4.3 Percent of a Number

Suggested Timing
80–100 minutes

Materials
• calculator
• hundred grids

Blackline Masters
Master 10 Hundred Grids
BLM 4–3 Chapter 4 Warm-Up
BLM 4–10 Section 4.3 Extra Practice
BLM 4–11 Section 4.3 Math Link

Mathematical Processes
✓ Communication (C)
✓ Connections (CN)
✓ Mental Mathematics and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes
N3 Demonstrate an understanding of percents greater than or equal to 0%.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, 10, Math Link
Typical	1–3, 5, 8–13, Math Link
Extension/Enrichment	1, 2, 14–16, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce material learned in previous sections.

Use the opening text to introduce the Explore the Math. Have students share examples of percents that include fractions. Consider showing some examples of the significance of a fractional percent with a large value.

Literacy Link Point out the Literacy Link on page 138 that explains the term *profit*. Have students use the term to describe a fundraising event with which they may be familiar.

Explore the Math

In this exploration, students solve problems involving percents. Consider having students share what they know about the projects that the fundraising team wants to support: emergency relief, food, clean drinking water, and oral rehydration therapy.

As a class, read the Did You Know? on page 139, which describes oral rehydration therapy. When discussing the purpose of salt and sugar, prompt students to realize that a state of dehydration results from a rapid loss of water, which also includes a loss of salts. Explain that giving a dehydrated person only a saline solution has no beneficial effect because the normal mechanism by which salts are absorbed is impaired when the body is in a dehydrated state. If salt cannot be absorbed, neither can water. Giving a dehydrated person salt, sugar, and water works because the intestinal wall is able to absorb the sugar, and the sugar carries the salt with it. An increase in the salt concentration across the intestinal wall pulls water through after it.

Method 1
Have students work individually or in pairs to complete the activity. Have them discuss their findings as a class. Since section 4.3 highlights using mental math techniques such as halving and dividing repeatedly by tens to solve problems and to check for reasonableness of calculated answers, you might walk through some examples using these techniques. You may need to reactivate some students' skills in using a calculator to check mental math calculations.

Method 2
Consider having students share what they know about various relief organizations. You may wish to change the organization to another that the school particularly supports. Provide students with **Master 10 Hundred Grids** and have them show all percents using the grids. On each grid, have them record what 100% represents (e.g., $10 000), then, what 50%, $\frac{1}{2}$%, and $50\frac{1}{2}$% represent. Do the same for the donation amounts. Discuss with students what mental math techniques they can use for the calculations required in these questions. Have students share their strategies. You may wish to do some other examples together as a class.

Example 1
Example 1 demonstrates using the mental math techniques of halving and dividing repeatedly by tens. As you walk through the example, help students recall the procedure for multiplying fractions. Model the technique in the Mental Math and Estimation box. Remind students that fractions of percents can be significant. Also, have students identify and discuss the pattern on page 139 and how it is used to solve the problem.

Literacy Link Discuss the Literacy Link on page 139 to clarify *halving* and *doubling*.

Example 2
Example 2 demonstrates calculating the percent of a number. Since the problems include text and numerical data, you might discuss how to read for understanding and tease out the numbers and procedures needed to solve each problem. You might walk through part a), and have students identify and record the numbers and corresponding units as they are encountered. Next, have them identify the

b) Determine $30\frac{3}{4}\%$ of 400.

Since $\frac{3}{4}\%$ is 0.75%, $30\frac{3}{4}\% = 30.75\%$.

Divide by 100 to write the percent as a decimal.

$30.75 \div 100 = 0.3075$

0.3075 of $400 = 0.3075 \times 400$
    $= 123$

C .3075 × 400 = 123

So, 123 of the 400 students own a cell phone.

c) Determine 120% of $40.12.

Divide by 100 to write the percent as a decimal.

$120 \div 100 = 1.2$

1.2 of $40.12 = 1.2 \times 40.12$
    ≈ 48.14

C 1.2 × 40.12 = 48.144

So, 120% of $40.12 is $48.14.

Mental Math and Estimation (margin):
10% of 400 is 40.
30% of 400 is 120.
1% of 400 is 4.
$\frac{1}{4}\%$ of 400 is 1.
$\frac{3}{4}\%$ of 400 is 3.
$30\frac{3}{4}\%$ of 400 is $120 + 3$.
$120 + 3 = 123$

To the nearest cent means to the nearest hundredth of a dollar.

100% of 40.12 is 40.12.
10% of 40.12 is 4.012.
20% of 40.12 is 8.024.
120% of 40.12 is
40.12 + 8.024.
40.12 + 8.024 = 48.144

Show You Know

Determine the percent of each number.
a) 160% of $53.27
b) $\frac{3}{4}\%$ of 135
c) $55\frac{8}{10}\%$ of 500

Key Ideas

- You can use mental math strategies such as halving, doubling, and dividing by ten to find the percents of some numbers.
- To calculate the percent of a number, write the percent as a decimal and then multiply by the number.

$12\frac{1}{2}\%$ of $50 = 0.125 \times 50$
    $= 6.25$

$12\frac{1}{2}\% = 12.5\%$
    $= 0.125$

Communicate the Ideas

1. Explain to a classmate how you could use mental math to find each of the following.
   a) 300% of 40   b) 0.5% of 120   c) $10\frac{1}{2}\%$ of 80

2. Describe two ways to find 6% of 120.

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problem and the information needed to solve it. It is very important for students to be able to show how they proceeded to solve a problem, since doing so enables them to identify errors. Promote using mental math and estimation to check answers for reasonableness. Model the techniques in the Mental Math and Estimation boxes on pages 140 and 141.

Literacy Link Point out the Literacy Link on page 140 and clarify that the word *of* often means to multiply.

Meeting Student Needs

- Have students who struggle with numbers while trying to learn the procedures to solve problems generate a chart of division by ten and a chart of multiplication by ten. Prompt students to recall that multiplying by ten moves the decimal to the right one place and dividing by ten moves the decimal to the left one place. This exercise is an application of using a pattern, and students may benefit from going through the process of developing the pattern themselves. Students can store the charts in their Foldable for reference.

- Have students who struggle with numbers while trying to learn the procedures to solve problems generate a conversion chart from percents to decimals to fractions. Consider having students work with a partner. Prompt them to isolate one type of conversion at a time by identifying it and the steps to solve it. For instance, when converting a decimal to a fraction, have them talk through the steps and record the number values. Students can store the chart in their Foldable for reference.

- Calculators with screens that show the procedures entered may help students to check their input for accuracy.

- The buttons on some calculators may be too small and close together for students with dexterity problems to use effectively. You may wish to provide a calculator with oversized keys.

- Encourage students who struggle with Example 2 to use the following steps as they work through the problem:
  – Highlight the relevant numbers and units.
  – Identify the conversion needed to solve the problem.
  – Refer back to the chapter Foldable for the steps to perform the conversion.
  – Solve the problem.

ELL

- English language learners may have difficulty with terms such as *relief organization*, *emergency relief*, *dehydration*, *survey*, *inline skates*, *cell phone*, *savings plan*, and *investment*. Have students add any new terms to their dictionary.

Gifted and Enrichment

- Have students research oral rehydration therapy and its use by relief organizations and medical practitioners in Canada. Have them explain the importance of the therapy, particularly for very young and elderly people who are dehydrated because of illness. See the related Web Link on TR page 179.

- Challenge students to research and report on access to clean drinking water worldwide. Have them develop a water-related math problem involving percents as part of the report. See the related Web Link on TR page 179.

Common Errors

- Some students may become confused when converting from fractions to percents and decimals.
- R_x Remind them to make one conversion at a time and to check for reasonableness using mental math.

Answers

Explore the Math

1. a) $5000 b) $100 c) $50 d) $5050

2. a) $505
   b) Answers will vary. Example: Move the decimal point one place to the left.

3. Since 20% is twice the value of 10%, simply multiply $505 by two, which equals $1010.

4. a) 500 L b) 125 adults

5. Answers will vary. Example: You can think of the percent you wish to calculate as the sum of several percents that are easier to calculate mentally. If you want to calculate 21% of a number, you can find 10%, then double that value, and finally add 1% more.

Show You Know: Example 1

a) 100% of $10 is $10; then 300% of $10 is $30; and 50% of $10 is $5. Therefore, 350% of $10 is $35.

b) 1% of $5000 is $50; then 0.1% of $5000 is $5.

c) 1% of $20 000 is $200; then 2% of $20 000 is $400; $\frac{1}{10}$% or 0.1% of $20 000 is $20. Therefore, $2\frac{1}{10}$% of $20 000 is $420.

Show You Know: Example 2

a) $85.23 b) 1.0125 c) 279

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings. Ask students to share the mental math techniques that they use.	• Some students may struggle with halving numbers mentally. Help them build this skill by encouraging them to visualize the answer and then check it using a calculator. • Encourage students to look for patterns and verbalize their observations. • Encourage students to identify and use the technique that they are most comfortable with, such as halving, dividing repeatedly by tens, and calculating 1% and then multiplying by what is needed.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may need to use a calculator. Encourage students to practise using mental math before using a calculator. Reinforce the importance of using mental math to check the reasonableness of answers. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking • You may wish to have students work with a partner. • You may need to help some students recall how to write a percent as a decimal. • Some students may need to use a calculator. Encourage students to use mental math before using a calculator. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.

The following is a reproduction of a student textbook page:

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So, 123 of the 400 students own a cell phone.

10% of 400 is 40. M E
30% of 400 is 120.
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$\frac{1}{4}\%$ of 400 is 1.
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Key Ideas

• You can use mental math strategies such as halving, doubling, and dividing by ten to find the percents of some numbers.
• To calculate the percent of a number, write the percent as a decimal and then multiply by the number.

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1. Explain to a classmate how you could use mental math to find each of the following.
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2. Describe two ways to find 6% of 120.

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Communicate the Ideas

These questions allow students to verbalize their thinking about using mental math techniques to find the percent of a number, which is an excellent way to reinforce understanding. Ask students to describe orally what features of a problem help them identify the mental math technique most appropriate to use.

Meeting Student Needs

• Allow visual learners to use hundred grids on **Master 10 Hundred Grids** to support their thinking for mental math processes. For example, they might cover 50% of a shaded region to show halving. Or, they might show that if 100% of 40 is 40 squares, then 300% of 40 squares is 3 sets of 40 squares, which equals 120 squares.

Common Errors

• Some students may fail to show their calculations, which may result in errors in procedure.

R_x Remind students to show all of their work.

Answers

Communicate the Ideas

1. a) Find 100% of 40, which is 40; then triple the value to get the answer, 120.
   b) Find 1% of 120, which is 1.2; then find half of that value to get the answer, 0.6.
   c) Find 1% of 80, which is 0.8; multiply this value by 10 to get 8; add half of 0.8, which is 0.4. The final answer is 8.4.

2. Look for two ways. Examples:
   • Using mental math, find 1% of 120, which is 1.2. Find 10% of 120, which is 12, and take half of it to get 5% or 6. Add 1.2 to 6 to get the answer, 7.2.
   • By direct calculation, convert 6% to a decimal, 0.06, and multiply by 120 to arrive at the answer, 7.2.

Key Ideas

The Key Ideas summarize mental math strategies and calculating the percent of a number. Have students read and discuss the Key Ideas. Ask them to explain in their own words how to calculate the percent of a number. Have students prepare and record their own summary of the Key Ideas in the notes on their chapter Foldable.

Encourage students to use mental math techniques to check the reasonableness of calculations. You may wish to have them record what mental math techniques they would use to check the reasonableness of the calculation in the Key Ideas. For example, 10% of 50 is 5; 1% of 50 is 0.5; 12% of 50 is 6. The answer will be a little larger than 6.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have students complete #1 and #2. As a class, have students share their responses for #1. Have students share their responses to #2 with a partner and listen to each other's explanations.	• Consider having students work in pairs. • Check each student's answers to the questions. These are key questions; make sure students understand the concepts before proceeding. • Have students who need help with #1 verbalize their thinking and process. Clarify any misunderstandings. You may need to help some students recall how to write a percent as a decimal before asking them to verbalize the solution for #1a). • Encourage students to use diagrams, hundred grids, and patterns to help solve #2.

Practise

For help with #3 and #4, refer to Example 1 on pages 139–140.

3. Use mental math to determine each of the following.
   a) 300% of 2000
   b) $1\frac{1}{4}\%$ of 60
   c) 0.1% of 40

4. Use mental math to find the following.
   a) 20% of 60
   b) 250% of 400
   c) $10\frac{1}{2}\%$ of 100

For help with #5 and #6, refer to Example 2 on pages 140–141.

5. Determine the percent of each number. Give your answer to the nearest hundredth.
   a) $\frac{2}{5}\%$ of 325
   b) $15\frac{1}{4}\%$ of 950
   c) 175% of $125.50

6. What is the percent of each number? Give your answer to the nearest hundredth.
   a) $\frac{5}{8}\%$ of 520
   b) $75\frac{2}{5}\%$ of 200
   c) 250% of $76.50

Apply

7. Two hundred tickets are being sold for a school draw.
   a) What is your chance of winning with one ticket? Express your answer as a percent.
   b) How many tickets would you need to purchase to have a 2.5% chance of winning?

8. The original price of a jacket was $84.00. A store manager marked the price down by $25\frac{1}{2}\%$. By how much was the price reduced?

9. The highest point in Canada is Mount Logan, which is in the Yukon Territory. Mount Logan is 159% as high as the highest point in Alberta, Mount Columbia. The elevation of Mount Columbia is 3747 m. What is the elevation of Mount Logan?

10. When water freezes, its volume increases by approximately 10%.
   a) By how much does the volume of 750 mL of water increase when it freezes?
   b) What is the volume of ice created?

11. The area of Canada is approximately 9 984 670 km². The area of Manitoba is about $6\frac{1}{2}\%$ of the area of Canada. What is the area of Manitoba to the nearest square kilometre?

12. A manufacturer of electric hybrid vehicles claims its vehicle will travel 200% as far as its regular vehicle on a full tank of gas. If the regular vehicle travels an average of 550 km on a full tank, how far will the hybrid go?

13. Suppose a real estate agent receives 5% commission on the first $200 000 of a house's selling price, and 6% on the remaining amount.
   a) What does *commission* mean?
   b) If a house sells for $345 000, how much commission does the real estate agent make on the sale of the house?

Extend

14. 4% of 100 is the same as 8% of what number? Explain how you arrived at your answer.

15. A new video gaming system was auctioned on the Internet. The starting bid was $100. The second bid was 135% of the first bid. The third bid was 257% of the second bid. There were then five more bids, each $10\frac{1}{2}\%$ over the previous bid. The winning bid came with only seconds left and was only 0.1% greater than the previous bid. What was the winning bid? What assumptions did you make to arrive at your answer?

16. Josephine scored 12 baskets out of 30 shots in her first basketball game this year. Her scoring average was then 40%. The next game, she made ten shots and raised her scoring average for both games to 50%. How many of the ten shots in the second game were baskets?

MATH LINK

Water conservation is very important to protect local fresh water supplies.

a) Research at least three ways that your home, school, and community could reduce water consumption.

b) Develop three water math problems that ask how much water you might save if you used some of these ways of conserving.

WWW Web Link
Did you know that a swimming pool cover can help reduce water loss by evaporation by 90%? To find data and tips on conserving water, go to www.mathlinks8.ca and follow the links.

Check Your Understanding

Practise

These questions involve using mental math and written processes to solve problems that involve percents. Encourage students to continue to use mental math and estimation to check the reasonableness of answers. Note that #3 and #4, and #5 and #6, are pairs of similar questions. Consider assigning one question from each pair initially. Assign the second question to students who would benefit from extra practice.

Apply

These questions provide a range of contexts in which students apply their understanding of solving problems involving percents less than 1%, percents greater than 100%, and fractional percents. While the procedures for solving these are the same as for more conventional percent problems, the presence of small and large values requires students to pay attention to the details of their procedures.

Extend

These questions allow students to extend their learning by looking for new procedures to solve problems involving percents. Consider having students work in pairs on these questions.

Math Link

The Math Link provides an opportunity for students to research practical ways to reduce water consumption in their home, school, and community. They use their findings to develop math problems involving percents related to water conservation. In advance, you may wish to check out some useful web sites and bring the data to class. See the related Web Link in the student resource and on TR page 182. Allow students to present their findings and the problems they generate using a format of their choice (e.g., poster, letter to the school paper, presentation using technology). You may wish to display students' work in the classroom to highlight the practical value of their findings.

Meeting Student Needs

- Remind students to use the techniques needed for #3 and #5, and use the examples as references to help solve problems.
- Encourage students who struggle with the problems to use the following steps:
  - Highlight the relevant numbers.
  - Identify the conversion needed to solve the problem.
  - Refer back to the chapter Foldable for the steps to perform the conversion.
  - Solve the problem.
- Have students draw a diagram to represent the situation in #9.
- Provide **BLM 4–10 Section 4.3 Extra Practice** to students who would benefit from more practice.
- Most activities in this chapter have been sit-down activities. Consider having students move around the classroom as they work on the Math Link.

ELL

- English language learners may have difficulty with terms such as *school draw*, *elevation*, *manufacturer*, *electric hybrid vehicles*, *video gaming system*, *auctioned*, *bid*, and *scoring average*. Have students add any new terms to their dictionary.
- For #5, you may need to remind English language learners of the meaning of *hundredth*. Emphasize the *th* sound at the end.
- For #10, explain the term *volume*. You might show students two identical containers of water (such as transparent water bottles with a fill-line marked), one frozen and the other liquid, to show the change in volume after freezing.

Gifted and Enrichment

- Challenge students to use a virtual manipulative and explore solving problems involving percent. Students can use the virtual manipulative in the Web Link below to input two of the three quantities in the equation, $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$, and then select the *Compute* button to find the remaining quantity.

WWW Web Link

For a virtual manipulative that allows students to solve problems involving percents, go to www.mathlinks8.ca and follow the links. The solution to each problem is displayed visually in a percent gauge and in a circle graph. There are several demonstrations of how to use the resource.

For additional information about water conservation practices, go to www.mathlinks8.ca and follow the links.

For information about urban water conservation practices in Western Canada, go to www.mathlinks8.ca and follow the links.

Answers

Math Link

a) Answers will vary. Look for at least three ways to reduce water consumption. Examples:
  - Install low-flow toilets and showerheads.
  - Limit car washing and garden watering.
  - Install sinks with automatic shut-off valves.

b) Answers will vary. Look for three water math problems and their solutions. Examples:
  - Jane's mother is installing a new low-flow toilet that will use 60% less water per flush. How much water is saved if the old toilet used 6 L per flush?
  - Anytown is thinking of limiting car washing to two days a month to save 7% more water. If car washing currently uses 10 000 L of water per month, how much water will be used if Anytown imposes this limit?
  - Central School has installed sinks with automatic shut-off valves. If the school used to use 2000 L of water each day and the sinks save 3.2%, how much water does the school now use?

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #3, #5, and #7. Students who have no problems with these questions can go on to the rest of the Apply questions.	• Provide additional coaching with Example 1 to students who need help with #3. Have students explain how they used mental math to solve #3a). Correct any misunderstandings. Coach students through #3, and then assign #4. Check back with them several times to make sure that they understand the concepts. • Encourage students to use mental math, diagrams, hundred grids, and patterns to help solve #5. Emphasize using mental math and estimation to check the reasonableness of their answers. • Provide additional coaching with Example 2 to students who need help with #5. Have students explain the process they used to determine percent. Correct any misunderstandings. Coach students through #5, and then assign #6. Check back with them several times to make sure that they understand the concepts. • Ask students who need help with #7 to identify the problem and the information provided to solve it. Coach them to develop a mathematical statement similar to #3 to help solve the problem. Coach students through #7, and then assign #8.
Math Link The Math Link on page 143 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 153.	• It is recommended that all students complete the Math Link. • Many students can be successful in this activity if they create visual representations of the impact of conserving water. Using visuals may also help them grasp the mathematical relationships they represent. • Some students may be overwhelmed by the openness of part a). Limit their choice to one of home, school, or community, and then have them develop the math problems based on the choice they made. • Students who need help getting started could use **BLM 4–11 Section 4.3 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students complete the following statement: • To determine the percent of a number, you …	• Depending on students' learning style, have them provide oral or written answers. • Encourage students to use the What I Need to Work On tab of their chapter Foldable to note what they continue to have difficulties with.