Multiplying a Fraction and a Whole Number

Multiplying a Fraction and a Whole Number Chess is one of the most popular board games. It h Focus on... been estimated that about $\frac{1}{2}$ After this lesson, you will be able to... of adult Canadians play multiply a fraction and a whole number chess at least once a year The chess board shown has black and white squares. What fraction of the total solve problems involving the multiplication of number of squares are black? If you were told the total area of all the squares, how could you determine the total area of the black squares? a fraction and a whole number Explore the Math How can you model the multiplication of a fraction and a Materials whole number? pattern blocks 1. a) How do the pattern blocks model an addition? Describe it. Literacy 🕄 Link Understanding Multiplication The product of 4 and 2 is 8, because $4 \times 2 = 8$. 00 one whole, what do a red sezoid, a blue rhombus, ar The equation $4 \times 2 = 8$ means that 4 groups of 2 make 8. You can also think of 4 \times 2 as the b) How do the pattern blocks also model a multiplication? Describe it. c) Work with a partner to explore other manipulatives you could use to model the multiplicati 2. a) Work with a partner to explore how you could use diagrams to model $4 \times \frac{1}{6}$, b) Write an equation to represent your model. 198 MHR + Chapter 6

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce material learned in previous sections.

Have students discuss the two questions in the section introduction about the chessboard. Even if they are unfamiliar with the board, they will see from the illustration that half the playing area is black. There are likely to be different opinions on how to determine the total area of the black squares. Some students may not consider fractions and may suggest that the total area should be divided by two. Other students may suggest determining half the playing area, without necessarily realizing that this process is a multiplication of the fraction $\frac{1}{2}$ and the playing area. You may wish to return to the second question after students have completed Example 3.

MathLinks 8, pages 198–203

Suggested Timing

50–60 minutes

50-00 minute:

Materials

- pattern blocks
- fraction strips (optional)
- transparent shapes or strips (optional)
- dry erase markers (optional)
- coloured pencils (optional)
- ruler

Blackline Masters

Master 13 Pattern Blocks Master 14 Fraction Strips BLM 6–3 Chapter 6 Warm-Up BLM 6–6 Section 6.1 Extra Practice BLM 6–7 Section 6.1 Math Link

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- Reasoning (R)
- Technology (T)
- ✓ Visualization (V)

Specific Outcomes

N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, Math Link
Typical	1-4, 6, 8-12, Math Link
Extension/Enrichment	1–3, 10–15



Explore the Math

Students develop a model for multiplying a fraction and a whole number by considering multiplication as repeated addition and using pattern blocks or diagrams.

Literacy Link Draw students' attention to the Literacy Links on pages 198 and 199. The first Literacy Link reviews concepts about multiplication. Discuss with students how the same concept applies to multiplying fractions. In this Explore the Math, they will work with groups of fractions.

The second Literacy Link reviews the meanings of *proper fraction*, *improper fraction*, and *mixed number*. To reactivate student learning related to these terms, have students classify a list of fractions and provide examples and nonexamples of each type.

Method 1 Have students work with a partner. Provide a set of pattern blocks and other fraction manipulatives, such as fraction strips or fraction circles. You may wish to hand out **Master 13 Pattern Blocks** if sets of pattern blocks are not available. As students work on the Explore the Math, circulate and ask questions such as

- What addition do the pattern blocks show?
- How might you use what you know about adding fractions to help you multiply fractions?
- What multiplication do the pattern blocks show?
- What other manipulatives could you use to multiply $3 \times \frac{1}{2}$?
- How could you use a number line (or rectangles or circles) to model $3 \times \frac{1}{2}$?
- What method can you use to model $2 \times \frac{4}{3}$?
- What can you learn from these models that will help you multiply fractions?

Method 2 Have students use other models, such as fraction strips, to model the multiplications as repeated additions, as shown in Example 1. You may wish to hand out **Master 14 Fraction Strips**. Have students consider what the models show and generalize a rule for multiplying a fraction with a whole number.

Example 1

Example 1 models using fraction strips to model the multiplication of a fraction and a whole number. Reinforce that the example uses a familiar problem solving strategy (i.e., Model It). Ask students to think of another strategy they could use (e.g., Draw



a Diagram). Point out the thought bubble that shows the same multiplication modelled using pattern blocks. Ask students which of the two methods shown in Example 1 they prefer, and why. Emphasize that $3 \times \frac{5}{6} = \frac{15}{6}$ would be a correct multiplication statement, but that the example requires the product to be expressed in lowest terms. This is the way in which fractions are most commonly expressed. Challenge students to express the answer as a mixed number $(2\frac{1}{2})$.

Example 2

Example 2 illustrates using a number line to model the multiplication of a fraction and a whole number. Reinforce that the example uses a familiar problem solving strategy (i.e., Draw a Diagram). Ask students to think of another strategy they could use (e.g., Model It). Point out the thought bubble that shows the same multiplication modelled using diagrams of rectangles. Point out that the product, $\frac{6}{5}$, is already in lowest terms, since the numerator and denominator have no common factor, other than one. Some students may report this as a mixed number $(1\frac{1}{5})$, which is not necessary at this stage. Students will work with mixed numbers later in the chapter. **Literacy Link** Draw students' attention to the Literacy Link on page 200 to help them recall that the word *of* often indicates multiplication.

Draw students' attention to the statement indicating that $\frac{3}{4} \times 8 = 8 \times \frac{3}{4}$ as an example of the commutative property: $a \times b = b \times a$, which is outlined in the Literacy Link on page 201. In other words, two numbers can be multiplied in either order. You might ask students to consider whether division, addition, and subtraction are commutative and to give examples with whole numbers.

Example 3

Example 3 illustrates an application of the multiplication of a fraction and a whole number. The number-line method used is the same method as in Example 2. You may wish to have students show how to solve the same problem by drawing rectangles.

You may wish to introduce a second number-line method, which has advantages when the whole number in the multiplication is large. For example, to model $\frac{3}{4} \times 8$, first cut eight into four equal parts on a number line. Determine the total value of three of the four parts.

When they know both number-line methods, you might ask students to decide which method they would prefer for determining the product of a fraction and a large who le number (e.g., $125 \times \frac{3}{5}$).

Meeting Student Needs

- Invite students to use their own data when exploring the concepts in this section. For example, what fraction of students in the class speaks one language, what fraction speaks two languages, and so on.
- Some students may benefit from using virtual manipulatives to practise working with fractions of a circle or a square. See the related Web Link below.

ELL

- Ensure students understand the following terms: *multiplication, sum, repeated addition, lowest terms, perimeter, area, product,* and *result (answer).* Have students add any new terms to their personal dictionary.
- When working through examples, ensure that students know the names of the different manipulatives used to describe how to multiply

a fraction. Give a list with pictures of different manipulatives. Also, make sure that students understand that a diagram is a picture.

• Example 3 refers to an ant and a spider. Students can add these pictures to their personal dictionary. Provide students with a visual on the board or on a computer.

Common Errors

- Some students may not understand how to write fractional products in lowest terms.
- $\mathbf{R}_{\mathbf{x}}$ Encourage students to examine fractional products to see if the numerator and denominator have a common factor other than 1. Encourage the use of concrete or semi-concrete models, such as fraction strips or diagrams of rectangles, to represent equivalent fractions for students when they are identifying a common factor and dividing the numerator and denominator by it.

- Some students may not consider whether answers are reasonable.
- $\mathbf{R}_{\mathbf{x}}$ Point out the use of mental reasoning beside the answers in Examples 1 and 3. Ask students to make up some other examples that illustrate the generalization beside Example 1. For example, half of 2 must be less than 2.



For an activity that allows students to explore fraction pieces using interactive software, go to www.mathlinks8.ca and follow the links. Students can use the software to work with fractions of a circle or a square. Note that the resource does not include print activities.

Answers

Explore the Math

- **1.** a) To the left of the equal sign, the three trapezoids model $\frac{1}{2}$ being added three times. To the right of the equal sign, the two stacked trapezoids model one whole and the other trapezoid models $\frac{1}{2}$.
 - **b)** To the left of the equal sign, the three trapezoids model $\frac{1}{2}$ being multiplied by 3. To the right of the equal sign, the two stacked trapezoids model one whole and the other trapezoid models $\frac{1}{2}$.
 - c) Examples of other manipulatives are fraction strips and fraction circles.
- **2.** a) Answers will vary. Example:

b) $4 \times \frac{1}{2} - \frac{4}{2}$

$$6 - 6$$

3. a) Answers will vary. Example:



4. b) Answers will vary. Example: I could use a number line by marking off fractional parts along the number line.

Show You Know: Example 1



Show You Know: Example 2



Show You Know: Example 3

4 scoops

Assessment	Supporting Learning	
Assessment <i>as</i> Learning		
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.	 Check for understanding of fractions or parts of a whole using fraction blocks or rectangles to ensure the concept is clear before moving to repeated addition. Some students may benefit from repeating the examples in the Explore the Math with different models of fraction blocks or rectangles, and verbalizing what they see. 	
Assessment <i>for</i> Learning		
Example 1 Have students do the Show You Know related to Example 1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Some students may benefit from recalling factors and divisibility rules. Having the rules taped to the top of their desk or a quick reference chart inserted into their chapter Foldable may be beneficial. Encourage the use of diagrams for determining the answer. Have students verbalize and write the algebraic statement with each model. 	
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Encourage students to use the kinds of diagrams that they find the easiest to work with. Encourage students to try at least one number line and to verbalize how they would solve it. Note that students should work with the process that is conceptually strong in their own minds, but they do need to be able to identify the operations in other models and approaches. 	
Example 3 Have students do the Show You Know related to Example 3.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Encourage students to use the kinds of diagrams that they find the easiest to work with. Note that students should work with the process that is conceptually strong in their own minds, but they do need to be able to identify the operations in other models and approaches . 	



Key Ideas

This section summarizes concrete and semi-concrete methods for multiplying a fraction and a whole number, and makes the point that they can be multiplied in either order. Students could prepare their own list of Key Ideas and include it in their Foldable, especially if they have used methods not included in the section (e.g., modelling with the second number-line method described in Example 3).

Communicate the Ideas

These questions allow students to explain ways of multiplying a fraction and a whole number. In #1, students interpret a diagram and observe a limitation of the use of pattern blocks for the same multiplication. In #2, students explain a modelling method that is similar to the use of fractions strips or diagrams. In #3, students have an opportunity to reach a generalization about the product of a fraction and a whole number.

Common Errors

- Some students may have difficulty with #1b).
- **R**_x Remind students of the values represented by the various pattern blocks, as shown in #1 of Explore the Math on page 198. Ask if there is a block that represents $\frac{1}{5}$.
- Some students may have difficulty devising examples in their explanation for #3.
- **R**_x Advise students to use the multiplications they have already seen in the exploration and the worked examples.

Answers

Communicate the Ideas

1. a) $3 \times \frac{6}{5} = \frac{18}{5}$

- **b)** Answers will vary. Example: A hexagon would be difficult to use because it is difficult to divide into five equal parts.
- **2.** a) Answers will vary. Example: He used 5-by-1 grids because the denominator is 5.
 - **b)** Answers will vary. Example: He used four grids because he was multiplying by 4.
 - c) Answers will vary. Example: Four groups of three counters yields 12 counters, where each counter represents $\frac{1}{5}$.
- **3.** Yes. Answers will vary. Example:



Assessment	Supporting Learning
Assessment as Learning	
Communicate the Ideas Have all students complete #1 to #3.	 Students who need assistance with #1 and #2 may benefit from referring back to Examples 1 and 2. Have students verbalize what they see in the diagrams and identify the value of each rectangle independently. For #3, have students diagram out the answer of ¹²/₅ using rectangles or another model. The visual image may support the solution for students working on the process. Discuss #3 with the entire class before moving on. This step is important to subsequent lessons.



Check Your Understanding

Practise

If students choose to use manipulatives to complete #6 and #7, give them a choice of manipulatives. You might have students discuss the methods they chose for these questions and explain their choices.

Apply

Encourage students to think about the reasonableness of their answers. For example, in #9, the minibus seats 12 people. If it is only $\frac{3}{4}$ full, there must be fewer than 12 people seated on the minibus. Some students may realize that if 6 people are on the



minibus, it is $\frac{1}{2}$ full. So, a bus that is $\frac{3}{4}$ full must contain between 6 and 12 people.

In #12, some students may be puzzled by how to solve $10\,000\,000 \times \frac{1}{5}$ with the concrete and semi-concrete methods they know. If so, point out that $10\,000\,000 \times \frac{1}{5} = 10 \times 1\,000\,000 \times \frac{1}{5}$. Because the order of multiplication does not matter, students can model $10 \times \frac{1}{5}$ and then multiply the whole-number result by 1000000. Some students may benefit from using a simpler case, such as $80 \times \frac{1}{2}$. For #13, you may wish to have students work in pairs and act out the question. Partner A could explain the given pattern to Partner B. They could work together to devise a pattern for part b) and then switch roles, so that B explains the new pattern to A or to another student in the class.

For #14, some students may use original contexts for their problem. Other students may adapt the wording of earlier problems (e.g., #9). You might point out that earlier problems that include real data (such as #12) cannot be used in this way. Encourage students to share their problems so that they are exposed to some that show originality.

Extend

Using the number-line method from Example 2 to complete #15 is laborious. You may wish to make sure that students complete #3 before #15, so that the latter can be completed symbolically. Alternatively, you may wish to introduce the second number-line method described above in connection with Example 3.

In #16, students can multiply $15 \times \frac{1}{5}$ to determine the shortest side length, and then use subtraction and division of whole numbers to determine the other two side lengths. You may wish to return to this problem and to request an alternative solution after students learn to divide a fraction by a whole number in section 6.2. Students will then be able to divide

 $\frac{4}{5}$ by 2 to determine the fraction of the perimeter accounted for by each of the equal sides in the triangle.

Ensure that students understand the physical situation in #17. The ball goes straight down to hit the ground for the first time. The ball then goes straight up and down before it hits the ground for the second time and for each subsequent time. Students must include all the distances that the ball travels up and down until it hits the ground for the fifth time.

Math Link

This Math Link allows students to apply the multiplication of a fraction and a whole number to data about Canada's ecozones.

Meeting Student Needs

- If spatial sense is an area of growth for some students, consider having them create rough sketches first, then try to label the drawing with the measurement criteria from the Math Link. Through conversation with you, students should begin to understand the relative sizes given. Other students may prefer to consider the perspective from above for their initial drawing.
- Provide **BLM 6–6 Section 6.1 Extra Practice** to students who would benefit from more practice.

ELL

• Demonstrate the term *width* by running a finger along the width of a flag, saying that it is the width. Do the same for *length*. Have students demonstrate their understanding by asking them to point out the width and length of objects in the classroom.

Answers

Math Link a) 5 b) 15

Assessment	Supporting Learning	
Assessment for Learning		
Practise and Apply Have students do #4, #6, #8, and #9. Students who have no problems with these questions can go on to the rest of the Apply questions.	 Provide additional coaching with Examples 1 and 2 for students who need help with #4 and #6. Encourage students to use the manipulative or model that they feel most comfortable with. Coach students through #4, and then have them complete either #5a) or #5b). Coach students through #6, and then have them complete #7 on their own. Check back with them several times to make sure that they understand the concepts. Encourage students to find at least two models that they feel comfortable with. Provide additional coaching to students who need help with #8. Refer them back to #3 in Communicate the Ideas. Provide additional coaching with Example 3 to students who need help with #9. Have students draw a diagram representing the problem first, verbalize the process they would use to solve it, and then record their work and solution. 	
Math Link The Math Link on page 203 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 239.	 Most students should complete this Math Link, as they will use these basic skills when they design and solve their own questions related to the ecozones in the Wrap It Up! It may be helpful to refer students to the Wrap It Up on page 239 so they know what their end goal will be. Some students may wish to include a diagram that reflects their questions and solutions for the Wrap It Up! Students who need help getting started could use BLM 6–7 Section 6.1 Math Link, which provides scaffolding. 	
Assessment <i>as</i> Learning		
 Math Learning Log Have students complete the following statements: The method I find the easiest to understand when multiplying a fraction and a whole number is The method I find the most confusing is because 	 Help students recall the use of pattern blocks or other manipulatives, rectangle diagrams, and number lines before completing the statements. Check to see what method students find most difficult to use. If there is a common strand, consider taking some additional instructional time to review it. Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with. 	