole was drifting at an average speed of 10 km/year. Since the 1970s, the speed at which the magnetic pole has been drifting has increased to about 50 km/year. The circumference of Earth is approximately 40 000 km.

a) What percent is the current speed of the original speed?

b) The circumference of Earth is approximately 40 000 km. At 50 km/year, what percent of Earth's circumference will the pole drift in one year?

Solution

a) The current speed is 50 km/year.

The original speed is 10 km/year.

Divide to find what percent the current speed is of the original speed.

$\frac{50}{10} = 5$

Percent means out of 100. So, $5 = \frac{500}{100}$.

So, 5 = 500%

The current speed is 500% of the original speed.

b) The circumference of Earth is 40 000 km.

The distance the pole drifts in one year is 50 km.

The amount of Earth's circumference travelled in one year is represented by

$\frac{50}{40\,000} = \frac{1}{800}$

$= 0.001\,25$

$0.00125 = 0.125\%$

At 50 km/year, the pole will drift 0.125% or $\frac{1}{8}\%$ of Earth's circumference in one year.

0.125 is equivalent to the fraction $\frac{1}{8}$.

Show You Know

Suppose that the speed at which the pole is drifting increased to 75 km/year.

a) What percent is 75 km/year of the original speed?

b) At 75 km/year, what percent of 40 000 km would the pole drift in one year?

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Teams can discuss their answers with each other and work together to identify what may be the problem if their answers differ.

Example 2

Have students use what they already know about percents and decimals to estimate which decimals will be

- more than 100%
- more or less than 50%
- more or less than 10%
- less than 1%

As they analyse the methods shown in Example 2, challenge students to notice what strategies are the same as and different from those in Example 1. Again, have students discuss why certain methods are shown and the benefits and possible drawbacks of these methods.

When discussing the method for part a), ask:

- Why are hundreds grids used here?
- Estimate the size of 3.26 as a fraction. (Encourage students to notice that 0.26 is close to $\frac{1}{4}$. So they are looking for a number around $3\frac{1}{4}$.)
- How else could you solve this question?
- How can you use mental math to help you here?

When discussing the method for part b), ask:

- How can you connect $\frac{125}{1000}$ to what you already know about percents and fractions? (For example, $100\% = \frac{1000}{1000}$, $50\% = \frac{500}{1000}$, and $25\% = \frac{250}{1000}$. $\frac{125}{1000}$ is half of $\frac{250}{1000}$. That must be 12.5%. Another way to write percent is to put the number over 100. So $12.5\% = \frac{12.5}{100}$. Continue to encourage students to connect what they are seeing in the suggested solutions to what they already know.)
- What method is being used here?
- How is it helpful?
- How does it work?
- What other strategy could you use to answer this question?
- How can you use mental math to help you here?

Discuss the following with individuals, small groups, or the entire class.

- Which strategy shown in Example 2 do you prefer? Why?
- Which strategy(ies) from Example 1 would you rather use? Explain.
- When might one strategy be better than another? Explain.
- Which strategy is the most efficient?

Have students solve the problems in the Show You Know using one method and then use a second method to check their answers. Ask:

- What methods do you find easier to use?
- What decimals do you find easier to convert? Why?
- How can you use your knowledge of the decimals you find easy to convert to help you convert others?

Example 3

To increase student understanding of how to use hundred grids to represent percents with decimal or fractional parts, you may wish to have students do the following activity:

- Divide the class into two or more large groups.
- Provide one set of groups with either one copy of **Master 9 0.5 Centimetre Grid Paper** or three copies of **Master 8 Centimetre Grid Paper,** scissors, tape or glue, and experience chart paper. Challenge them to develop a grid they could use to represent 0.6%. For example, they might display 100 strips of 10 blocks each to make a thousand grid and colour in 6 of them.
- Provide the second set of groups with about 13 copies of **Master 10 Hundred Grids**, scissors, tape or glue, and several pieces of experience chart paper. Challenge group members to develop a grid they could use to represent 0.35%. For example, they might use 100 hundred grids to make a ten thousand grid and colour in 35 grid squares.

Have groups share their large grids and thinking with the class.

- As individuals or groups look at the thousand grid, help students to better understand this grid by asking:
  - Show me 1% on this grid. (1 strip of 10 blocks)
  - How much of this grid would be needed to represent 6%? (6 groups of 10 squares)
  - Show me 0.1% on this grid. (1 block from a strip of 10 blocks)
  - How much of this grid would be needed to represent 0.6%? (6 blocks from a strip of 10 blocks)
  - Show me 1.6% on this grid. (1 strip of 10 blocks and 6 single blocks)
- As individuals or groups look at the ten thousand grid, help students to better understand this grid by asking:
  - Show me 1% on this grid. (1 hundred grid)
  - How much of this grid would be needed to represent 6%? (6 hundred grids)
  - Show me 0.1% on this grid. (1 row of a hundred grid)
  - Show me 0.01% on this grid. (1 block of a hundred grid)
  - How much of this grid would be needed to represent 0.3%? (3 rows of a hundred grid)
  - How much of this grid would be needed to represent 0.35%? (3 rows of a hundred grid plus 5 more squares)
  - How much of this large grid would be needed to represent 0.6%? (6 rows in one of the 100 squares)
  - How much of this grid would be needed to represent 6%? (6 of the hundred grids)
- Consider in groups or as a class why it takes more squares to show 6% in the ten thousand grid than it does in the thousand grid or the hundred grid.

You may wish to post these large grids in the classroom for students to continue to use to model other fractional and decimal percents such as those used in the Show You Know. Once students have completed the activity described above, have them consider how mental math might help them solve the problems in this Example.

Discuss the following with individuals, small groups, or the entire class.

- How does a hundred grid, a thousand grid, and a ten thousand grid help you understand what is happening during conversions such as the ones in Example 3?
- Which strategy(ies) from Examples 1 and 2 would you rather use? Explain.
- When might one strategy be better than another? Explain.

When discussing the method for part a), you may also wish to ask:

- How else could you use fractions to represent what you see in the hundred grids? (For example, there are 16 rows coloured in. One grid has 10 rows. So $\frac{16}{10}$ grids are filled in. That can be simplified to $\frac{8}{5}$.)

When discussing the method for parts b) and c), you may also wish to ask:

- Is the fraction in lowest terms? How do you know?
- If the fraction is not in lowest terms, how could you put it in lowest terms?

Have students solve the problems in the Show You Know using the method of their choice, and compare their answer with someone who used a different method. You may wish to have students show the answers to parts b) and c) on the class thousand grid and ten thousand grid.

Example 4

Students can use a number of different strategies to solve problems such as those shown in Example 4. The student resource provides a couple of strategies. Challenge students to come up with alternative strategies by asking and discussing questions such as the following:

- What strategies used in Examples 1 to 3 could you use to solve these problems? (Discuss each strategy and how it might be used.)
- What other strategies could be used to solve these problems? (Encourage students to consider ones they may not have used previously, including ones from other aspects of math, such as geometry. For example:

  40 000 km represents the circumference of a circle.

  There are 360° in a circle.

  What fraction of a degree represents one kilometre?
  $360 \div 40\,000 = 0.009$

  0.009° represent 1 km.

  How many degrees represent 50 km?
  $50 \times 0.009 = 0.45$

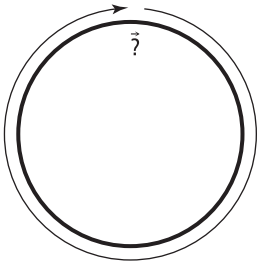
  0.45° represent 50 km.

  How many groups of 0.45° are in 360°?
  $360 \div 0.45 = 800$

  Therefore 50 km is $\frac{1}{800}$ of the circumference of the Earth.

  $$\frac{1}{800} = \frac{x}{100}$$
  $$x = 0.125$$

  Since percent means out of 100, x represents 0.125%.)



Challenge student pairs to use different techniques to solve the Show You Know and then to compare their answers.

Meeting Student Needs

- Concrete and kinesthetic learners may benefit from working with manipulatives to help them make conversions in the Explore the Math. Other students may benefit from creating conversion tables to help develop procedures. For example, representing $\frac{1}{2}$% as a fraction, a decimal, and a zoomed-in drawing may help students to determine values close to $\frac{1}{2}$%, such as $\frac{3}{4}$%.
- Provide additional practice to students who need help with identifying place values in decimals. They may find using a place value chart helpful.
- Students who have low math skills and English language learners may find this section challenging. The examples are complex and involve six different types of conversions: fractions to decimals, fractions to percents, decimals to percents, decimals to fractions, percents to decimals, and percents to fractions. In addition, there are different steps to solve each type of conversion. Consider isolating one type of conversion at a time. Teach students to identify each type of conversion and the steps to solve it. For instance, when converting a fraction to a decimal, record each step along with the number values. Ask students to restate the step in their own words. If they use non-mathematical language, use both forms. For example, if a student says to divide the top number by the bottom number to change a fraction to a decimal, record this, but add the words *numerator* and *denominator* beside the words *top* and *bottom*. This helps students to bridge from their own language to math terms. Either post the procedures or have students copy the steps and store them in their chapter Foldable for future reference.
- Consider teaching this section over three lessons and focusing on one example per lesson. Give students at least two additional sets of questions for each example.
- Invite someone who does beading to share bead loom patterns. For patterns that use 100 beads, have students determine the percent of each colour in the pattern, and then convert the percents to decimals and fractions. For patterns that use more than or less than 100 beads, have students determine the fraction of each colour in the pattern, and then convert the fractions to decimals and percents.

ELL

- Discuss the role of sports commentators and provide some examples of sports reports. Ask students who are familiar with hockey to explain how to play the game and explain terms such as *period*, *shot on goal*, *saves*, *goals against*, and *save percent*.
- For #4a), encourage students to use a chart to list the similarities and the differences between decimals, percents, and fractions.
- Clarify that *convert* means to change.
- Clarify each of the following terms by providing a contextual example: *north magnetic pole*, *drifting*, and *circumference*.

Gifted and Enrichment

- Challenge students to research and report on why the poles move. See the related Web Link on this page. Their report may help students who struggle with the math in Example 4 to gain a better understanding.

Common Errors

- Some students may become confused with values that are represented as being equivalent, especially in the case of fractional percents and percents over 100%.

R_x Walk students through some examples to reinforce that values such as 50% can be represented as 50%, 0.50, and $\frac{1}{2}$.

Web Link

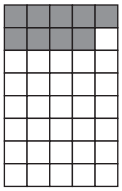
For NHL hockey statistics you might use when discussing the section opener, go to www.mathlinks8.ca and follow the links.

For information about the north magnetic pole, go to www.mathlinks8.ca and follow the links.

Answers

Explore the Math

1. a) $\frac{9}{20}$ **b)** $\frac{45}{100}$ **c)** 0.45 **d)** 45%

2.  ; $\frac{9}{40}$ or 0.225 or 22.5%

3. two squares; You will need to shade 1.35 squares. $\frac{27}{20}$ or 1.35 or 135%.

4. a) Answers will vary. Example: Decimals and percents use the same number but the decimal place is different. Percents are the same as the numerator of the equivalent fraction with 100 for a denominator.

b) Answers will vary. Example: Fractions are easy to work with because there are no decimal points or repeating decimals.

Show You Know: Example 1

a) 0.075 or 7.5% **b)** 0.57 or 57% **c)** 1.76 or 176%

Show You Know: Example 2

a) 0.64% or $\frac{64}{10\,000} = \frac{4}{625}$

b) 26.8% or $\frac{268}{1000} = \frac{67}{250}$

c) 598% or $\frac{598}{100} = \frac{299}{50}$

Show You Know: Example 3

a) 7.5 or $\frac{750}{150} = \frac{15}{2}$

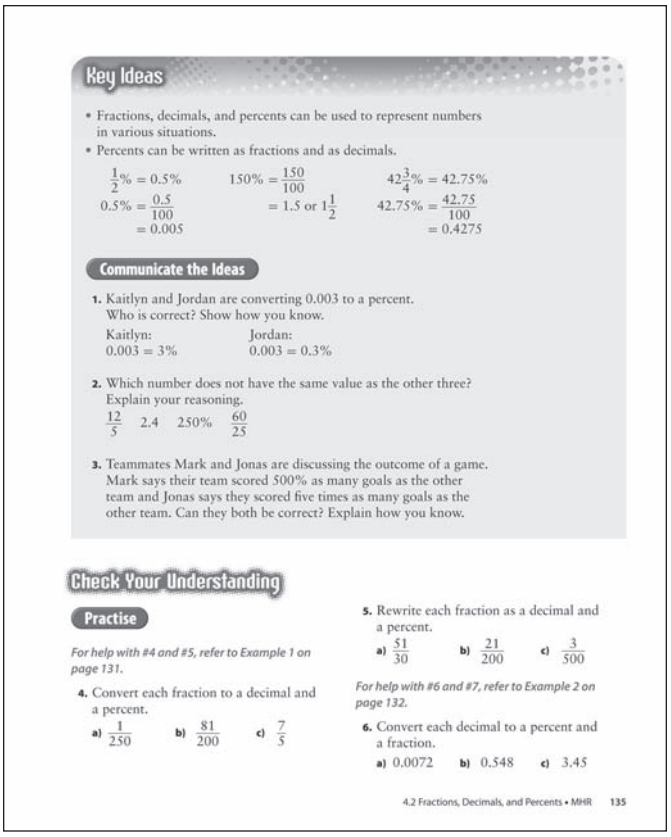
b) 0.003 or $\frac{3}{1000}$

c) 0.1275 or $\frac{1275}{10\,000} = \frac{51}{400}$

Show You Know: Example 4

a) 750% **b)** 0.1875%

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize any conclusions about their findings. Have students add their most successful conversion procedures to their chapter Foldable.	• Have students share their procedures for converting between fractions, decimals, and percents. Students may benefit from viewing other students' methods, as long as they make mathematical sense. • Having students share their answer to #4b) may help clarify thinking for other students.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Consider allowing students to use a calculator. Ensure they know the correct keystrokes. • You may need to reactivate some students' skills in using mathematical language to explain the process for converting fractions to decimals. • Have students share the methods they use successfully to convert fractions to decimals and percents. Correct any misinterpretations. • Give additional questions to students who would benefit from them. Allow them to work with a partner and talk through their thinking.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Provide additional coaching to students who need help with converting decimals to percents. It may be helpful to provide a rule, such as *multiplying by 100 moves the decimal to the right two places*. • Have students who need help converting decimals to fractions verbalize the place values of each decimal. • Have students share the methods they use successfully to convert decimals to percents and fractions. Correct any misinterpretations. • Give additional questions to students who would benefit from them. Allow them to work with a partner and talk through their thinking.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Provide additional coaching to students who need help with converting percents to decimals. It may be helpful to provide a rule, such as *dividing by 100 moves the decimal to the left two places*. • You may need to help reactivate some students' skills in reducing fractions to lowest terms. • Have students share the methods they use successfully to convert percents to decimals and fractions. Correct any misinterpretations. • Give additional questions to students who would benefit from them. Allow them to work with a partner and talk through their thinking.
Example 4 Have students do the Show You Know related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to check Example 4 and use the procedures to help them answer the questions. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.



Key Ideas

- Fractions, decimals, and percents can be used to represent numbers in various situations.
- Percents can be written as fractions and as decimals.

$\frac{1}{2}\% = 0.5\%$ $\quad 150\% = \frac{150}{100} = 1.5 \text{ or } 1\frac{1}{2}$ $\quad 42\frac{3}{4}\% = 42.75\%$

$0.5\% = \frac{0.5}{100} = 0.005$ $\quad 42.75\% = \frac{42.75}{100} = 0.4275$

Communicate the Ideas

1. Kaitlyn and Jordan are converting 0.003 to a percent. Who is correct? Show how you know.
   Kaitlyn: $0.003 = 3\%$ Jordan: $0.003 = 0.3\%$
2. Which number does not have the same value as the other three? Explain your reasoning.
   $\frac{12}{5}$ 2.4 250% $\frac{60}{25}$
3. Teammates Mark and Jonas are discussing the outcome of a game. Mark says their team scored 500% as many goals as the other team and Jonas says they scored five times as many goals as the other team. Can they both be correct? Explain how you know.

Check Your Understanding

Practise

For help with #4 and #5, refer to Example 1 on page 131.

4. Convert each fraction to a decimal and a percent.
   a) $\frac{1}{250}$ b) $\frac{81}{200}$ c) $\frac{7}{5}$
5. Rewrite each fraction as a decimal and a percent.
   a) $\frac{51}{30}$ b) $\frac{21}{200}$ c) $\frac{3}{500}$

For help with #6 and #7, refer to Example 2 on page 132.

6. Convert each decimal to a percent and a fraction.
   a) 0.0072 b) 0.548 c) 3.45

4.2 Fractions, Decimals, and Percents • MHR 135

Key Ideas

The Key Ideas summarize the procedures used to convert between fractions, decimals, and percents less than 1% and greater than 100%. Have students read and review the Key Ideas. Then, have them prepare and record their own summary of the Key Ideas in their chapter Foldable.

Communicate the Ideas

These questions encourage students to "speak the math," which will strengthen their understanding of conversion procedures. Have students work individually or in groups to answer the questions.

In #1 and #3, they apply their understanding of converting between fractions, decimals, and percents to identify and correct errors. In #2, students determine which number has a different value from the others. Have students share their answers for #3 in a class discussion.

Meeting Student Needs

ELL

- Clarify that *express as a percent*, *convert to a percent*, and *change to a percent* can be used interchangeably to ask for a conversion.
- Allow English language learners to show the math rather than use words to communicate their understanding.

Common Errors

- Some students may confuse decimals and decimal percents.

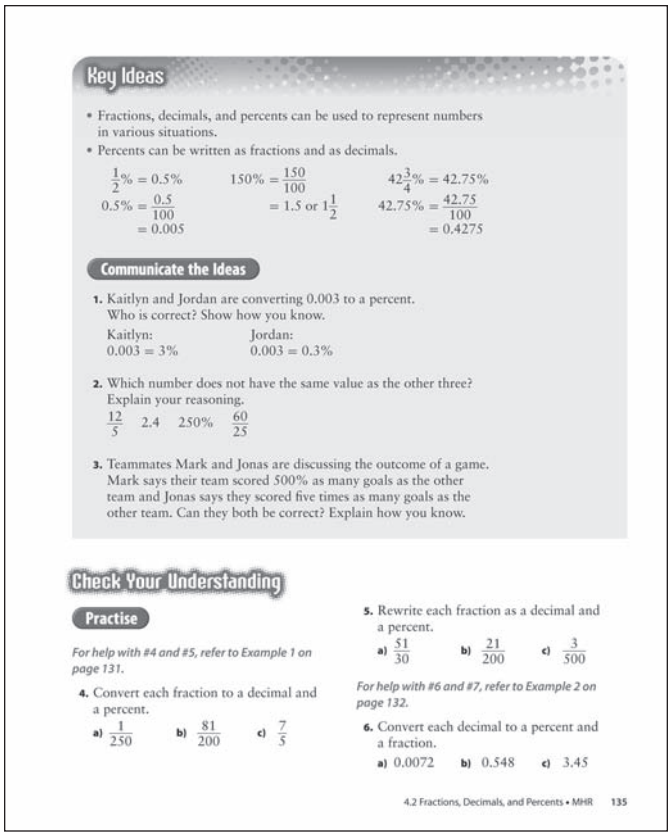
R$_x$ It may be helpful to show some examples, such as $\frac{1}{2}$ and 50%, on a number line. Or, you might use a hundred grid and have students point out half a square and say, "$\frac{1}{2}$ of 1%." Then, have them point out 50% of a grid and say, "50% of 100." Have students compare the two values.

Answers

Communicate the Ideas

1. Jordan is correct. Jordan correctly multiplied the decimal by 100 to arrive at the correct answer of 0.3%.
2. 250%, 250% equals 2.5 as a decimal. The other three numbers are equal to 2.4 when they are converted to decimal form.
3. They are both correct. Five times as many goals is equivalent to 500% as many goals since $5.0 = 500\%$.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have students complete #1 and #2.	• Check each student's answers to #1 and #2. These are key questions; make sure students understand how to convert between fractions, decimals, and percents before proceeding. • Refer students who need help with #1 to Example 2. • Have students verbalize their thinking for #2. Correct any misunderstandings. Then, coach them through corrections and have students record their errors and the corrections on their chapter Foldable or in their learning log. • Ask students who need coaching with #3 to show both numbers on hundred grids.



Key Ideas

- Fractions, decimals, and percents can be used to represent numbers in various situations.
- Percents can be written as fractions and as decimals.

$\frac{1}{2}\% = 0.5\%$ $\quad 150\% = \frac{150}{100}$ $\quad 42\frac{3}{4}\% = 42.75\%$

$0.5\% = \frac{0.5}{100}$ $\quad = 1.5 \text{ or } 1\frac{1}{2}$ $\quad 42.75\% = \frac{42.75}{100}$

$= 0.005$ $\quad = 0.4275$

Communicate the Ideas

1. Kaitlyn and Jordan are converting 0.003 to a percent. Who is correct? Show how you know.

Kaitlyn: $0.003 = 3\%$ $\quad$ Jordan: $0.003 = 0.3\%$

2. Which number does not have the same value as the other three? Explain your reasoning.

$\frac{12}{5}$ $\quad 2.4$ $\quad 250\%$ $\quad \frac{60}{25}$

3. Teammates Mark and Jonas are discussing the outcome of a game. Mark says their team scored 500% as many goals as the other team and Jonas says they scored five times as many goals as the other team. Can they both be correct? Explain how you know.

Check Your Understanding

Practise

For help with #4 and #5, refer to Example 1 on page 131.

4. Convert each fraction to a decimal and a percent.

a) $\frac{1}{250}$ $\quad$ b) $\frac{81}{200}$ $\quad$ c) $\frac{7}{5}$

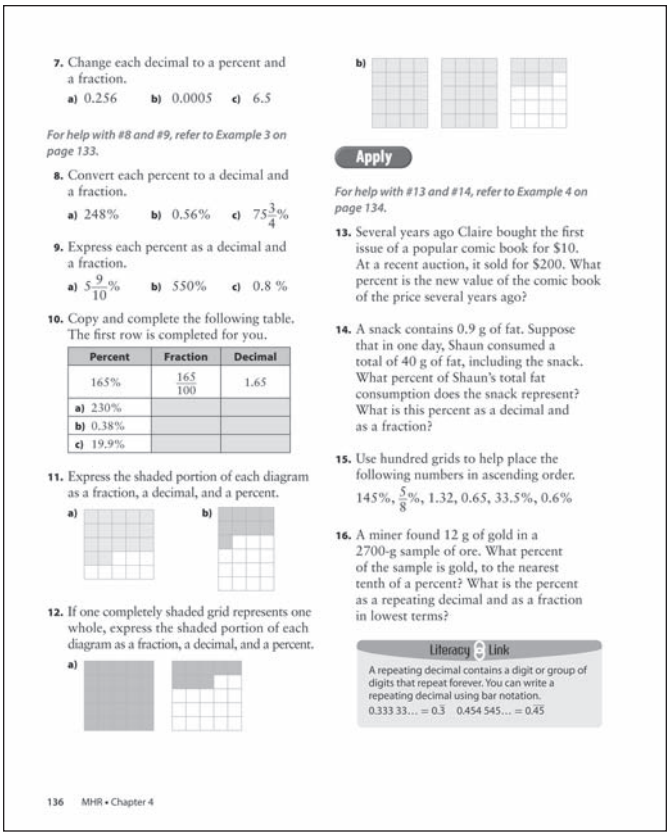
5. Rewrite each fraction as a decimal and a percent.

a) $\frac{51}{30}$ $\quad$ b) $\frac{21}{200}$ $\quad$ c) $\frac{3}{500}$

For help with #6 and #7, refer to Example 2 on page 132.

6. Convert each decimal to a percent and a fraction.

a) 0.0072 $\quad$ b) 0.548 $\quad$ c) 3.45



7. Change each decimal to a percent and a fraction.

a) 0.256 $\quad$ b) 0.0005 $\quad$ c) 6.5

For help with #8 and #9, refer to Example 3 on page 133.

8. Convert each percent to a decimal and a fraction.

a) 248% $\quad$ b) 0.56% $\quad$ c) $75\frac{3}{4}\%$

9. Express each percent as a decimal and a fraction.

a) $5\frac{9}{10}\%$ $\quad$ b) 550% $\quad$ c) 0.8 %

10. Copy and complete the following table. The first row is completed for you.

Percent	Fraction	Decimal
165%	$\frac{165}{100}$	1.65
a) 230%		
b) 0.38%		
c) 19.9%		

11. Express the shaded portion of each diagram as a fraction, a decimal, and a percent.

a) b)

12. If one completely shaded grid represents one whole, express the shaded portion of each diagram as a fraction, a decimal, and a percent.

a) b)

Apply

For help with #13 and #14, refer to Example 4 on page 134.

13. Several years ago Claire bought the first issue of a popular comic book for $10. At a recent auction, it sold for $200. What percent is the new value of the comic book of the price several years ago?

14. A snack contains 0.9 g of fat. Suppose that in one day, Shaun consumed a total of 40 g of fat, including the snack. What percent of Shaun's total fat consumption does the snack represent? What is this percent as a decimal and as a fraction?

15. Use hundred grids to help place the following numbers in ascending order.

145%, $\frac{5}{8}\%$, 1.32, 0.65, 33.5%, 0.6%

16. A miner found 12 g of gold in a 2700-g sample of ore. What percent of the sample is gold, to the nearest tenth of a percent? What is the percent as a repeating decimal and as a fraction in lowest terms?

Literacy Link

A repeating decimal contains a digit or group of digits that repeat forever. You can write a repeating decimal using bar notation.

$0.333\,33\ldots = 0.3$ $\quad 0.454\,545\ldots = 0.45$

Check Your Understanding

The emphasis in this section is to solve problems involving percents by multiplying or dividing by powers of ten. Most students should be able to do this without using a calculator.

Practise

Consider allowing students to choose between #4 and #5, and #6 and #7. Consider using #4, #6, and #8 to check students' understanding of the key concepts in this section.

Apply

The Apply questions provide a wide range of contexts in which fractions, decimals, and percents are used. Have students share their strategies for identifying the important mathematical information in these questions.

For #20, students who have this information may prefer to use their own heart rate data.

Literacy Link For #16, refer to the Literacy Link on page 136 that explains how to write a repeating decimal using bar notation.

Extend

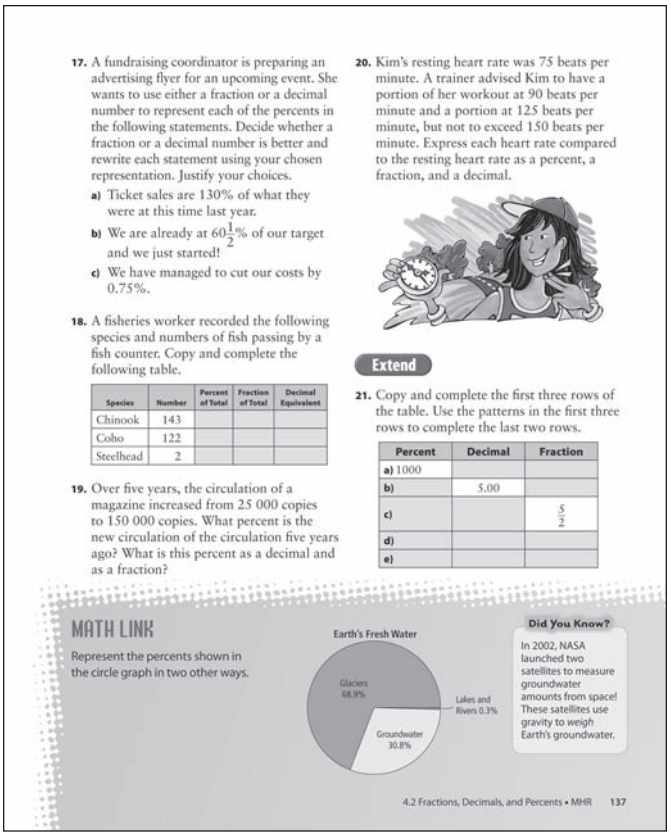
The Extend question pushes students' ability to convert number values that are very large or very small.

Math Link

The Math Link gives students an opportunity to apply their understanding of converting percents to fractions and decimals. Have students note the percent of Earth's supply of fresh water that is held as ice and ask how climate change might affect the percent of fresh water held as ice. Push students to explore how percent values might change if the ice melts into the ocean.

Meeting Student Needs

- Consider allowing students to work in pairs. They might work on one question together and then work individually on the next one, or work on questions individually, then discuss the procedures they used. Ensure that students complete a number of questions individually.
- Provide **BLM 4–8 Section 4.2 Extra Practice** to students who would benefit from more practice.



17. A fundraising coordinator is preparing an advertising flyer for an upcoming event. She wants to use either a fraction or a decimal number to represent each of the percents in the following statements. Decide whether a fraction or a decimal number is better and rewrite each statement using your chosen representation. Justify your choices.
a) Ticket sales are 130% of what they were at this time last year.
b) We are already at $60\frac{1}{2}$% of our target and we just started!
c) We have managed to cut our costs by 0.75%.

18. A fisheries worker recorded the following species and numbers of fish passing by a fish counter. Copy and complete the following table.

Species	Number	Percent of Total	Fraction of Total	Decimal Equivalent
Chinook	143			
Coho	122			
Steelhead	2			

19. Over five years, the circulation of a magazine increased from 25 000 copies to 150 000 copies. What percent is the new circulation of the circulation five years ago? What is this percent as a decimal and as a fraction?

20. Kim's resting heart rate was 75 beats per minute. A trainer advised Kim to have a portion of her workout at 90 beats per minute and a portion at 125 beats per minute, but not to exceed 150 beats per minute. Express each heart rate compared to the resting heart rate as a percent, a fraction, and a decimal.

Extend

21. Copy and complete the first three rows of the table. Use the patterns in the first three rows to complete the last two rows.

Percent	Decimal	Fraction
a) 1000		
b)	5.00	
c)		$\frac{5}{2}$
d)		
e)		

MATH LINK

Represent the percents shown in the circle graph in two other ways.

Earth's Fresh Water

Glaciers 68.9%
Lakes and Rivers 0.3%
Groundwater 30.8%

Did You Know?

In 2002, NASA launched two satellites to measure groundwater amounts from space! These satellites use gravity to *weigh* Earth's groundwater.

4.2 Fractions, Decimals, and Percents • MHR 137

ELL

- English language learners may have difficulty with terms such as *comic book*, *recent*, *auction*, *miner*, *ore*, *fundraising coordinator*, *ticket sales*, *target*, *fisheries worker*, and *circulation of a magazine*. Have student add new terms to their dictionary.

Gifted and Enrichment

- Give students the following problem to solve: Johnny's average on his first four math tests was 75%. What does Johnny need to score on his next test so that his overall average will be 80%?
- Have students use spreadsheet software and model the growth of an investment at various interest rates.
- Challenge students to research and report on how scientists weigh Earth's water from space. Have them use fractions, decimals, and percents in their report. See the related Web Link on this page.

WWW Web Link

The Did You Know? on page 137 features information about weighing Earth's water from space. For more information, go to www.mathlinks8.ca and follow the links.

Answers

Math Link

Glaciers: 0.689 or $\frac{689}{1000}$

Groundwater: 0.308 or $\frac{308}{1000} = \frac{77}{250}$

Lakes and Rivers: 0.003 or $\frac{3}{1000}$

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #4, #6, #8, and #13. Students who have no problems with these questions can go on to the rest of the Apply questions.	• For #4, #6, and #8, encourage students to verbalize their understanding of each type of conversion. • Encourage partners to check each other's procedures for consistency. If procedures are inconsistent, have students check with you. • Provide additional coaching with Example 1 to students who need help with #4. Coach students through #4 and then have them complete #5 on their own. Check back with them several times to make sure that they understand the concepts. • Provide additional coaching with Example 2 to students who need help with #6. Coach students through #6 and then have them complete #7 on their own. Check back with them several times to make sure that they understand the concepts. • Provide additional coaching with Example 3 to students who need help with #8. Coach students through #8 and then have them complete #9 on their own. Check back with them several times to make sure that they understand the concepts. • Students who need assistance with #4, #6, and #8 should complete #10 or #11 before moving on. • Provide additional coaching with Example 4 to students who need help with #13. Have students highlight the mathematical terms and number values in the problem and verbalize what the problem is asking them to find. Coach students through #13 and then have them complete #14 on their own.
Math Link The Math Link on page 137 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 153.	• It is recommended that most students complete the Math Link. • Emphasize that the circle graph represents only fresh water. • Students who need help getting started could use **BLM 4–9 Section 4.2 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students answer the following questions: • What do you find easy about converting fractions, decimals, and percents? Why? • What do you find difficult about converting fractions, decimals, and percents? Why? • What can you do to help you remember the different conversion methods?	• Depending on students' learning style, have them provide oral or written answers. • Have students reflect on some of their classmates' different, yet correct, ways of converting percents to fractions and decimals. • Encourage students to use the What I Need to Work On tab of their chapter Foldable to note what they continue to have difficulties with.