5.4 Surface Area of a Cylinder

***MathLinks 8*, pages 182–187**

Suggested Timing
80–100 minutes

Materials
- glow stick or other cylindrical object (optional)
- grid paper
- ruler

Blackline Masters
Master 8 Centimetre Grid Paper
BLM 5–3 Chapter 5 Warm-Up
BLM 5–13 Compare the Surface Area of a Prism and a Cylinder
BLM 5–14 Section 5.4 Extra Practice
BLM 5–15 Section 5.4 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
SS2 Draw and construct nets for 3-D objects.
SS3 Determine the surface area of:
- right rectangular prisms
- right triangular prisms
- right cylinders

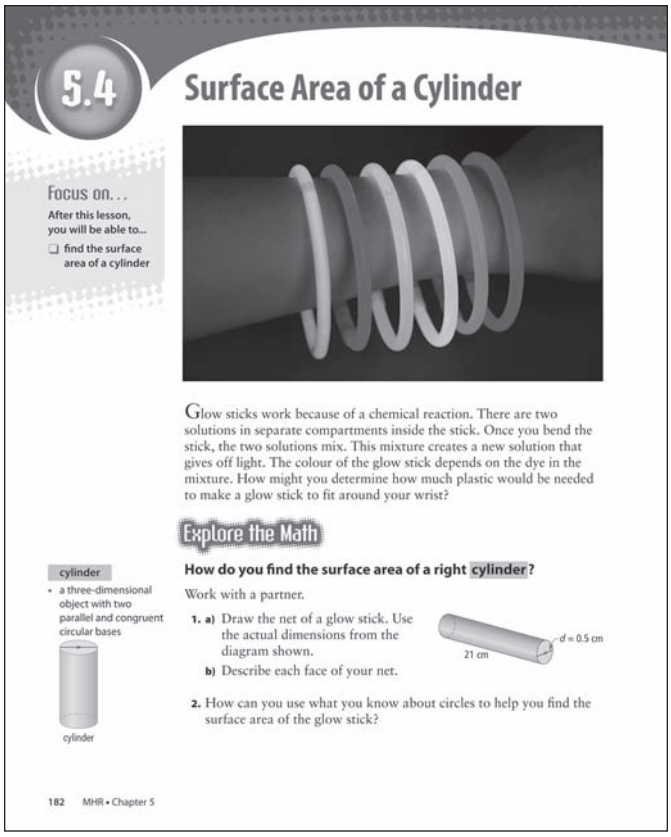
to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, Math Link
Typical	1–3, 4 *or* 5, 6, 8–10, Math Link
Extension/Enrichment	1, 2, 6a) *or* b), 9, 11–13, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

You may wish to bring in a glow stick to show to any students who have not seen one. Students could then measure the actual glow stick for more accurate dimensions or for a comparison.


5.4 Surface Area of a Cylinder

Focus on...
After this lesson, you will be able to...
- find the surface area of a cylinder

Glow sticks work because of a chemical reaction. There are two solutions in separate compartments inside the stick. Once you bend the stick, the two solutions mix. This mixture creates a new solution that gives off light. The colour of the glow stick depends on the dye in the mixture. How might you determine how much plastic would be needed to make a glow stick to fit around your wrist?

Explore the Math

cylinder
- a three-dimensional object with two parallel and congruent circular bases

cylinder

How do you find the surface area of a right cylinder?

Work with a partner.

1. a) Draw the net of a glow stick. Use the actual dimensions from the diagram shown.
   b) Describe each face of your net.

21 cm, d = 0.5 cm

2. How can you use what you know about circles to help you find the surface area of the glow stick?

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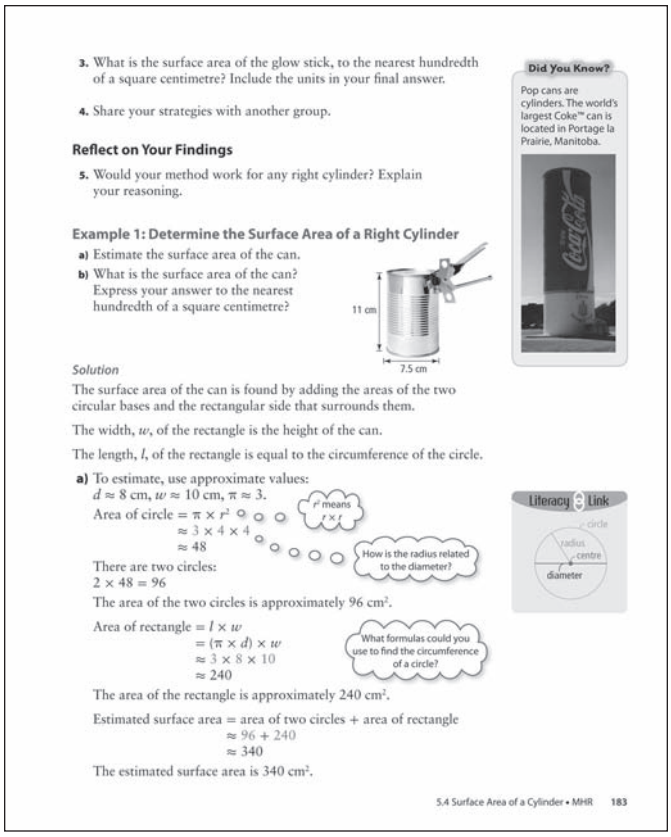
Explore the Math

Method 1 Distribute **Master 8 Centimetre Grid Paper**. Have students work in partners or small groups. Circulate and listen to students talking about the problem. If a pair or group comes to a dead end, you may wish to use guiding questions such as:
- Identify the shapes that make up your net.
- Label the measurements of each shape.
- Which measurements have not been provided?
- How can you find each of these measurements?
- How can you use what you know about circles to help you find the missing measurement(s)?
- What do you already know about finding the area of each shape?
- Is there something you need to review about finding the area of any shape?
- How can you use the strategies we developed for calculating the surface area of rectangular and triangular prisms to help you with the surface area of a cylinder?



3. What is the surface area of the glow stick, to the nearest hundredth of a square centimetre? Include the units in your final answer.

4. Share your strategies with another group.

Reflect on Your Findings

5. Would your method work for any right cylinder? Explain your reasoning.

Example 1: Determine the Surface Area of a Right Cylinder

a) Estimate the surface area of the can.

b) What is the surface area of the can? Express your answer to the nearest hundredth of a square centimetre?

11 cm, 7.5 cm

Solution

The surface area of the can is found by adding the areas of the two circular bases and the rectangular side that surrounds them.

The width, w, of the rectangle is the height of the can.

The length, l, of the rectangle is equal to the circumference of the circle.

a) To estimate, use approximate values:
$d \approx 8$ cm, $w \approx 10$ cm, $\pi \approx 3$.

Area of circle $= \pi \times r^2$
$\approx 3 \times 4 \times 4$
≈ 48

r^2 means $r \times r$

How is the radius related to the diameter?

There are two circles:
$2 \times 48 = 96$

The area of the two circles is approximately 96 cm².

Area of rectangle $= l \times w$
$= (\pi \times d) \times w$
$\approx 3 \times 8 \times 10$
≈ 240

What formulas could you use to find the circumference of a circle?

The area of the rectangle is approximately 240 cm².

Estimated surface area = area of two circles + area of rectangle
$\approx 96 + 240$
≈ 340

The estimated surface area is 340 cm².

Did You Know?

Pop cans are cylinders. The world's largest Coke™ can is located in Portage la Prairie, Manitoba.

Literacy Link

circle, radius, centre, diameter

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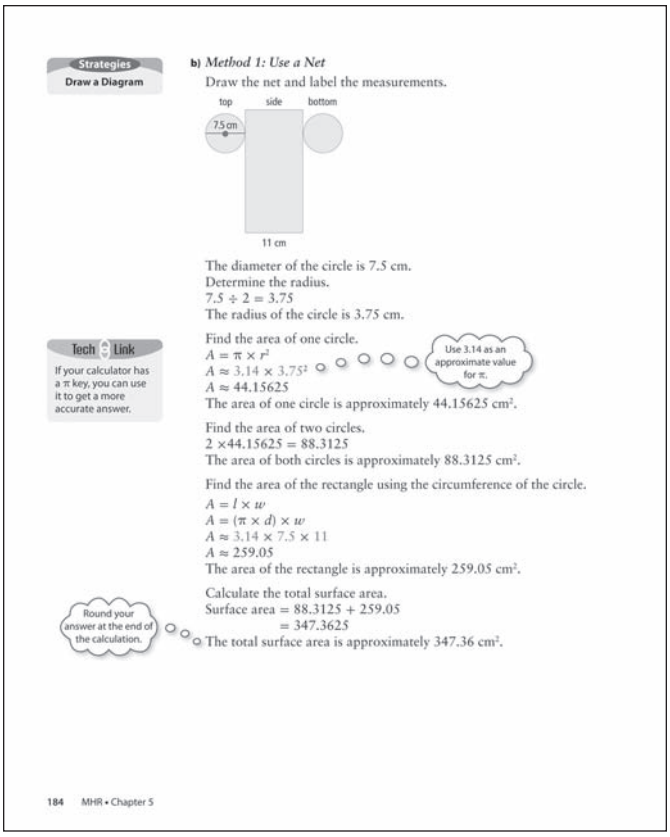

Strategies

Draw a Diagram

b) *Method 1: Use a Net*

Draw the net and label the measurements.

top, side, bottom; 7.5 cm; 11 cm

The diameter of the circle is 7.5 cm.
Determine the radius.
$7.5 \div 2 = 3.75$
The radius of the circle is 3.75 cm.

Tech Link

If your calculator has a π key, you can use it to get a more accurate answer.

Find the area of one circle.
$A = \pi \times r^2$
$A \approx 3.14 \times 3.75^2$
$A \approx 44.15625$

Use 3.14 as an approximate value for π.

The area of one circle is approximately 44.15625 cm².

Find the area of two circles.
$2 \times 44.15625 = 88.3125$
The area of both circles is approximately 88.3125 cm².

Find the area of the rectangle using the circumference of the circle.
$A = l \times w$
$A = (\pi \times d) \times w$
$A \approx 3.14 \times 7.5 \times 11$
$A \approx 259.05$
The area of the rectangle is approximately 259.05 cm².

Calculate the total surface area.
Surface area $= 88.3125 + 259.05$
$= 347.3625$

Round your answer at the end of the calculation.

The total surface area is approximately 347.36 cm².

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Have pairs and small groups discuss their strategies and answers with another pair or small group. Discuss the different strategies used and have students post at least one example of each strategy.

- How are the strategies similar?
- How do they differ?
- What skills do you need to know to use each strategy?
- Would your method work for other right cylinders? Test your response.

Method 2 Provide each pair of students with an empty can and have them roll that to draw and label all parts of a cylinder net. Challenge students to identify the shapes used to make up a cylinder and find the total surface area of the cylinder. How does a net help them calculate the surface area?

Challenge students to identify the missing measurement(s) and how to get them. Discuss student ideas in small groups, and then with the class, as in Method 1.

Method 3 Provide each pair of students with a right cylinder such as an empty cardboard juice can, complete with lid, or an empty paper towel or toilet paper tube for which lids have been improvised. Have them cut apart the cylinder to display the net, and then trace and label all parts. Challenge students to identify the shapes used to make up a cylinder and find the total surface area of the cylinder. How does a net help them calculate the surface area?

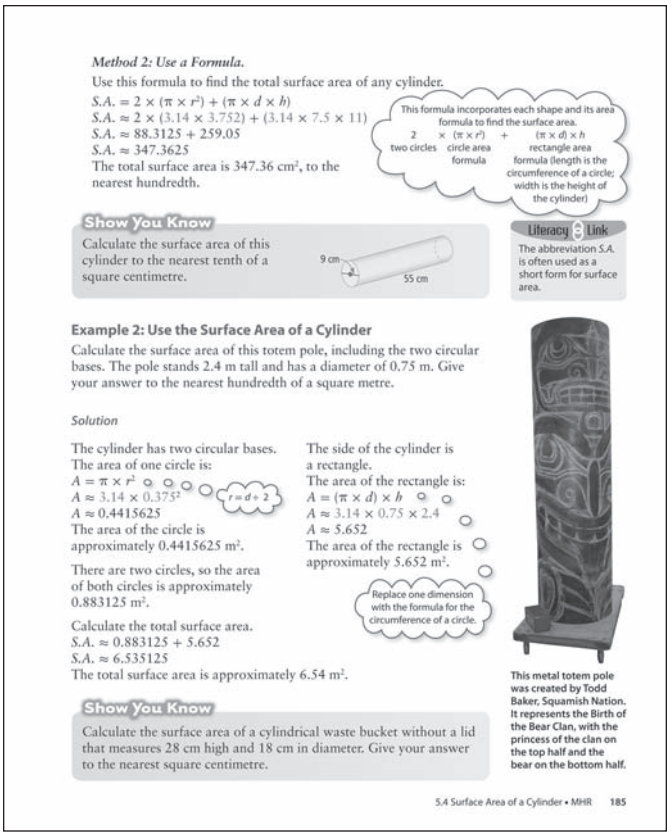
Challenge students to identify the missing measurement(s) and how to get them. Ask them how taking the cylinder apart to make a net helped them develop one or more personal strategies for solving this problem.

Example 1

This example demonstrates two strategies for determining the surface area of a cylinder. Method 1 uses a strategy similar to the one used in section 5.3, where students find the area of each face and add them all together. Method 2 uses a formula to find the surface area.

Literacy Link Direct students to the Literacy Link on page 183 to help clarify the terms *radius* and *diameter* of a circle.

Note the use of the abbreviation, *S.A.*, in Method 2. Refer students to the Literacy Link on page 185 that explains the abbreviation for surface area.



Method 2: Use a Formula.

Use this formula to find the total surface area of any cylinder.

$S.A. = 2 \times (\pi \times r^2) + (\pi \times d \times h)$
$S.A. \approx 2 \times (3.14 \times 3.752) + (3.14 \times 7.5 \times 11)$
$S.A. \approx 88.3125 + 259.05$
$S.A. \approx 347.3625$
The total surface area is 347.36 cm², to the nearest hundredth.

This formula incorporates each shape and its area formula to find the surface area.
2 two circles × $(\pi \times r^2)$ circle area formula + $(\pi \times d) \times h$ rectangle area formula (length is the circumference of a circle; width is the height of the cylinder)

Show You Know

Calculate the surface area of this cylinder to the nearest tenth of a square centimetre.

9 cm, 55 cm

Literacy Link

The abbreviation *S.A.* is often used as a short form for surface area.

Example 2: Use the Surface Area of a Cylinder

Calculate the surface area of this totem pole, including the two circular bases. The pole stands 2.4 m tall and has a diameter of 0.75 m. Give your answer to the nearest hundredth of a square metre.

Solution

The cylinder has two circular bases.
The area of one circle is:
$A = \pi \times r^2$
$A \approx 3.14 \times 0.375^2$ ($r = d \div 2$)
$A \approx 0.4415625$
The area of the circle is approximately 0.4415625 m².

There are two circles, so the area of both circles is approximately 0.883125 m².

Calculate the total surface area.
$S.A. \approx 0.883125 + 5.652$
$S.A. \approx 6.535125$
The total surface area is approximately 6.54 m².

The side of the cylinder is a rectangle.
The area of the rectangle is:
$A = (\pi \times d) \times h$
$A \approx 3.14 \times 0.75 \times 2.4$
$A \approx 5.652$
The area of the rectangle is approximately 5.652 m².

Replace one dimension with the formula for the circumference of a circle.

Show You Know

Calculate the surface area of a cylindrical waste bucket without a lid that measures 28 cm high and 18 cm in diameter. Give your answer to the nearest square centimetre.

This metal totem pole was created by Todd Baker, Squamish Nation. It represents the Birth of the Bear Clan, with the princess of the clan on the top half and the bear on the bottom half.

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You may wish to do a hands-on demonstration to clarify where and why circumference is used, so that students can see how the length of the rectangle is equivalent to the circumference of the circle. Remind students that the symbol ≈ means an approximate answer due to an estimation or because 3.14 is a value of pi to the nearest hundredth.

Example 2

Example 2 demonstrates how a strategy using surface area can apply to a real-life context. Remind students how to find the area of a circle.

Meeting Student Needs

- Before beginning this section, help students recall how to find the area of a circle.
- Provide nets for students to cut out and manipulate, if needed.
- Encourage students to draw a net and label the dimensions for clarification.
- Create a poster for the classroom showing the net for a cylinder. On the net, write the steps for finding the surface area.
- Before working on the Show You Know for Example 1, provide students with additional practice. Work as a whole class and measure a can. Demonstrate rounding each measurement to the nearest whole number. Use the measurements to calculate the surface area of the can. Use both methods (using a net and using a formula) to calculate surface area. Have students work with a partner and repeat the activity several times with different-sized cans or other cylindrical objects.

ELL

- Students may have difficulty with the following vocabulary: *glow sticks*, *chemical reaction*, *separate compartments*, *identical*, and *surrounds*.
- Use the visual on page 182 and/or a real glow stick to help describe a glow stick. Help students understand how a glow stick works by using a drawing on the chalkboard of the inside of a glow stick with two different compartments and explain how bending the glow stick causes the compartments to break, which allows the solutions to mix.

Gifted and Enrichment

- Challenge students to calculate the surface area of a hand drum or a ceremonial drum. Have students work in small groups and discuss how to develop a pattern or use a net for showing someone how to make a drum.

Common Errors

- Some students may include the area of only one circle in the total surface area.

R_x Remind students to add the areas of all shapes in the total surface area.

- Some students may not follow the order of operations when calculating the area of a circle.

R_x Ensure students square the radius before multiplying by pi.

- Some students may use the diameter instead of the radius to calculate the area of a circle.

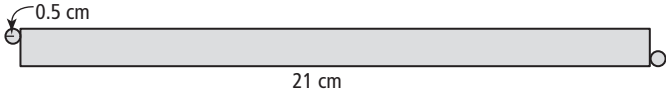
R_x Help students recall the formula for calculating the area of a circle.

Answers

Explore the Math

1. a) Drawings will vary. Look for the following:
- the width of the rectangle should be just over three times the diameter of the circle
- the length of the rectangle represents the height of the cylinder

Example:



b) Answers may vary. Example: The net shows two circles and one rectangle.

2. Answers may vary. Example:
- Calculate the area of one circular base using the formula $A = \pi \times r^2$.
- Add the areas of the two circular bases.
- Use the formula $C = \pi \times d$ to find the length of the rectangular area that goes around the circular ends of the cylinder.
- Use the circumference as the length of the rectangle that makes up the net.

3. 33.37 cm^2

4. Answers may vary. Provide time for students to share and compare their strategies.

5. Answers may vary depending on methods. Expect students to demonstrate how their method works for a specific cylinder. Have them check their calculations using a different method.

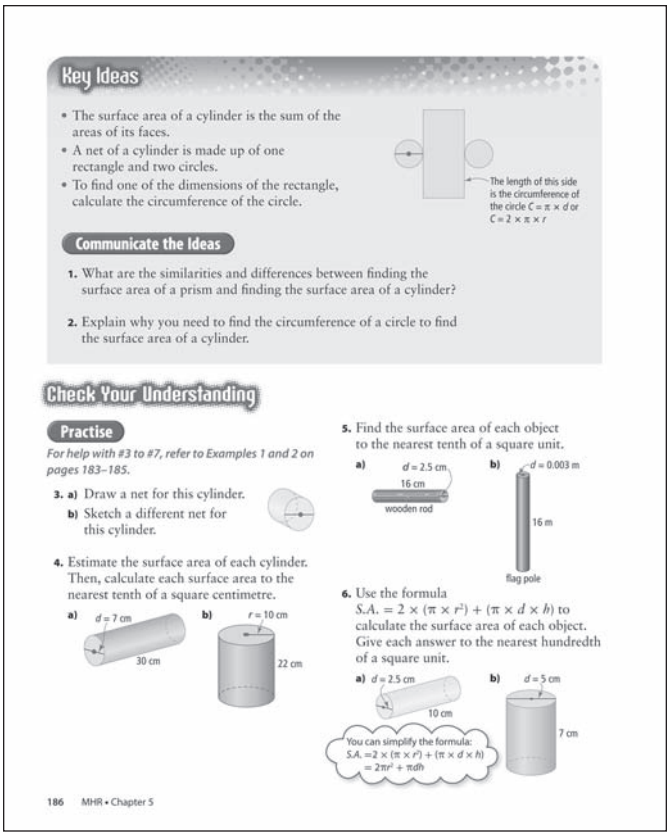
Show You Know: Example 1

1681.5 cm^2

Show You Know: Example 2

1837 cm^2

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize a conclusion from their findings. A critical point for students is to use the circumference formula when calculating the area of the rectangle to determine the total surface area.	• Have students try to find alternatives to using the circumference formula by giving them the answer for the surface area of a cylinder, and encourage them to work backwards from the answer. (There are no alternatives. Students may make fewer mistakes or be less likely to forget to use the circumference formula if they exhaust all other possibilities.) • Some students may benefit from having the surface area formula for the cylinder written using only radius so they do not have two different values (d and r) in the formula. $S.A. = 2 \times (\pi \times r^2) + 2 \times (\pi \times r \times h)$ • Discussing the answer may benefit students who can use the thinking to apply to future questions.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to estimate their answers first to help them determine whether the actual answer is reasonable. • Encourage students to use the strategy they prefer.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from drawing a diagram and labelling it with the given dimensions. • Have students try to use the strategy they didn't choose for the Example 1 Show You Know. • Consider having students check each other's answer.



Key Ideas

- The surface area of a cylinder is the sum of the areas of its faces.
- A net of a cylinder is made up of one rectangle and two circles.
- To find one of the dimensions of the rectangle, calculate the circumference of the circle.

The length of this side is the circumference of the circle $C = \pi \times d$ or $C = 2 \times \pi \times r$

Communicate the Ideas

1. What are the similarities and differences between finding the surface area of a prism and finding the surface area of a cylinder?
2. Explain why you need to find the circumference of a circle to find the surface area of a cylinder.

Check Your Understanding

Practise

For help with #3 to #7, refer to Examples 1 and 2 on pages 183–185.

3. a) Draw a net for this cylinder.
   b) Sketch a different net for this cylinder.
4. Estimate the surface area of each cylinder. Then, calculate each surface area to the nearest tenth of a square centimetre.
   a) $d = 7$ cm, 30 cm
   b) $r = 10$ cm, 22 cm
5. Find the surface area of each object to the nearest tenth of a square unit.
   a) $d = 2.5$ cm, 16 cm, wooden rod
   b) $d = 0.003$ m, 16 m, flag pole
6. Use the formula $S.A. = 2 \times (\pi \times r^2) + (\pi \times d \times h)$ to calculate the surface area of each object. Give each answer to the nearest hundredth of a square unit.
   a) $d = 2.5$ cm, 10 cm
   b) $d = 5$ cm, 7 cm

You can simplify the formula: $S.A. = 2 \times (\pi \times r^2) + (\pi \times d \times h) = 2\pi r^2 + \pi dh$

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Have students summarize the Key Ideas in their own words, using a diagram and a worked example. Encourage students to use the method they prefer to calculate the surface area of a cylinder.

Communicate the Ideas

These questions provide an opportunity for students to explain their understanding of calculating the surface area of prisms and cylinders.

You may wish to have students work with a partner to discuss their answers and present them to the class. Record all similarities and differences on chart paper for students to refer to throughout this section.

Literacy Link Identifying similarities and differences has proven to be one of the most effective strategies for learning. Developing a comparison using a graphic organizer, such as a double bubble organizer, provides students with a visual and memory aid. You may wish to put the double bubble organizer on **BLM 5–13 Compare the Surface Area of a Prism and a Cylinder** on an overhead transparency and fill in the blanks with students.

Meeting Student Needs

- Have students work in groups of three or four to answer #1 and #2.
- Consider having groups hand in their answers instead of reporting to the whole class.
- Double bubble organizers are particularly useful for visual learners. You may wish to have students fill out **BLM 5–13 Compare the Surface Area of a Prism and a Cylinder** on their own.

ELL

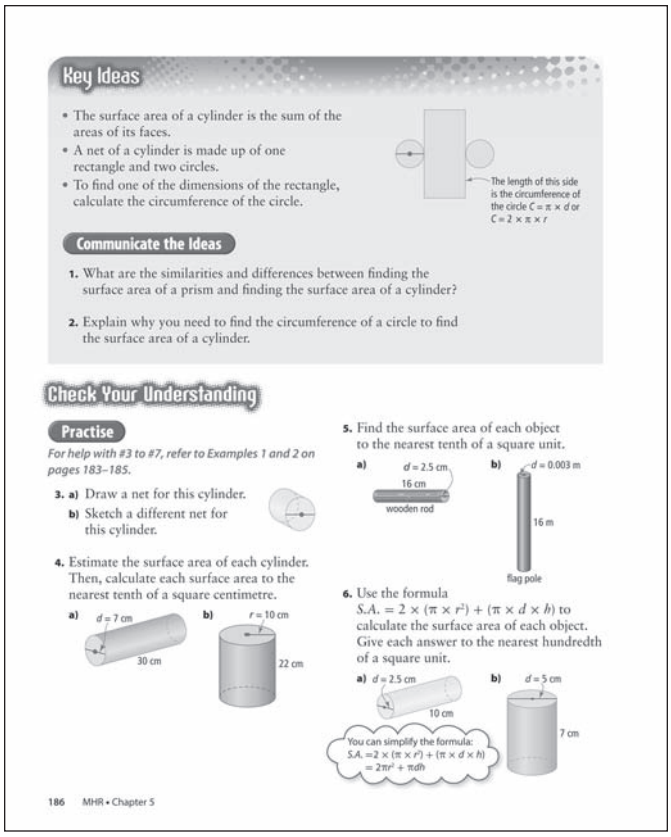
- For #1, have students fill out a T-chart with the headings Same and Different and visuals of the surface area of a prism and a cylinder. Graphic organizers are a good way for students to access the information being taught.

Answers

Communicate the Ideas

1. Answers may vary. Examples:
   Similarities – The surface area can be found by finding the total area of the shapes in its net; the surface area requires calculating the area of a rectangle.
   Differences – The surface area of a cylinder requires calculating the area and circumference of a circle, which is not required for a prism.
2. In the net of a cylinder, one side of the rectangle is equal to the circumference of the circular base of the cylinder.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and #2.	• Have students work in groups to develop ideas and then write their answers on their own. • Sharing the answers in a class discussion will benefit students who are still unsure.



Key Ideas

- The surface area of a cylinder is the sum of the areas of its faces.
- A net of a cylinder is made up of one rectangle and two circles.
- To find one of the dimensions of the rectangle, calculate the circumference of the circle.

The length of this side is the circumference of the circle $C = \pi \times d$ or $C = 2 \times \pi \times r$

Communicate the Ideas

1. What are the similarities and differences between finding the surface area of a prism and finding the surface area of a cylinder?
2. Explain why you need to find the circumference of a circle to find the surface area of a cylinder.

Check Your Understanding

Practise

For help with #3 to #7, refer to Examples 1 and 2 on pages 183–185.

3. a) Draw a net for this cylinder.
   b) Sketch a different net for this cylinder.
4. Estimate the surface area of each cylinder. Then, calculate each surface area to the nearest tenth of a square centimetre.
   a) $d = 7$ cm, 30 cm
   b) $r = 10$ cm, 22 cm
5. Find the surface area of each object to the nearest tenth of a square unit.
   a) $d = 2.5$ cm, 16 cm, wooden rod
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6. Use the formula $S.A. = 2 \times (\pi \times r^2) + (\pi \times d \times h)$ to calculate the surface area of each object. Give each answer to the nearest hundredth of a square unit.
   a) $d = 2.5$ cm, 10 cm
   b) $d = 5$ cm, 7 cm

You can simplify the formula: $S.A. = 2 \times (\pi \times r^2) + (\pi \times d \times h) = 2\pi r^2 + \pi dh$

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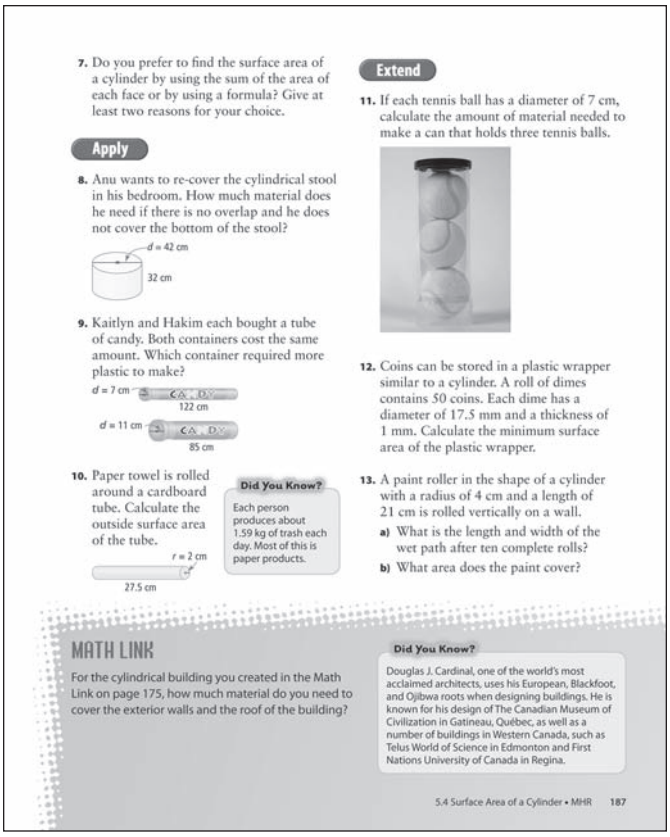

7. Do you prefer to find the surface area of a cylinder by using the sum of the area of each face or by using a formula? Give at least two reasons for your choice.

Apply

8. Anu wants to re-cover the cylindrical stool in his bedroom. How much material does he need if there is no overlap and he does not cover the bottom of the stool?

9. Kaitlyn and Hakim each bought a tube of candy. Both containers cost the same amount. Which container required more plastic to make?

10. Paper towel is rolled around a cardboard tube. Calculate the outside surface area of the tube.

Did You Know?

Each person produces about 1.59 kg of trash each day. Most of this is paper products.

Extend

11. If each tennis ball has a diameter of 7 cm, calculate the amount of material needed to make a can that holds three tennis balls.

12. Coins can be stored in a plastic wrapper similar to a cylinder. A roll of dimes contains 50 coins. Each dime has a diameter of 17.5 mm and a thickness of 1 mm. Calculate the minimum surface area of the plastic wrapper.
13. A paint roller in the shape of a cylinder with a radius of 4 cm and a length of 21 cm is rolled vertically on a wall.
    a) What is the length and width of the wet path after ten complete rolls?
    b) What area does the paint cover?

MATH LINK

For the cylindrical building you created in the Math Link on page 175, how much material do you need to cover the exterior walls and the roof of the building?

Did You Know?

Douglas J. Cardinal, one of the world's most acclaimed architects, uses his European, Blackfoot, and Ojibwa roots when designing buildings. He is known for his design of The Canadian Museum of Civilization in Gatineau, Québec, as well as a number of buildings in Western Canada, such as Telus World of Science in Edmonton and First Nations University of Canada in Regina.

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Check Your Understanding

Practise

You may wish to assign parts a) or b) for #4 to #6, instead of both, then use the remaining parts for extra practice. For #4 and #5, it is assumed that students will use the Method 1 strategy from Example 1 to answer the questions. Then, #6 specifies that students are to use a formula. In #7, students are asked to state which method they prefer.

Apply

The Apply questions provide a range of contexts for students to solve problems involving surface area. Work with students to identify actual objects with the shapes given in the questions. You may have them calculate the surface area of specific shapes instead of the ones pictured in the student resource. For example, the cardboard tube in #10 could be replaced by a toilet paper roll.

Extend

The Extend questions require students to perform multiple steps to find the solutions.

You may wish to have students compare their calculations for #12 to the actual dimensions of a plastic wrapper for dimes.

To extend #13, have students do the calculation for several rollers of various sizes.

Math Link

The Math Link provides an opportunity for students to calculate the surface area of a cylinder. It is a continuation of the Math Link on page 175. Students take the cylindrical building they designed and find the amount of material needed to cover the sides and the roof. If students have not completed this earlier Math Link, have them go to page 175 and complete parts a) and b) for the cylinder, before doing the calculations for this Math Link.

Meeting Student Needs

- Encourage students to estimate first if they are using decimal numbers.
- Provide **BLM 5–14 Section 5.4 Extra Practice** to students who would benefit from more practice.

ELL

- For #4, use the chalkboard to model how to estimate and then calculate an answer to a question.
- Ensure that students understand the following terms: *re-cover*, *stool*, *overlap*, *tube*, *container*, *coins*, *plastic wrapper*, and *paint roller*.

Answers

Math Link

Answers may vary. Example: For a cylindrical hotel building that is 60 m tall and 36 m in diameter, you would need about 7799.76 m^2 of material for the roof and the exterior walls.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #3, #4, #6, and #8. Students who can readily do these questions can go on to the rest of the Apply questions, then try the Extend questions.	• Provide grid paper to students to assist them with #3. Have students compare their nets to other students' nets. • Have students attempt #4 using the method with which they are most comfortable. Grid paper may be of assistance. Remind students to estimate and explain why estimating provides a link to the actual calculation. Provide assistance to students who need help with #4. Coach them through it and clarify any misunderstandings. Encourage students to try one of the question parts in #5 before going on. • In #6, students are asked to use a formula. Students may not be comfortable with the use of the formula but they should be aware of what each part represents. To complete the question, allow students to use the method they are most comfortable with.
Math Link The Math Link on page 187 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 191.	• Clarify that for the calculation of total surface area, students do not need to include maaterial needed to cover a floor, as the floor is not on the outside, but in the ground. • Students may benefit from checking each other's work for errors. • **BLM 5–15 Section 5.4 Math Link** provides scaffolding that will help some students complete the Math Link.
Assessment *as* Learning	
Math Learning Log Have students answer #7 on page 187. Also, have them develop their own problem and solve it using their preferred method. Have students complete the following statement: • The part I find most confusing in finding the surface area of a cylinder is …	• Have students discuss the answer with a partner before recording the answer on their own. • Encourage students to use the list of steps to solve a problem. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.