5.3 Surface Area of a Prism

***MathLinks 8*, pages 176–181**

Suggested Timing
80–100 minutes

Materials
- small empty cardboard box
- scissors
- ruler
- scrap paper

Blackline Masters
BLM 5–3 Chapter 5 Warm-Up
BLM 5–11 Section 5.3 Extra Practice
BLM 5–12 Section 5.3 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
SS2 Draw and construct nets for 3-D objects.
SS3 Determine the surface area of:
- right rectangular prisms
- right triangular prisms
- right cylinders

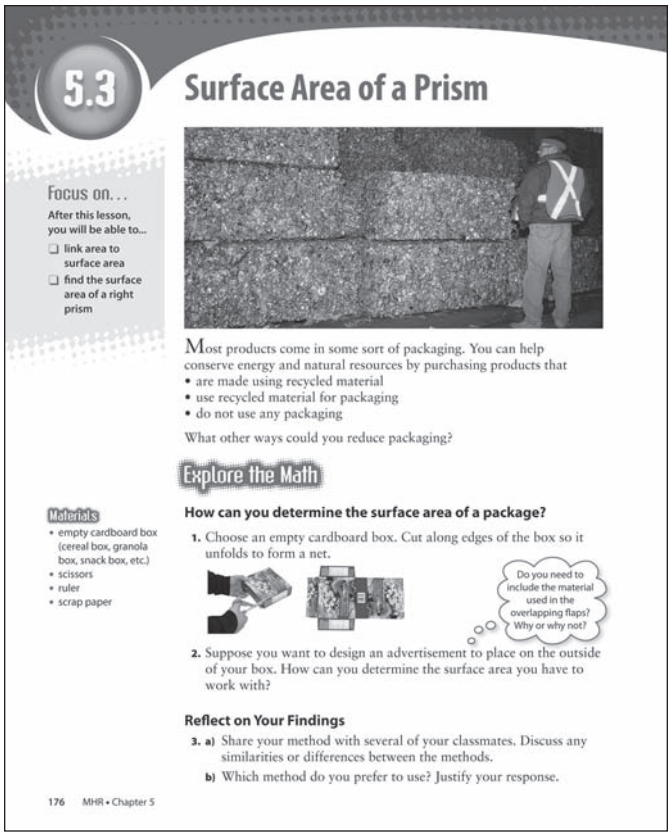
to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 8, Math Link
Typical	1, 2, 3 *or* 4, 5 *or* 6, 8, 10, 12, Math Link
Extension/Enrichment	1–3, 5, 10–16, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

Begin this section by having a discussion about recycling and why it is important to recycle packaging. Discuss ways that students could reduce packaging. Ask students to bring in a small empty cardboard box to use for the Explore the Math. Collect extra boxes for students who forget.



5.3 Surface Area of a Prism

Focus on...
After this lesson, you will be able to...
- ❑ link area to surface area
- ❑ find the surface area of a right prism

Most products come in some sort of packaging. You can help conserve energy and natural resources by purchasing products that
- are made using recycled material
- use recycled material for packaging
- do not use any packaging

What other ways could you reduce packaging?

Explore the Math

Materials
- empty cardboard box (cereal box, granola box, snack box, etc.)
- scissors
- ruler
- scrap paper

How can you determine the surface area of a package?

1. Choose an empty cardboard box. Cut along edges of the box so it unfolds to form a net.

Do you need to include the material used in the overlapping flaps? Why or why not?

2. Suppose you want to design an advertisement to place on the outside of your box. How can you determine the surface area you have to work with?

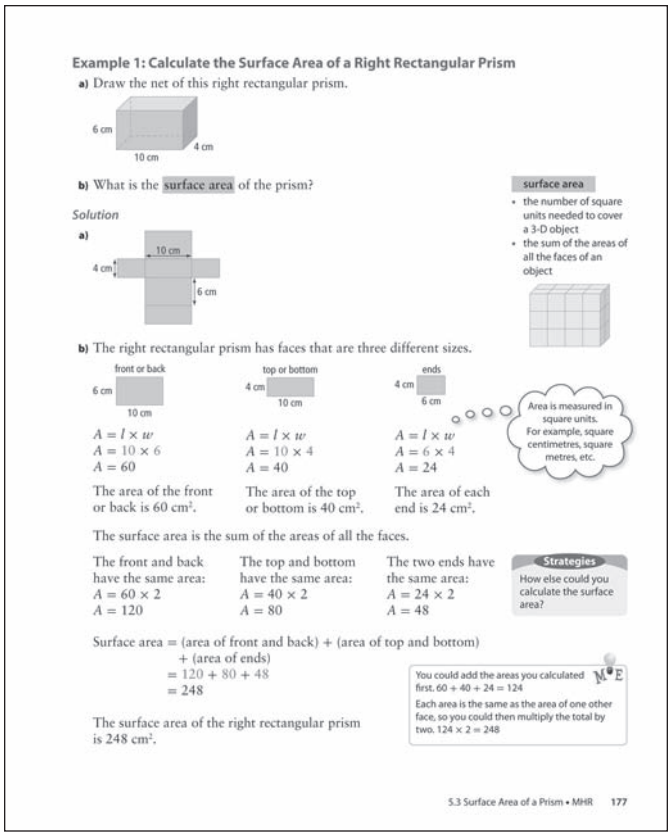
Reflect on Your Findings

3. a) Share your method with several of your classmates. Discuss any similarities or differences between the methods.
b) Which method do you prefer to use? Justify your response.

176 MHR • Chapter 5

Explore the Math

As students work on this investigation, have them consider the following in groups or individually. At appropriate times, ask probing questions about how boxes are made and what surface area involves as students take apart their boxes and see that the material for the box includes flaps:
- Now that you have your box taken apart, what does the material for the box include?
- Show me the parts you couldn't see when the box was put together.
- What do you call these parts? (flaps)
- What is the purpose of these flaps?
- Are the flaps part of the surface area? Explain why or why not.
- If they are not part of the surface area, when and why would box makers need to know the size of the flaps? (If students think that flaps are part of the surface area, have them put the box back together and ask them to show you the outside or "surface" of the box. Do they see the flaps now?)



Example 1: Calculate the Surface Area of a Right Rectangular Prism

a) Draw the net of this right rectangular prism.

6 cm, 10 cm, 4 cm

b) What is the surface area of the prism?

surface area
- the number of square units needed to cover a 3-D object
- the sum of the areas of all the faces of an object

Solution

a)

10 cm, 4 cm, 6 cm

b) The right rectangular prism has faces that are three different sizes.

front or back: 6 cm, 10 cm
top or bottom: 4 cm, 10 cm
ends: 4 cm, 6 cm

Area is measured in square units. For example, square centimetres, square metres, etc.

$A = l \times w$
$A = 10 \times 6$
$A = 60$

The area of the front or back is 60 cm².

$A = l \times w$
$A = 10 \times 4$
$A = 40$

The area of the top or bottom is 40 cm².

$A = l \times w$
$A = 6 \times 4$
$A = 24$

The area of each end is 24 cm².

The surface area is the sum of the areas of all the faces.

The front and back have the same area:
$A = 60 \times 2$
$A = 120$

The top and bottom have the same area:
$A = 40 \times 2$
$A = 80$

The two ends have the same area:
$A = 24 \times 2$
$A = 48$

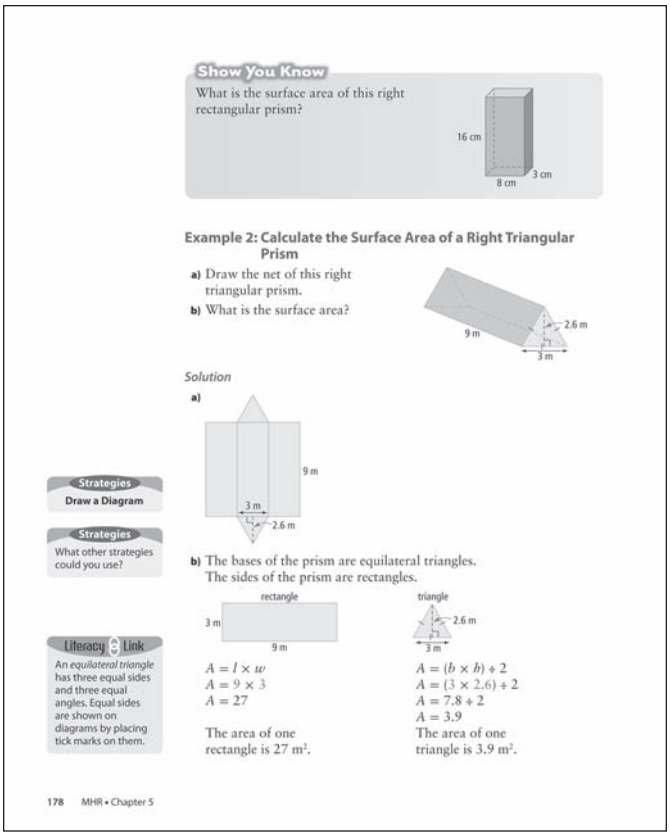
Strategies
How else could you calculate the surface area?

Surface area = (area of front and back) + (area of top and bottom) + (area of ends)
$= 120 + 80 + 48$
$= 248$

You could add the areas you calculated first. $60 + 40 + 24 = 124$
Each area is the same as the area of one other face, so you could then multiply the total by two. $124 \times 2 = 248$

The surface area of the right rectangular prism is 248 cm².

5.3 Surface Area of a Prism • MHR 177



Show You Know

What is the surface area of this right rectangular prism?

16 cm, 8 cm, 3 cm

Example 2: Calculate the Surface Area of a Right Triangular Prism

a) Draw the net of this right triangular prism.

b) What is the surface area?

9 m, 3 m, 2.6 m

Solution

a)

9 m, 3 m, 2.6 m

Strategies
Draw a Diagram

Strategies
What other strategies could you use?

b) The bases of the prism are equilateral triangles. The sides of the prism are rectangles.

rectangle: 3 m, 9 m
triangle: 2.6 m, 3 m

Literacy Link
An *equilateral triangle* has three equal sides and three equal angles. Equal sides are shown on diagrams by placing tick marks on them.

$A = l \times w$
$A = 9 \times 3$
$A = 27$

The area of one rectangle is 27 m².

$A = (b \times h) \div 2$
$A = (3 \times 2.6) \div 2$
$A = 7.8 \div 2$
$A = 3.9$

The area of one triangle is 3.9 m².

178 MHR • Chapter 5

- When would box makers need to know the surface area of a box? (Look for suggestions such as the amount of space for advertising, the amount of ink needed to colour the box, the amount of wrapper needed if the box is to have a paper cover, the space for a bar code, etc. Connect what students are doing in the Explore to this purpose.)

Method 1 Have students work on the exploration in pairs or small groups. Have students carefully dismantle their empty cardboard box to show the net plus the flaps. After groups have discussed their methods for finding the surface area with each other, have pairs or groups present and post their different strategies for the class. Ask:

- Did you get the same surface area? Explain why or why not. (Since students worked on different boxes, most will likely have different surface areas. Discuss how it is not the value of the surface area that is important, but how they determined the value. This will lead to a discussion of the strategies they used.)
- Explain the strategy you used and how it worked.
- (to other members of the class) Does this strategy determine the surface area of a box? Justify your thinking.
- How might you modify this strategy?
- (after discussion of several strategies) How are the strategies similar?
- How do they differ?
- Which is the most creative?
- Which is the most efficient?
- Which will likely work with the greatest variety of prisms?

Method 2 Have students complete the exploration individually, and share the strategy they used with a small group, and then the class. Again, discuss and post the different strategies. Use similar discussion questions to those in Method 1.

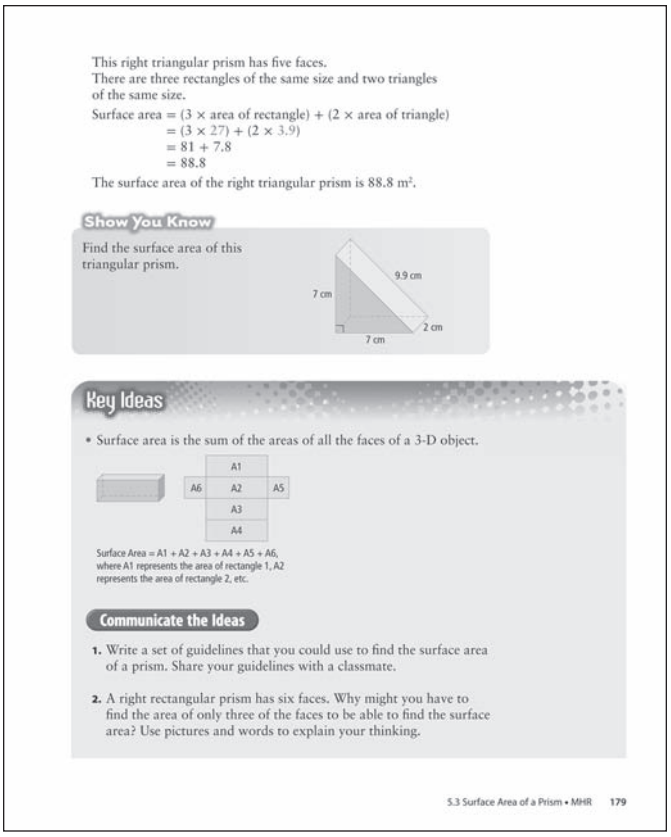
Example 1

Before considering Example 1, have students review the different strategies they developed in the Explore. Challenge them to review the strategy used in Example 1 and suggest a more efficient method for calculating the answer. Have them solve the problem using this more efficient strategy, then discuss what other strategies could be used.

Ask students to use a personal strategy to solve the Show You Know.

Example 2

Rectangular prisms were used in the Explore and Example 1. Challenge students to identify the prism they see in Example 2 and explain how that type



This right triangular prism has five faces.
There are three rectangles of the same size and two triangles of the same size.
Surface area = (3 × area of rectangle) + (2 × area of triangle)
= (3 × 27) + (2 × 3.9)
= 81 + 7.8
= 88.8
The surface area of the right triangular prism is 88.8 m².

Show You Know

Find the surface area of this triangular prism.

Key Ideas

- Surface area is the sum of the areas of all the faces of a 3-D object.

Surface Area = A1 + A2 + A3 + A4 + A5 + A6, where A1 represents the area of rectangle 1, A2 represents the area of rectangle 2, etc.

Communicate the Ideas

1. Write a set of guidelines that you could use to find the surface area of a prism. Share your guidelines with a classmate.
2. A right rectangular prism has six faces. Why might you have to find the area of only three of the faces to be able to find the surface area? Use pictures and words to explain your thinking.

5.3 Surface Area of a Prism • MHR 179

of prism might affect any calculation of surface area. Discuss their suggestions for modifying any strategies developed during the Explore.

Challenge students to read the example and identify a more efficient way to find the total surface area. Would this strategy work with all right triangular prisms?

You may wish to discuss how the strategies students use may change for each of the following scenarios:

- They have a triangular prism with a right triangle for the face and know the length of the two sides but not the hypotenuse.
- They have a triangular prism with a scalene triangle for the face.

Have them solve the Show You Know using both an efficient strategy and a less efficient strategy of their choice. Which strategy do they prefer? Why?

Literacy Link Refer students to the Literacy Link on page 178 that explains the term *equilateral triangle*.

Meeting Student Needs

- Before beginning this section, reactivate students' skills in finding the area of squares, rectangles, and triangles.
- Students may need to calculate the area of each face instead of doubling or adding it twice.
- Create a poster for the classroom showing the nets of a rectangular prism and a triangular prism. For each net, write the steps for finding the surface area.

ELL

- Ensure that students understand the following vocabulary: *packaging*, *conserve energy*, *natural resources*, *purchasing*, *unfold*, *dimensions*, *equilateral*, and *set of guidelines*.
- Orally explain the meaning of #5 to ensure understanding.

Common Errors

- Students may forget to include the area of one or more faces in the total.

R_x Remind students that surface area means the *total* of the areas of all the faces. Have them review the strategy they are using and make sure that it identifies all surfaces of the prism.

- Students may assume that all triangular prisms have an equilateral triangle as their base.

R_x Remind them not to make this assumption.

- Some students may use incorrect area formulas.

R_x Ensure students use the correct area formulas.

Answers

Explore the Math

2. Answers may vary. Example: Find the sum of the areas of the six faces. Consider a cardboard box with length 20 cm, width 10 cm, and height 4 cm. There are two faces, each measuring 20 cm by 10 cm, with an area of 200 cm^2. There are another two faces, each measuring 20 cm by 4 cm, with an area of 80 cm^2. The two remaining faces, each measuring 10 cm by 4 cm, have an area of 40 cm^2.

3. a) Methods may vary. Example: Find the sum of the areas of the six faces.

b) Answers may vary. Look for a method and a justification. Example: Since there are three pairs of equal faces, add the areas of the three different faces, and double the sum to find the total area. This is an efficient method to determine the total surface area.

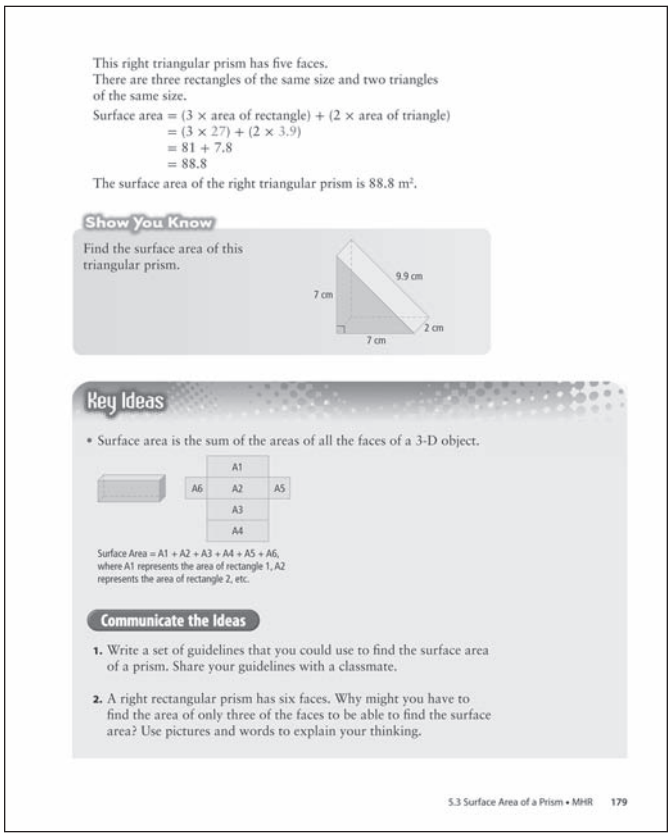
Show You Know: Example 1

400 cm^2

Show You Know: Example 2

96.8 cm^2

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize a conclusion from their findings. These questions are intended to help students discover that the sum of the areas of each face equals the surface area, and identify a strategy to find surface area.	• Students may benefit from working through an example using a 3-D object that you cut apart and manipulate. Alternatively, consider pairing students so they can explain their thinking to each other. • Some students may benefit from the class discussion for #3b) to help them understand how to calculate total area of a prism.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to show all their work, so you can identify any mistakes. • If students need additional practice, have them measure any rectangular prism in the classroom (e.g., the top of a desk) and calculate its surface area.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Notice that this is an isosceles triangle, which is different than the triangle shown in the example. • Drawing a net and labelling the dimensions may benefit some students, while others may find it more confusing. • Having students identify each different shape and its measurements may make it easier for them to calculate the area of each face separately and then add the area of all the faces together. • Encourage students to check their answers with a classmate to help clarify their understanding or catch anything that is missing.



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Surface area = (3 × area of rectangle) + (2 × area of triangle)
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Key Ideas

- Surface area is the sum of the areas of all the faces of a 3-D object.

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Communicate the Ideas

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2. A right rectangular prism has six faces. Why might you have to find the area of only three of the faces to be able to find the surface area? Use pictures and words to explain your thinking.

Key Ideas

Have students relate the Key Ideas to the Communicate the Ideas by having them examine the guidelines in the Key Ideas before writing the set of guidelines for #2. In addition, you may wish to have students rewrite the Key Ideas in their own words and include a worked example of their own.

Communicate the Ideas

These questions give students an opportunity to explain their understanding of how to calculate surface area of prisms.

Meeting Student Needs

- Some students may benefit from group discussions prior to writing down their own ideas.

ELL

- Give students a simple model of a set of guidelines. An example of such a model set might be "Come into class after recess. You must come in quietly. Put away jackets and be seated. Have a pen, your notebook, and your student resource, and sit quietly."

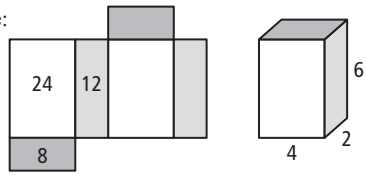
- For #1, you may wish to allow students to write the set of guidelines in their first language. This offers students the opportunity to activate their knowledge using familiar language. After doing so, it may be easier for students to ask for the missing vocabulary to express their thinking in English.
- For #2, encourage students to use visuals to illustrate their thinking and then use the visuals to explain their thinking orally.

Answers

Communicate the Ideas

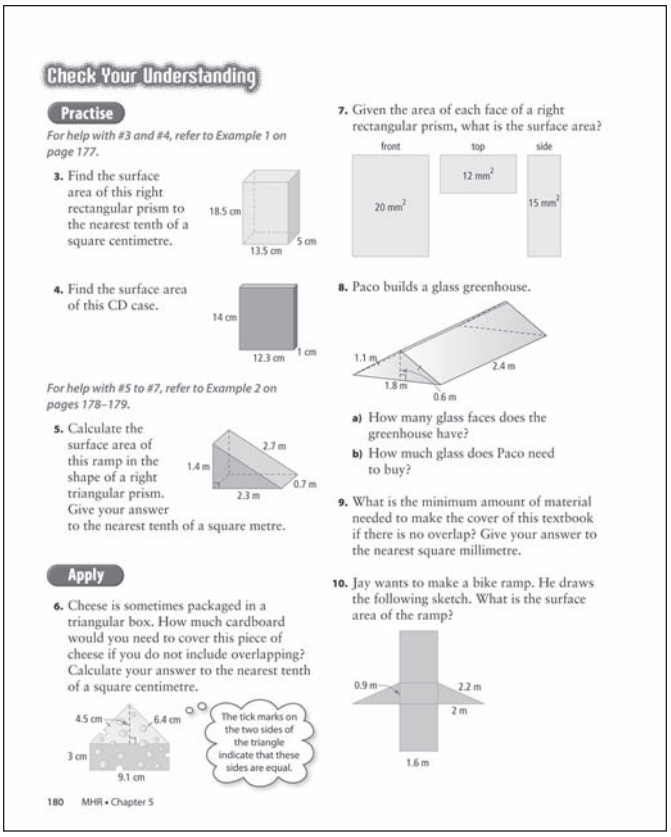
1. Answers may vary. Example:
- Draw a net for the prism.
- Find the area of each face.
- Find the sum of the areas of all the faces.

2. Answers may vary. Example:



In a right rectangular prism, the opposite faces are the same. When you know the total area of the three faces of different sizes, you can double the result to find the surface area of the prism. The net of this rectangular prism shows that the sum of the three faces on the left is 44 square units. The surface area of the prism is double that of 44 square units, or 88 square units.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and #2.	• Encourage students to share their ideas orally before recording them. • Encourage students to draw a sketch for #2.



Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on page 177.

3. Find the surface area of this right rectangular prism to the nearest tenth of a square centimetre.

4. Find the surface area of this CD case.

For help with #5 to #7, refer to Example 2 on pages 178–179.

5. Calculate the surface area of this ramp in the shape of a right triangular prism. Give your answer to the nearest tenth of a square metre.

Apply

6. Cheese is sometimes packaged in a triangular box. How much cardboard would you need to cover this piece of cheese if you do not include overlapping? Calculate your answer to the nearest tenth of a square centimetre.

7. Given the area of each face of a right rectangular prism, what is the surface area?

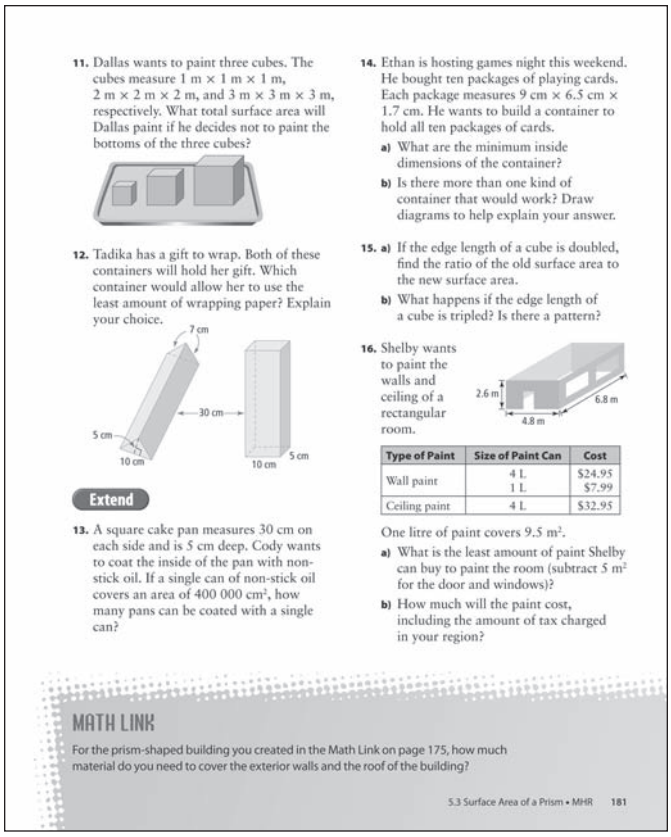
8. Paco builds a glass greenhouse.

a) How many glass faces does the greenhouse have?
b) How much glass does Paco need to buy?

9. What is the minimum amount of material needed to make the cover of this textbook if there is no overlap? Give your answer to the nearest square millimetre.

10. Jay wants to make a bike ramp. He draws the following sketch. What is the surface area of the ramp?

180 MHR • Chapter 5



11. Dallas wants to paint three cubes. The cubes measure 1 m × 1 m × 1 m, 2 m × 2 m × 2 m, and 3 m × 3 m × 3 m, respectively. What total surface area will Dallas paint if he decides not to paint the bottoms of the three cubes?

12. Tadika has a gift to wrap. Both of these containers will hold her gift. Which container would allow her to use the least amount of wrapping paper? Explain your choice.

Extend

13. A square cake pan measures 30 cm on each side and is 5 cm deep. Cody wants to coat the inside of the pan with non-stick oil. If a single can of non-stick oil covers an area of 400 000 cm², how many pans can be coated with a single can?

14. Ethan is hosting games night this weekend. He bought ten packages of playing cards. Each package measures 9 cm × 6.5 cm × 1.7 cm. He wants to build a container to hold all ten packages of cards.
a) What are the minimum inside dimensions of the container?
b) Is there more than one kind of container that would work? Draw diagrams to help explain your answer.

15. a) If the edge length of a cube is doubled, find the ratio of the old surface area to the new surface area.
b) What happens if the edge length of a cube is tripled? Is there a pattern?

16. Shelby wants to paint the walls and ceiling of a rectangular room.

Type of Paint	Size of Paint Can	Cost
Wall paint	4 L	$24.95
	1 L	$7.99
Ceiling paint	4 L	$32.95

One litre of paint covers 9.5 m².
a) What is the least amount of paint Shelby can buy to paint the room (subtract 5 m² for the door and windows)?
b) How much will the paint cost, including the amount of tax charged in your region?

MATH LINK

For the prism-shaped building you created in the Math Link on page 175, how much material do you need to cover the exterior walls and the roof of the building?

5.3 Surface Area of a Prism • MHR 181

Check Your Understanding

Practise

You may need to help students recall how to round to the tenth and hundredth decimal place prior to assigning questions. There are only three Practise questions because each one asks students to find the surface area, which involves many calculations. You may wish to have students do #3 or #4, and #5 so that they practise calculating surface area of both rectangular and triangular prisms.

Apply

The Apply questions require students to find surface area. For #7, students work with the area of each face, but have to consider how many faces there are.

Question 8, which is similar to #5, is an example of a real-world application. It has a progressive approach, in that students are prompted by the question through each step of solving the problem.

For #10, you may wish to refer students back to the two nets they enlarged in #10 on page 175 in section 5.2.

For #12, have students consider the amount of wrapping paper used, without considering the amount of waste.

Extend

The Extend questions invite students to apply their knowledge of surface area to solving problems.

Several questions in this section include job-related skills. You may wish to discuss how designers, chefs, and interior decorators use surface area.

Math Link

The Math Link provides an opportunity for students to calculate surface area of a prism. It requires students to use the building they created in the Math Link on page 175. If students have not completed this earlier Math Link, have them go to page 175 and complete parts a) and b) for the prism, before doing the calculations for this Math Link.

Meeting Student Needs

- Have students who have not done the Math Link on page 175 work with a partner to complete parts a) and b) for a prism. This will decrease the number of buildings to choose from when completing the Wrap It Up!
- For #8, some students may not be familiar with greenhouses. You may wish to provide a photo of one.
- Provide **BLM 5–11 Section 5.3 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure that students understand the following terms: *bike ramp*, *wrapping paper*, *cake pan*, and *least amount*.

Answers

Math Link

Answers may vary. Example: A miniature hospital building in the shape of a right rectangular prism that is 30 cm wide, 8 cm deep, and 15 cm tall would need 1380 cm^2 of material.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #3, #5, and #8. Students who can readily answer these questions can go on to the rest of the Apply questions.	• Refer students who need help with #3 back to Example 1. They may also find it easier to sketch three of the faces, find the areas of each, and add them together. Ask students how they could use the work they have done to find the surface area and how they could generalize the process. Have students try #4 before proceeding. • Provide additional coaching with Example 2 to students who need help with #5.
Math Link The Math Link on page 181 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 191.	• Clarify that the building will have sides and the roof will be covered, as the floor is not on the outside, but in the ground. • Encourage students to check each other's work for errors. • **BLM 5–12 Section 5.3 Math Link** provides scaffolding that will help some students complete the Math Link.
Assessment *as* Learning	
Math Learning Log Have students answer the following question: • Explain the similarities and differences between the processes of finding the surface area of a rectangular prism and a triangular prism.	• Students may benefit from a class discussion about the steps they need to take to find the surface area of each prism. • Consider recording the steps on chart paper. Then, have students use the notes to identify the similarities and differences between the two processes. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.