6.2 Dividing a Fraction by a Whole Number

***MathLinks 8*, pages 204–209**

Suggested Timing
50–60 minutes

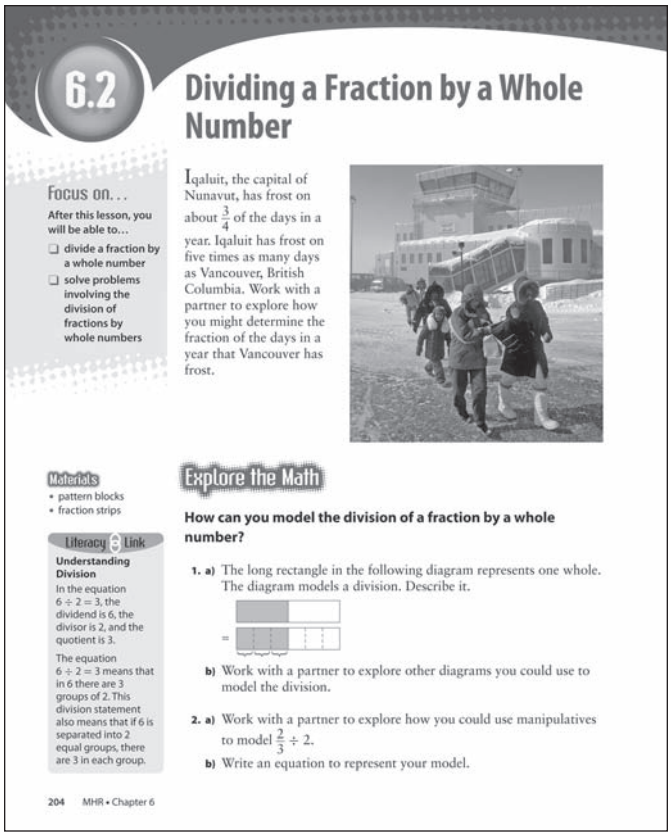
Materials
- pattern blocks
- ruler
- fraction strips (optional)
- transparent shapes or strips (optional)
- dry erase markers (optional)
- coloured pencils (optional)

Blackline Masters
Master 3 Integer Number Lines
Master 13 Pattern Blocks
Master 14 Fraction Strips
BLM 6–3 Chapter 6 Warm-Up
BLM 6–8 Rectangles
BLM 6–9 Fraction Number Lines
BLM 6–10 Section 6.2 Extra Practice
BLM 6–11 Section 6.2 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☐ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 4, 6, 8, Math Link
Typical	1, 2, 4, 6, 8–13, Math Link
Extension/Enrichment	1, 2, 12–15



6.2 Dividing a Fraction by a Whole Number

Focus on...
After this lesson, you will be able to...
- divide a fraction by a whole number
- solve problems involving the division of fractions by whole numbers

Iqaluit, the capital of Nunavut, has frost on about $\frac{3}{4}$ of the days in a year. Iqaluit has frost on five times as many days as Vancouver, British Columbia. Work with a partner to explore how you might determine the fraction of the days in a year that Vancouver has frost.

Materials
- pattern blocks
- fraction strips

Literacy Link
Understanding Division
In the equation $6 \div 2 = 3$, the dividend is 6, the divisor is 2, and the quotient is 3.
The equation $6 \div 2 = 3$ means that in 6 there are 3 groups of 2. This division statement also means that if 6 is separated into 2 equal groups, there are 3 in each group.

Explore the Math
How can you model the division of a fraction by a whole number?

1. a) The long rectangle in the following diagram represents one whole. The diagram models a division. Describe it.
b) Work with a partner to explore other diagrams you could use to model the division.
2. a) Work with a partner to explore how you could use manipulatives to model $\frac{2}{3} \div 2$.
b) Write an equation to represent your model.

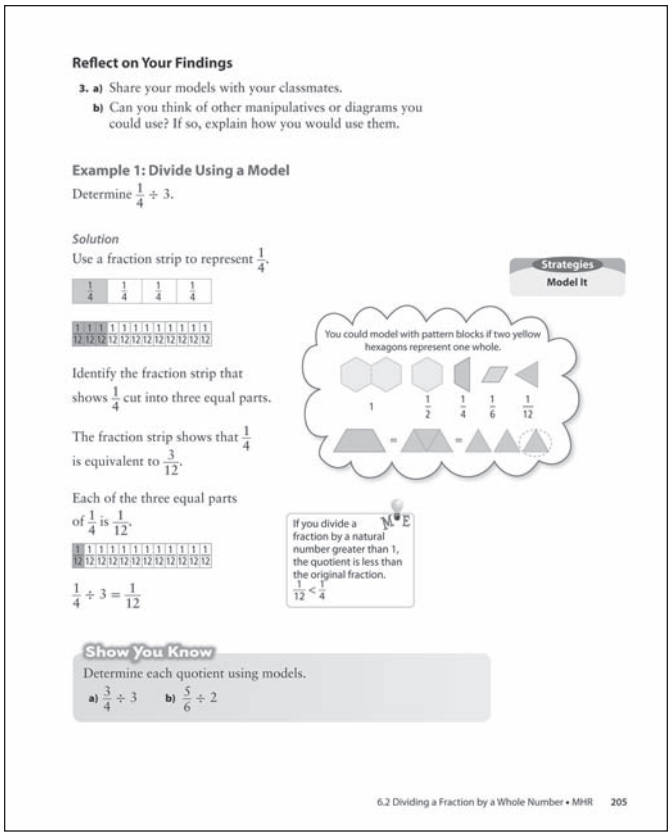
204 MHR • Chapter 6

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce material learned in previous sections.

Have students work together to develop answers for the question in the introduction. At this point, they might use multiplication of fractions and suggest that the problem they need to solve is $5 \times \blacksquare = \frac{3}{4}$. Discussing how to solve such an equation could lead to the Explore the Math. (At this stage, students have not learned how to carry out the division of $\frac{3}{4}$ by 5, but they will complete this division in Apply #11 in this section.)

Students could also solve the problem in the section opener using whole numbers. Discuss this strategy. If they come up with it first, challenge them to consider how the question could be answered using fractions.



Reflect on Your Findings

3. a) Share your models with your classmates.

b) Can you think of other manipulatives or diagrams you could use? If so, explain how you would use them.

Example 1: Divide Using a Model

Determine $\frac{1}{4} \div 3$.

Solution

Use a fraction strip to represent $\frac{1}{4}$.

Strategies: Model It

Identify the fraction strip that shows $\frac{1}{4}$ cut into three equal parts.

The fraction strip shows that $\frac{1}{4}$ is equivalent to $\frac{3}{12}$.

Each of the three equal parts of $\frac{1}{4}$ is $\frac{1}{12}$.

$\frac{1}{4} \div 3 = \frac{1}{12}$

You could model with pattern blocks if two yellow hexagons represent one whole.

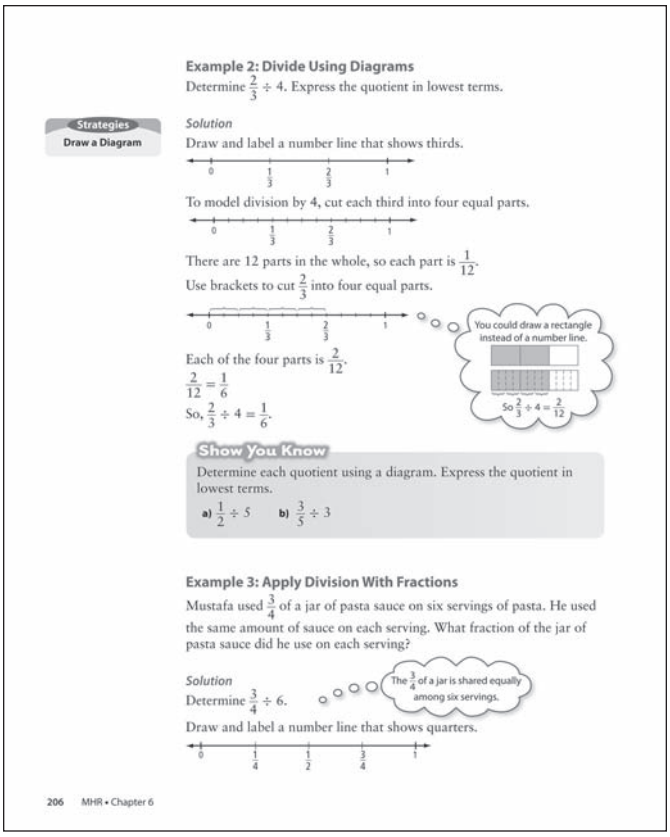
If you divide a fraction by a natural number greater than 1, the quotient is less than the original fraction. $\frac{1}{12} < \frac{1}{4}$

Show You Know

Determine each quotient using models.

a) $\frac{3}{4} \div 3$ **b)** $\frac{5}{6} \div 2$

6.2 Dividing a Fraction by a Whole Number • MHR 205



Example 2: Divide Using Diagrams

Determine $\frac{2}{3} \div 4$. Express the quotient in lowest terms.

Strategies: Draw a Diagram

Solution

Draw and label a number line that shows thirds.

To model division by 4, cut each third into four equal parts.

There are 12 parts in the whole, so each part is $\frac{1}{12}$.

Use brackets to cut $\frac{2}{3}$ into four equal parts.

You could draw a rectangle instead of a number line.

Each of the four parts is $\frac{2}{12}$.

$\frac{2}{12} = \frac{1}{6}$

So, $\frac{2}{3} \div 4 = \frac{1}{6}$.

Show You Know

Determine each quotient using a diagram. Express the quotient in lowest terms.

a) $\frac{1}{2} \div 5$ **b)** $\frac{3}{5} \div 3$

Example 3: Apply Division With Fractions

Mustafa used $\frac{3}{4}$ of a jar of pasta sauce on six servings of pasta. He used the same amount of sauce on each serving. What fraction of the jar of pasta sauce did he use on each serving?

Solution

Determine $\frac{3}{4} \div 6$.

The $\frac{3}{4}$ of a jar is shared equally among six servings.

Draw and label a number line that shows quarters.

206 MHR • Chapter 6

Explore the Math

Students use models or diagrams to develop a model for dividing a fraction by a whole number.

Literacy Link Draw students' attention to the Literacy Link on page 204, which deals with concepts about division. Have students consider how this Literacy Link might apply to fractions. For example, if $\frac{1}{2}$ were separated into three groups, how big would each group be? Students will readily realize that they cannot use halves since there is only one group of halves present.

Method 1 Have students work with a partner. Provide a set of pattern blocks or other fraction manipulatives, such as fraction strips or fraction circles. You may wish to hand out **Master 13 Pattern Blocks** if sets of pattern blocks are not available and **Master 14 Fraction Strips** if fraction strips are not available.

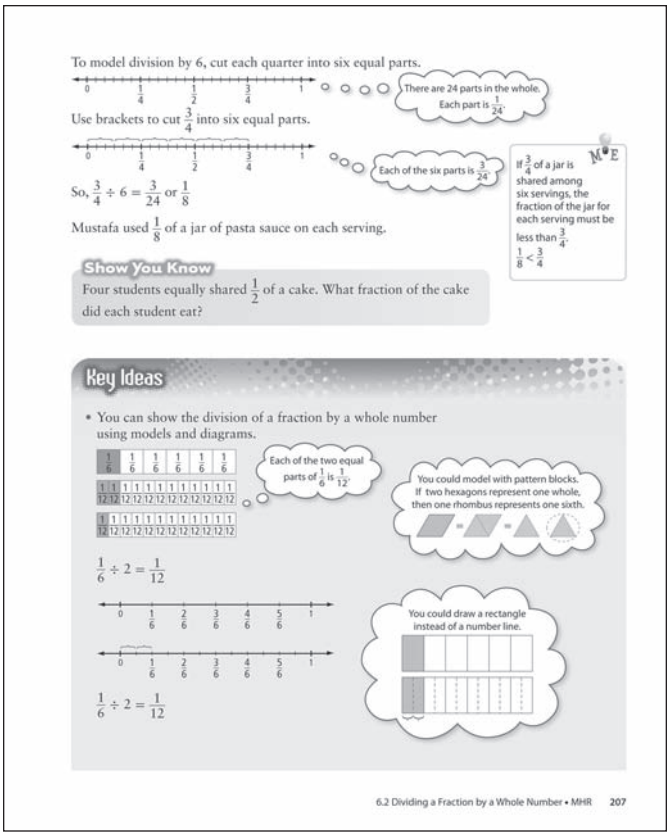
As students work on the Explore the Math, circulate and ask questions such as

- What division does the diagram show? How do you know?
- How might you use what you know about multiplication of fractions to divide fractions?
- What model could you use to solve $\frac{2}{3} \div 2$?
- How could you use diagrams to solve $\frac{2}{3} \div 2$?
- What can you learn from these models that will help you divide a fraction by a whole number?

Method 2 Have students use other models, such as fraction strips or number lines, to model the division, as shown in Example 1. You may wish to hand out **Master 14 Fraction Strips** or **BLM 6–9 Fraction Number Lines**.

Example 1

This example illustrates the use of fraction strips to model the division of a fraction by a whole number. Reinforce that the example uses a familiar problem solving strategy (i.e., Model It). Ask students to think of another strategy they could use (e.g., Draw a Diagram). Point out the thought bubble that shows the same division modelled using pattern blocks. To reinforce #3 in Explore the Math, you might ask students which of the two methods shown in Example 1 they prefer, and why.



Example 2

This example illustrates the use of a number line to model the division of a fraction by a whole number. Reinforce that the example uses a familiar problem solving strategy (i.e., Draw a Diagram). Ask students to think of another strategy they could use (e.g., Model It). Point out the thought bubble that shows the same division modelled using diagrams of rectangles. You might ask students which of the two methods shown in Example 2 they prefer, and why.

Example 3

This example illustrates an application of the division of a fraction by a whole number. The number-line method used is the same method as in Example 2. You may wish to have students show how to solve the same problem by drawing rectangles.

Meeting Student Needs

- When considering the method in Example 1, some students may benefit from doing the following to see the relationship between $\frac{1}{4}$ and $\frac{3}{12}$. Have them
  – fold a long strip of paper into quarters
  – colour one quarter
  – refold the paper into quarters
  – leaving it folded into quarters, fold it further into thirds

  When students open up the strip, ask them how many pieces the strip is now divided into. How many of these smaller pieces make up one quarter?
- Some students may benefit from using **BLM 6–9 Fraction Number Lines** when completing the Show You Know questions for Examples 2 and 3.
- Invite students to use population data when exploring the concepts in this section. For example, they can use local data, such as within their classroom or community, or provincial or territorial data that they find through research on the Internet. Allow students to explore the material in small groups before moving on to the next concept.

ELL

- Ensure that students understand the following words by orally explaining them in context: *operations (multiplication*, *division*, *addition*, *and subtraction)*, *quotient*, *whole*, *parts*, *coconut*, *pitcher (jar)*, *fraction strip*, *gasoline*, and *ribbon*. Have students add any new terms to their personal dictionary.
- Some students may not be familiar with frost. Have volunteers describe frost. Discuss the name *Jack Frost* with students.

Gifted and Enrichment

- Once students have completed the Explore the Math, challenge them to solve the problem in the section opener using fractions and more than one strategy. Have them post their strategies.

Common Errors

- Some students may have difficulty with pattern-block models in which more than one hexagon represents one whole, as in Example 1.

R$_x$ Ask students to explain the fractional values represented by the pattern blocks in Example 1. You may also wish to have students interpret and explain the values represented by a set of pattern blocks when three hexagons represent one whole and four hexagons represent one whole.

- Some students may not consider whether their answers are reasonable.

R$_x$ Point out the use of mental reasoning beside the answers in Examples 1 and 3. Ask students to make up some other examples that illustrate the generalization beside Example 1. For example, the quotient of $\frac{1}{2}$ divided by 2 must be less than $\frac{1}{2}$.

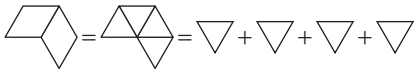
Answers

Explore the Math

1. a) $\frac{1}{2} \div 3 = \frac{1}{6}$. Answers will vary. Example: $\frac{1}{2}$ of the rectangle is divided into three equal parts.

b) Answers will vary.

2. a) Answers will vary. Example:

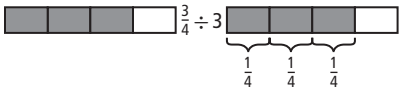


b) $\frac{2}{3} \div 2 = \frac{1}{3}$

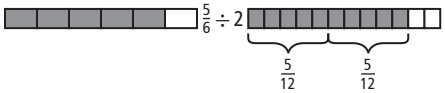
3. b) Answers will vary. Example: I could use a number line by dividing it into sections.

Show You Know: Example 1

a) $\frac{1}{4}$

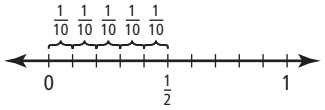


b) $\frac{5}{12}$

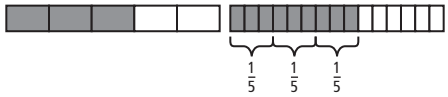


Show You Know: Example 2

a) $\frac{1}{10}$



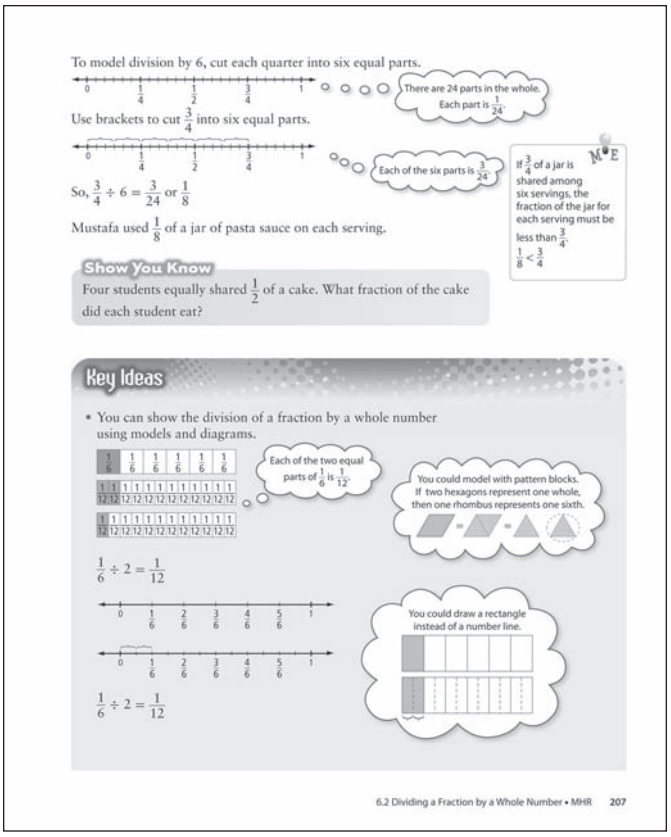
b) $\frac{1}{5}$



Show You Know: Example 3

Each student ate $\frac{1}{8}$ of the cake.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.	• Encourage students to divide using a variety of models. • Encourage students to use one method that they feel comfortable with and explain why. Have them identify the one that they have the most difficulty with and explain why in their chapter Foldable.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. Have partners describe to each other how they would determine each quotient. • Remind students that the denominator tells how many pieces the whole will be divided into and the divisor tells how many pieces an individual unit is divided into.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students choose an alternative model to a number line to work through the process. • Some students may benefit from using **BLM 6–8 Rectangles**, which provides rectangles that students can use to determine each quotient. • Have students diagram $\frac{1}{2}$ or $\frac{3}{5}$ of any model they wish to use. • Then, have them divide one of each unit ($\frac{1}{2}$ or $\frac{3}{5}$) into the parts identified in questions a) and b). • Have students verbally identify the total number of parts, thus showing the division. As they use their model, have them write down the algebraic terms that are associated with the step. • Some students may benefit from using **BLM 6–9 Fraction Number Lines**, which provides number lines that students can use to help them visualize the fractions.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. As they do so, have them write the algebraic equivalent. • You may wish to have students work with a partner. • Have students choose an alternative model to a number line to work through the process.



To model division by 6, cut each quarter into six equal parts.

There are 24 parts in the whole. Each part is $\frac{1}{24}$.

Use brackets to cut $\frac{3}{4}$ into six equal parts.

Each of the six parts is $\frac{3}{24}$.

So, $\frac{3}{4} \div 6 = \frac{3}{24}$ or $\frac{1}{8}$

Mustafa used $\frac{1}{8}$ of a jar of pasta sauce on each serving.

MOE

If $\frac{3}{4}$ of a jar is shared among six servings, the fraction of the jar for each serving must be less than $\frac{3}{4}$.

$\frac{1}{8} < \frac{3}{4}$

Show You Know

Four students equally shared $\frac{1}{2}$ of a cake. What fraction of the cake did each student eat?

Key Ideas

- You can show the division of a fraction by a whole number using models and diagrams.

Each of the two equal parts of $\frac{1}{6}$ is $\frac{1}{12}$.

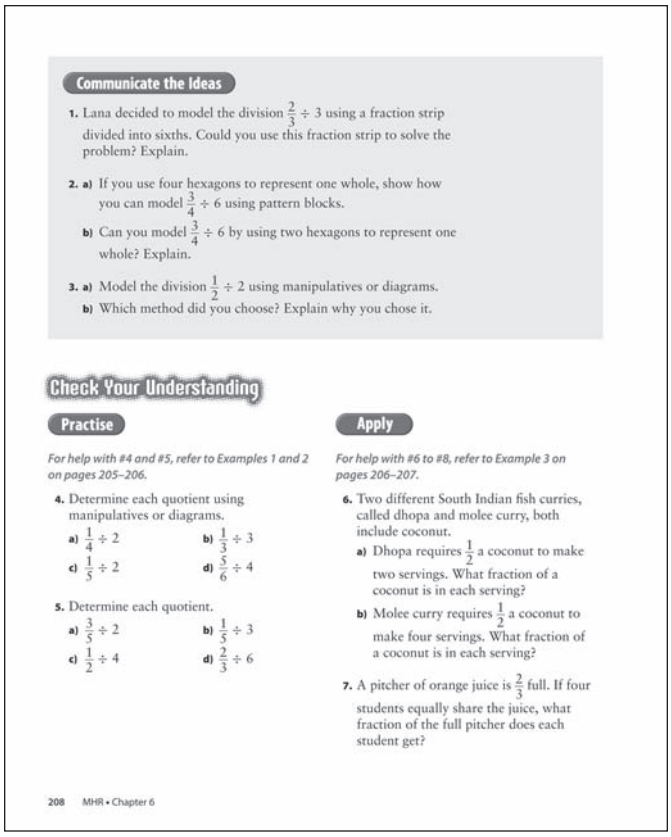
You could model with pattern blocks. If two hexagons represent one whole, then one rhombus represents one sixth.

$\frac{1}{6} \div 2 = \frac{1}{12}$

You could draw a rectangle instead of a number line.

$\frac{1}{6} \div 2 = \frac{1}{12}$

6.2 Dividing a Fraction by a Whole Number • MHR 207



Communicate the Ideas

1. Lana decided to model the division $\frac{2}{3} \div 3$ using a fraction strip divided into sixths. Could you use this fraction strip to solve the problem? Explain.

2. a) If you use four hexagons to represent one whole, show how you can model $\frac{3}{4} \div 6$ using pattern blocks.

   b) Can you model $\frac{3}{4} \div 6$ by using two hexagons to represent one whole? Explain.

3. a) Model the division $\frac{1}{2} \div 2$ using manipulatives or diagrams.

   b) Which method did you choose? Explain why you chose it.

Check Your Understanding

Practise

For help with #4 and #5, refer to Examples 1 and 2 on pages 205–206.

4. Determine each quotient using manipulatives or diagrams.

   a) $\frac{1}{4} \div 2$ b) $\frac{1}{3} \div 3$

   c) $\frac{1}{5} \div 2$ d) $\frac{5}{6} \div 4$

5. Determine each quotient.

   a) $\frac{3}{5} \div 2$ b) $\frac{1}{5} \div 3$

   c) $\frac{1}{2} \div 4$ d) $\frac{2}{3} \div 6$

Apply

For help with #6 to #8, refer to Example 3 on pages 206–207.

6. Two different South Indian fish curries, called dhopa and molee curry, both include coconut.

   a) Dhopa requires $\frac{1}{2}$ a coconut to make two servings. What fraction of a coconut is in each serving?

   b) Molee curry requires $\frac{1}{2}$ a coconut to make four servings. What fraction of a coconut is in each serving?

7. A pitcher of orange juice is $\frac{2}{3}$ full. If four students equally share the juice, what fraction of the full pitcher does each student get?

208 MHR • Chapter 6

Key Ideas

This section summarizes concrete and semi-concrete methods for dividing a fraction by a whole number. Students could prepare their own list of Key Ideas and include it in their chapter Foldable. Advise them to develop alternative examples for each point.

Communicate the Ideas

These questions allow students to explain ways of modelling the division of a fraction by a whole number. In #1, students consider a possible error in the use of fraction strips. In #2, they consider difficulties that can arise in the use of pattern blocks. In #3, students choose among all the methods they have seen and then explain their choice.

Meeting Student Needs

- Some students may benefit from using **BLM 6–9 Fraction Number Lines** to answer Communicate the Ideas #3.

ELL

- Demonstrate what #2 is asking by using hexagon pattern blocks.

Answers

Communicate the Ideas

1. No, she needs to divide the strip into ninths.

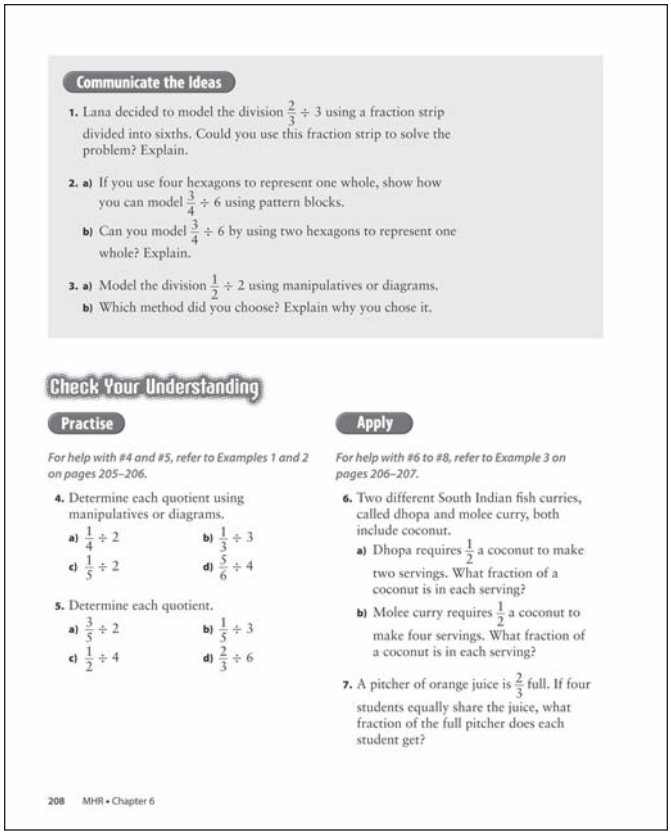
2. a) $\frac{3}{4}$ ÷ 6 ÷ 6 ÷ 6 $= \frac{3}{24}$

b) No, because the whole would have 12 pieces and $\frac{3}{4}$ would have 9 pieces, and 9 cannot be divided evenly by 6.

3. a) Answers will vary. Example: $\frac{1}{4}$ $\frac{1}{4}$

b) Answers will vary. Example: I chose rectangles because it is easy to divide $\frac{1}{2}$ of a rectangle into two pieces.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and #2.	• Students may need to use the method that works best for them as opposed to those suggested in #1 or #2. If so, have students solve the problem with a method they are comfortable with, but then have them compare how these models would work using the method suggested in the questions. • It is important for students to demonstrate their understanding of the method that they find easiest to use. However, they should have some understanding of alternative methods. • Some students may benefit from referring back to Examples 1 and 2.



Communicate the Ideas

1. Lana decided to model the division $\frac{2}{3} \div 3$ using a fraction strip divided into sixths. Could you use this fraction strip to solve the problem? Explain.

2. a) If you use four hexagons to represent one whole, show how you can model $\frac{3}{4} \div 6$ using pattern blocks.
   b) Can you model $\frac{3}{4} \div 6$ by using two hexagons to represent one whole? Explain.

3. a) Model the division $\frac{1}{2} \div 2$ using manipulatives or diagrams.
   b) Which method did you choose? Explain why you chose it.

Check Your Understanding

Practise

For help with #4 and #5, refer to Examples 1 and 2 on pages 205–206.

4. Determine each quotient using manipulatives or diagrams.
   a) $\frac{1}{4} \div 2$ b) $\frac{1}{3} \div 3$
   c) $\frac{1}{5} \div 2$ d) $\frac{5}{6} \div 4$

5. Determine each quotient.
   a) $\frac{3}{5} \div 2$ b) $\frac{1}{5} \div 3$
   c) $\frac{1}{2} \div 4$ d) $\frac{2}{3} \div 6$

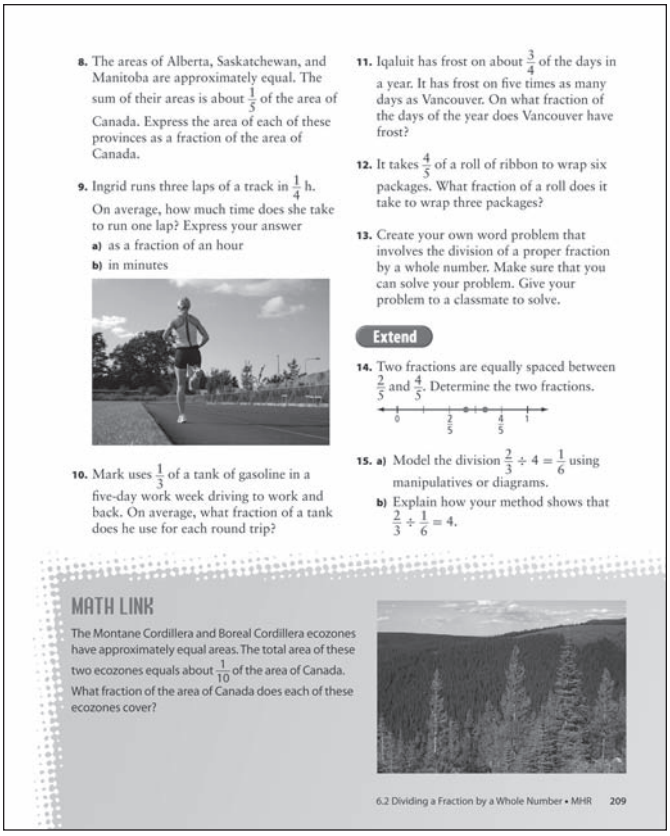
Apply

For help with #6 to #8, refer to Example 3 on pages 206–207.

6. Two different South Indian fish curries, called dhopa and molee curry, both include coconut.
   a) Dhopa requires $\frac{1}{2}$ a coconut to make two servings. What fraction of a coconut is in each serving?
   b) Molee curry requires $\frac{1}{2}$ a coconut to make four servings. What fraction of a coconut is in each serving?

7. A pitcher of orange juice is $\frac{2}{3}$ full. If four students equally share the juice, what fraction of the full pitcher does each student get?

208 MHR • Chapter 6



8. The areas of Alberta, Saskatchewan, and Manitoba are approximately equal. The sum of their areas is about $\frac{1}{5}$ of the area of Canada. Express the area of each of these provinces as a fraction of the area of Canada.

9. Ingrid runs three laps of a track in $\frac{1}{4}$ h. On average, how much time does she take to run one lap? Express your answer
   a) as a fraction of an hour
   b) in minutes

10. Mark uses $\frac{1}{3}$ of a tank of gasoline in a five-day work week driving to work and back. On average, what fraction of a tank does he use for each round trip?

11. Iqaluit has frost on about $\frac{3}{4}$ of the days in a year. It has frost on five times as many days as Vancouver. On what fraction of the days of the year does Vancouver have frost?

12. It takes $\frac{4}{5}$ of a roll of ribbon to wrap six packages. What fraction of a roll does it take to wrap three packages?

13. Create your own word problem that involves the division of a proper fraction by a whole number. Make sure that you can solve your problem. Give your problem to a classmate to solve.

Extend

14. Two fractions are equally spaced between $\frac{2}{5}$ and $\frac{4}{5}$. Determine the two fractions.

15. a) Model the division $\frac{2}{3} \div 4 = \frac{1}{6}$ using manipulatives or diagrams.
   b) Explain how your method shows that $\frac{2}{3} \div \frac{1}{6} = 4$.

MATH LINK

The Montane Cordillera and Boreal Cordillera ecozones have approximately equal areas. The total area of these two ecozones equals about $\frac{1}{10}$ of the area of Canada. What fraction of the area of Canada does each of these ecozones cover?

6.2 Dividing a Fraction by a Whole Number • MHR 209

Check Your Understanding

Practise

If students choose to use manipulatives to complete #4 and #5, give them a choice of manipulatives. You might have students compare the methods they chose for these questions and explain their choices.

Apply

Encourage students to think about the reasonableness of their answers. For example, in #7, four students equally share $\frac{2}{3}$ of a pitcher of juice, so the share that each student gets must be less than $\frac{2}{3}$.

In #10, students may not allow for the fact that there are two trips per workday (i.e., ten trips per week, not five). You might also ask what fraction of a tank Lynn uses per workday.

Remind students that they discussed the context of #11 at the beginning of the section.

Some students may need assistance with #12, which involves two operations: division to determine the fraction of a roll for one package, and then multiplication to determine the fraction of a roll for three packages. Some students may be able to reason that the answer can also be found by calculating $\frac{4}{5} \div 2$. You may wish to return to this question after students learn to multiply two proper fractions in section 6.3. You could then ask students to provide another method for solving the problem. Students will be able to determine $\frac{3}{6} \times \frac{4}{5}$ or $\frac{1}{2} \times \frac{4}{5}$.

For #13, some students may use original contexts for their problems. Other students may adapt the wording of earlier problems (e.g., #7 or #9) to include the required numbers. You might encourage students to share their problems so that they are exposed to some that show originality.

Extend

In #14, students need to realize that the two fractions cut the difference between $\frac{2}{5}$ and $\frac{4}{5}$ into three equal parts. When students have determined each part, they can determine the two fractions by addition or subtraction. If students use only one of these operations, you might ask them how they could use the other instead.

To help students with #15b), remind them of the Literacy Link on page 204 about understanding division. The division statement $\frac{2}{3} \div \frac{1}{6} = 4$ indicates that there are four groups of $\frac{1}{6}$ in $\frac{2}{3}$, as the manipulatives or diagrams used in #15a) will show.

Math Link

This Math Link allows students to apply the division of a fraction by a whole number to data concerning Canada's ecozones. In the Wrap It Up! section on page 239 in this chapter, students will develop original questions using given data. To help prepare students, you might discuss if other questions could be written using the data in the Math Link. For example, What fraction of the area of Canada is outside the Montane Cordillera (or the Boreal Cordillera)? Subtracting the answer to the Math Link question from 1 gives an answer of $\frac{19}{20}$.

Meeting Student Needs

- Some students may benefit from using **BLM 6–9 Fraction Number Lines** and/or **BLM 6–8 Rectangles** to help them model questions.
- Provide **BLM 6–10 Section 6.2 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure students understand the words *lap* and *track*. Use the picture in the student resource to demonstrate, or have a volunteer walk a lap around the classroom.
- Invite the class as a whole to create word problems together before students create questions on their own.

Gifted and Enrichment

- Although enrichment students likely do not have to complete the Math Link, you may wish to challenge them to determine what fraction of the area of Canada is outside the Montane Cordillera or the Boreal Cordillera.

Answers

Math Link

$\frac{1}{20}$

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #4, #6, and #8. Students who have no problems with these questions can go on to the rest of the Apply questions.	• Provide additional coaching with Examples 1 and 2 to students who need help with #4. Coach students through #4a) and #4d) and then have students complete #5 on their own. Check back with them several times to make sure that they understand the concepts. • Provide additional coaching with Example 3 to students who need help with #6 or #8. • Encourage students to use the method that is easiest for them to understand and model.
Math Link The Math Link on page 209 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 239.	• Have most students do this Math Link, as they will use these basic skills when they design and solve their own questions related to the ecozones in the Wrap It Up! • Some students may benefit from using **BLM 6–9 Fraction Number Lines** to help them visualize the fractions. • Students who need help getting started could use **BLM 6–11 Section 6.2 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students answer the following question: • A pitcher of milk is $\frac{3}{4}$ full. If six students want to share the milk, how much will each student get? Explain how you would solve this problem.	• Many students may use a model to solve the problem. Encourage them to verbalize their thinking. • Students may find it helpful to refer to #6 in the Practise questions to assist them in their thinking. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with.