6.3 Multiplying Proper Fractions

***MathLinks 8*, pages 210–215**

Suggested Timing
50–60 minutes

Materials
- six sheets of plain paper
- yellow and blue coloured pencils or crayons
- ruler
- fraction strips (optional)
- transparent strips or diagrams of rectangles (optional)
- dry erase markers (optional)

Blackline Masters
Master 14 Fraction Strips
BLM 6–3 Chapter 6 Warm-Up
BLM 6–8 Rectangles
BLM 6–9 Fraction Number Lines
BLM 6–12 Section 6.3 Extra Practice
BLM 6–13 Section 6.3 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

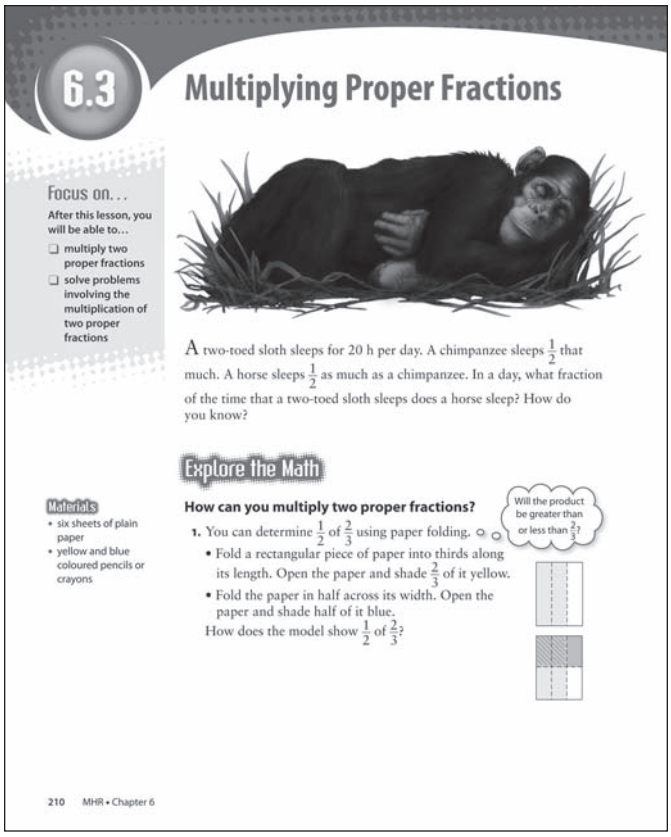
Specific Outcomes
N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, Math Link
Typical	1–3, 5, 7–11, Math Link
Extension/Enrichment	11–15

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce material learned in previous sections.

Have students answer the questions in the introduction and discuss their answers. At this stage, students do not know how to multiply two fractions. However, the problem can be solved by multiplying fractions and whole numbers. A chimpanzee sleeps $\frac{1}{2}$ of 20 h, which is 10 h. A horse sleeps $\frac{1}{2}$ of 10 h, which is 5 h. Expressing 5 as a fraction of 20 in lowest terms gives $\frac{1}{4}$. You may wish to return to this problem after students have answered some of the Apply questions in this section. You might then ask students to suggest another way to solve the problem. Students will then be able to determine $\frac{1}{2} \times \frac{1}{2}$, and the information that a two-toed sloth sleeps for 20 h per day will become unnecessary.


6.3 Multiplying Proper Fractions

Focus on...
After this lesson, you will be able to...
- multiply two proper fractions
- solve problems involving the multiplication of two proper fractions

A two-toed sloth sleeps for 20 h per day. A chimpanzee sleeps $\frac{1}{2}$ that much. A horse sleeps $\frac{1}{2}$ as much as a chimpanzee. In a day, what fraction of the time that a two-toed sloth sleeps does a horse sleep? How do you know?

Explore the Math

Materials
- six sheets of plain paper
- yellow and blue coloured pencils or crayons

How can you multiply two proper fractions?

1. You can determine $\frac{1}{2}$ of $\frac{2}{3}$ using paper folding.
   - Fold a rectangular piece of paper into thirds along its length. Open the paper and shade $\frac{2}{3}$ of it yellow.
   - Fold the paper in half across its width. Open the paper and shade half of it blue.

   How does the model show $\frac{1}{2}$ of $\frac{2}{3}$?

Will the product be greater than or less than $\frac{2}{3}$?

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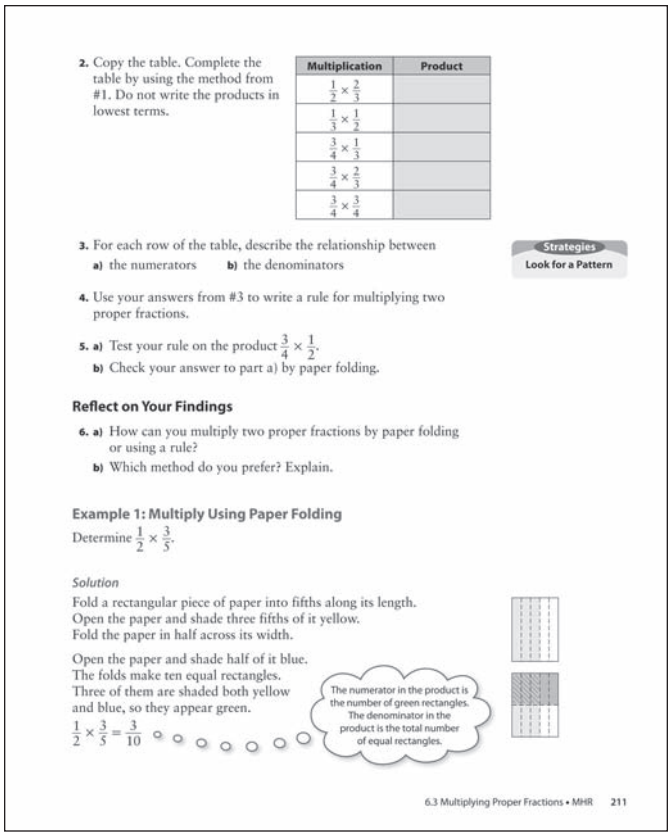
Explore the Math

Students use paper folding to develop a rule for multiplying two proper fractions.

Method 1 Have students work with a partner and check each other's paper folding and products. As students work, circulate and observe what students are doing. You may need to assist some students with the paper-folding instructions by making sure that they fold the paper in thirds and then fold the other way when they fold it in half.



2. Copy the table. Complete the table by using the method from #1. Do not write the products in lowest terms.

Multiplication	Product
$\frac{1}{2} \times \frac{2}{3}$	
$\frac{1}{3} \times \frac{1}{2}$	
$\frac{3}{4} \times \frac{1}{3}$	
$\frac{3}{4} \times \frac{2}{3}$	
$\frac{3}{4} \times \frac{3}{4}$	

3. For each row of the table, describe the relationship between
a) the numerators b) the denominators

Strategies
Look for a Pattern

4. Use your answers from #3 to write a rule for multiplying two proper fractions.

5. a) Test your rule on the product $\frac{3}{4} \times \frac{1}{2}$.
b) Check your answer to part a) by paper folding.

Reflect on Your Findings

6. a) How can you multiply two proper fractions by paper folding or using a rule?
b) Which method do you prefer? Explain.

Example 1: Multiply Using Paper Folding

Determine $\frac{1}{2} \times \frac{3}{5}$.

Solution

Fold a rectangular piece of paper into fifths along its length.
Open the paper and shade three fifths of it yellow.
Fold the paper in half across its width.

Open the paper and shade half of it blue.
The folds make ten equal rectangles.
Three of them are shaded both yellow and blue, so they appear green.

$\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

The numerator in the product is the number of green rectangles. The denominator in the product is the total number of equal rectangles.

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Use questions such as the following to help students think through what they are doing:

- Is $\frac{1}{2}$ of $\frac{2}{3}$ greater than or less than $\frac{2}{3}$? Explain your thinking.
- How does your paper folding model $\frac{1}{2}$?
- How does it model $\frac{2}{3}$?
- What operation is suggested by the statement $\frac{1}{2}$ of $\frac{2}{3}$?
- Using the model, what is $\frac{1}{2}$ of $\frac{2}{3}$? How do you know?
- How can you use the products in your table to help you develop a rule for multiplying two fractions?
- How can you test your rule?

You may wish to have students check that the order of multiplication does not matter since the multiplication of two fractions is commutative (e.g., by showing in #1 that folding into halves and then thirds gives the same result as folding into thirds and then halves). Emphasize that students should not write the products in #2 in lowest terms. If they do, they will not be able to see the relationships in #3. Make sure that the products recorded in the table are correct before students attempt to make a generalization in #3. Have students test their generalization by solving the question in the section opener ($\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$).

Method 2 Have students model the multiplication using other manipulatives, such as fraction strips. You may wish to hand out **Master 14 Fraction Strips**.

It may be beneficial to photocopy **Master 14 Fraction Strips** on overhead transparencies so that students can line up the strips on the overhead and place one strip over another to see the relationships. Questions such as the following may benefit students:

- What fraction strip can you use to show both $\frac{1}{2}$ and $\frac{2}{3}$?
- Show $\frac{2}{3}$ on that strip.
- How can you show half of that amount?

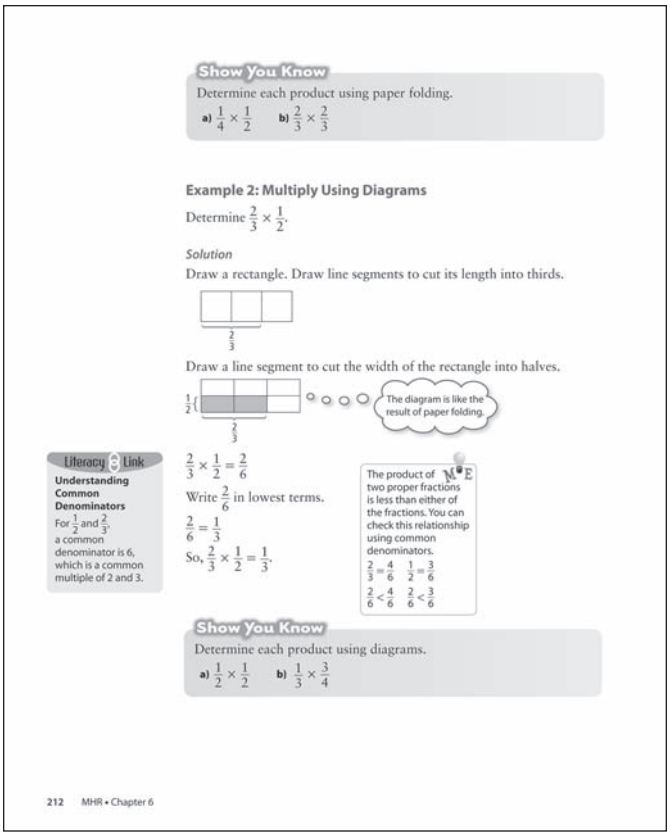
Method 3 Have students model the multiplication using diagrams, as shown in Example 2.

Example 1

This example illustrates the use of paper folding to model the multiplication of two proper fractions. Reinforce that the example uses a familiar problem solving strategy (i.e., Model It). Ask students to think of another strategy they could use (e.g., Draw a Diagram). Emphasize the relationship between the number of equal green rectangles and the numerator of the product, and between the total number of equal rectangles and the denominator of the product, as indicated in the thought bubble in the solution.

You might point out that because the order of multiplication does not matter, the order in which the dimensions of the paper are folded does not matter. The paper could be folded in half first (along its length or its width).

If you wish, introduce fraction strips as an alternative way to model the multiplication concretely in Example 1, as follows: Use a fraction strip to represent $\frac{3}{5}$. A common denominator of $\frac{1}{2}$ and $\frac{3}{5}$ is 10, so use a fraction strip that has ten sections in total to represent $\frac{3}{5}$. To multiply by $\frac{1}{2}$, colour on top of half as much of the strip as you coloured before. $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$.
You might point out that because the order of multiplication does not matter, the first fraction strip used could represent $\frac{1}{2}$ and the second $\frac{5}{10}$. Multiplying by $\frac{3}{5}$ would then be shown by colouring on top of $\frac{3}{5}$ as much of the strip as you did before to give the same result as before.



Show You Know

Determine each product using paper folding.

a) $\frac{1}{4} \times \frac{1}{2}$ b) $\frac{2}{3} \times \frac{2}{3}$

Example 2: Multiply Using Diagrams

Determine $\frac{2}{3} \times \frac{1}{2}$.

Solution

Draw a rectangle. Draw line segments to cut its length into thirds.

Draw a line segment to cut the width of the rectangle into halves.

The diagram is like the result of paper folding.

$\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$

Write $\frac{2}{6}$ in lowest terms.

$\frac{2}{6} = \frac{1}{3}$

So, $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

Literacy Link

Understanding Common Denominators

For $\frac{1}{2}$ and $\frac{2}{3}$, a common denominator is 6, which is a common multiple of 2 and 3.

The product of two proper fractions is less than either of the fractions. You can check this relationship using common denominators.

$\frac{2}{3} = \frac{4}{6}$ $\frac{1}{2} = \frac{3}{6}$

$\frac{2}{6} < \frac{4}{6}$ $\frac{2}{6} < \frac{3}{6}$

Show You Know

Determine each product using diagrams.

a) $\frac{1}{2} \times \frac{1}{2}$ b) $\frac{1}{3} \times \frac{3}{4}$

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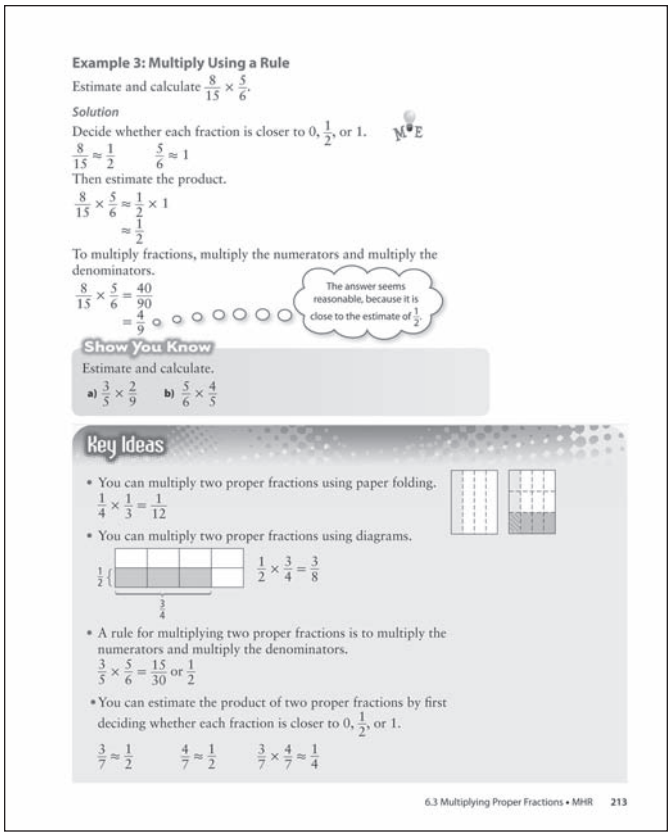

Example 3: Multiply Using a Rule

Estimate and calculate $\frac{8}{15} \times \frac{5}{6}$.

Solution

Decide whether each fraction is closer to 0, $\frac{1}{2}$, or 1.

$\frac{8}{15} \approx \frac{1}{2}$ $\frac{5}{6} \approx 1$

Then estimate the product.

$\frac{8}{15} \times \frac{5}{6} \approx \frac{1}{2} \times 1$

$\approx \frac{1}{2}$

To multiply fractions, multiply the numerators and multiply the denominators.

$\frac{8}{15} \times \frac{5}{6} = \frac{40}{90}$

$= \frac{4}{9}$

The answer seems reasonable, because it is close to the estimate of $\frac{1}{2}$.

Show You Know

Estimate and calculate.

a) $\frac{3}{5} \times \frac{2}{9}$ b) $\frac{5}{6} \times \frac{4}{5}$

Key Ideas

- You can multiply two proper fractions using paper folding.

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

- You can multiply two proper fractions using diagrams.

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

- A rule for multiplying two proper fractions is to multiply the numerators and multiply the denominators.

$\frac{3}{5} \times \frac{5}{6} = \frac{15}{30}$ or $\frac{1}{2}$

- You can estimate the product of two proper fractions by first deciding whether each fraction is closer to 0, $\frac{1}{2}$, or 1.

$\frac{3}{7} \approx \frac{1}{2}$ $\frac{4}{7} \approx \frac{1}{2}$ $\frac{3}{7} \times \frac{4}{7} \approx \frac{1}{4}$

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Example 2

This example illustrates the use of diagrams to model the multiplication of two proper fractions. Reinforce that the example uses a familiar problem solving strategy (i.e., Draw a Diagram). Ask students to think of another strategy they could use (e.g., Model It). Point out the thought bubble that indicates the similarity of the second diagram to the result of paper folding.

You might point out that because the order of multiplication does not matter, the order in which the dimensions of the large rectangle are drawn does not matter. The representation of $\frac{1}{2}$ could be completed before the representation of $\frac{2}{3}$.

Literacy Link Use the Literacy Link on page 212 to help students recall the meaning of *common denominator*. Have students find the common denominator of $\frac{2}{3}$ and $\frac{1}{6}$ to show their understanding.

Example 3

This example shows the use of a rule to multiply two proper fractions and presents a technique for estimating the product. If necessary, plot the fractions on a number line to demonstrate why $\frac{8}{15}$ is approximated as $\frac{1}{2}$ and why $\frac{5}{6}$ is approximated as 1. Stress the importance of comparing the estimate to the calculated product to check that the calculated value is reasonable. If necessary, plot $\frac{4}{9}$ and $\frac{1}{2}$ on a number line to demonstrate how close they are.

The calculation method used in Example 3 is to multiply before writing the product in lowest terms. You may wish to introduce the following alternative method, which involves removing common factors before multiplying $\frac{8}{15} \times \frac{5}{6} = \frac{4}{9}$. You may wish to explain this method by factoring the numerators and denominators to identify the common factors: $\frac{4 \times 2}{5 \times 3} \times \frac{5}{3 \times 2} = \frac{4 \times \cancel{2}}{\cancel{5} \times 3} \times \frac{\cancel{5}}{3 \times \cancel{2}} = \frac{4}{3 \times 3} = \frac{4}{9}$.

It is important for students to understand the process of multiplying fractions and clearly demonstrate this understanding before being shown how to simplify the multiplication by factoring the numerators and denominators and removing the common factors.

Meeting Student Needs

- Help students explore the concept by discussing sleeping patterns of other animals, such as rabbits, ducks, deer, and cows. Have students research the information on the Internet, and discuss in small groups or as a whole class what fraction of time one of the animals sleeps compared to a sloth.
- Help students remember how to multiply fractions and whole numbers by asking them how they might modify wording for the multiplication rule in order to incorporate multiplying by fractions.
- Some students may benefit from using a virtual manipulative to explore and practise multiplying proper and improper fractions. See the related Web Link below.

ELL

- Ensure students understand the words *denominator* and *numerator*.
- Read through the instructions orally as you demonstrate each one. First take the sheet of paper and say, "Here is a rectangular piece of paper. How do I know it is a rectangle?" Allow students in the class to answer. While folding the paper say, "Fold the paper into thirds, along its length."
- Invite the class as a whole to create a rule for multiplying fractions before they create their own.

Common Errors

- Some students may not understand how many equal sections to create when they use paper folding or a diagram to multiply two proper fractions.

R_x By referring to Examples 1 and 2 and other examples, point out that the number of sections along each dimension of the sheet of paper or large rectangle is given by the denominator of each fraction.

- Some students may not consider whether answers are reasonable.

R_x Point out the use of mental reasoning beside the solution to Example 2. Ask students to make up some other examples that illustrate this generalization. For example, the product $\frac{1}{2} \times \frac{1}{2}$ is $\frac{1}{4}$, which is less than $\frac{1}{2}$.

Web Link

For a virtual manipulative that allows students to explore and practise multiplying proper and improper fractions, go to www.mathlinks8.ca and follow the links. The Show Me option demonstrates and explores the multiplication of fractions. The Test Me option provides problems to solve. After specifying the answer on the grid, students can check to see if they are correct.

Answers

Explore the Math

1. Answers will vary. Example: The section of paper that is shaded both yellow and blue represents $\frac{1}{2}$ of $\frac{2}{3}$.

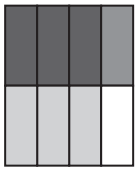
2. $\frac{2}{6}$; $\frac{1}{6}$; $\frac{3}{12}$; $\frac{6}{12}$; $\frac{9}{16}$

3. a) The numerators are multiplied to obtain the product.

b) The denominators are multiplied to obtain the product.

4. Answers will vary. Example: To find the product of two proper fractions, multiply the numerators and multiply the denominators.

5. a) $\frac{3}{8}$ **b)**

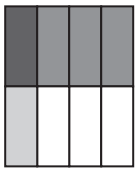


6. a) Answers will vary. Example: Fold the paper lengthwise into the number of sections indicated by the denominator of the first fraction. Colour the number of sections indicated by the numerator of the first fraction. Then, fold the paper widthwise into the number of sections indicated by the denominator of the second fraction. Colour the number of sections indicated by the numerator of the second fraction a different colour. The number of sections that contain both colours is the numerator of the product. The total number of sections is the denominator. A rule for multiplying two proper fractions is to multiply the numerators and multiply the denominators.

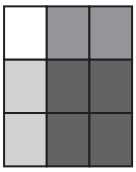
b) Answers will vary. Example: I prefer the rule because it is less time consuming.

Show You Know: Example 1

a) $\frac{1}{8}$

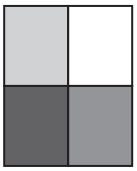


b) $\frac{4}{9}$

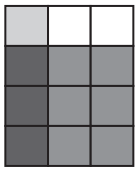


Show You Know: Example 2

a) $\frac{1}{4}$



b) $\frac{3}{12}$

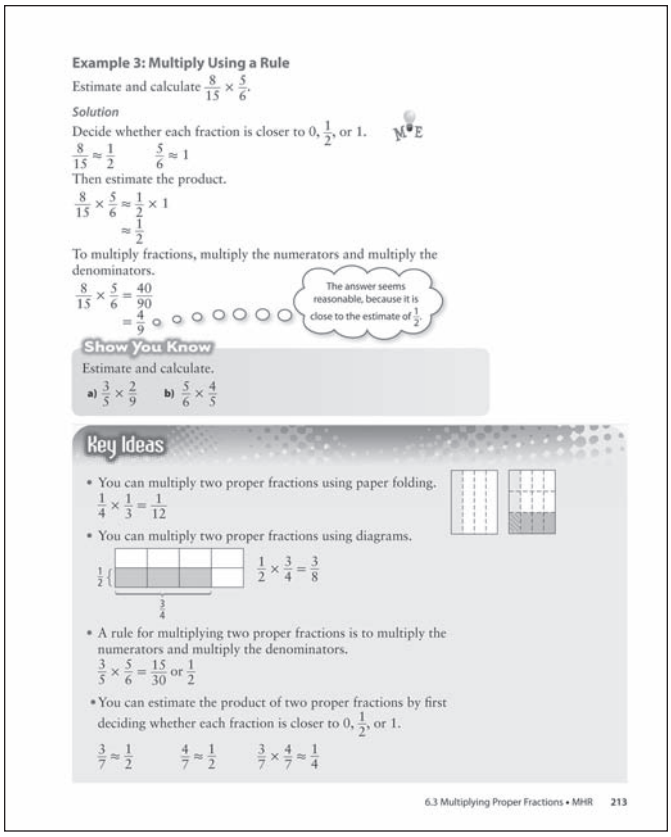


Show You Know: Example 3

a) Estimates will vary. Example: 0; Answer: $\frac{2}{15}$

b) Estimates will vary. Example: 1; Answer: $\frac{2}{3}$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.	• When initially folding paper, some students may benefit from prompts, such as How do you show thirds in folding? How do you show quarters? What do overlapping colours represent? • Some students should be encouraged to model the multiplication using more familiar manipulatives. Allow students to decide.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking as they colour each piece. • You may wish to have students work with a partner. • Help students recall possible folds for quarters and halves. • Consider coaching students through a problem without using paper folding or manipulatives. Find out if they are able to verbalize the steps. If not, allow them to continue using the manipulatives of their choice.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking as they colour each piece. • You may wish to have students work with a partner. • Help students recall possible folds for thirds and halves. • Some students may benefit from using **BLM 6–8 Rectangles** to model questions.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Coach students through a problem without using paper folding or manipulatives. Find out if they are able to verbalize the steps. If not, allow them to continue using the manipulatives of their choice. • Have students solve a similar problem using manipulatives, and have them record the algebraic steps as they work through each step using the manipulatives. Coach students to understand the parallels of each step. • Encourage students to estimate before solving. Some students may benefit from showing fractions on **BLM 6–9 Fraction Number Lines** as they consider whether a fraction is closer to 0, $\frac{1}{2}$, or 1.



Example 3: Multiply Using a Rule

Estimate and calculate $\frac{8}{15} \times \frac{5}{6}$.

Solution

Decide whether each fraction is closer to 0, $\frac{1}{2}$, or 1. ME

$\frac{8}{15} \approx \frac{1}{2}$ $\frac{5}{6} \approx 1$

Then estimate the product.

$\frac{8}{15} \times \frac{5}{6} \approx \frac{1}{2} \times 1$

$\approx \frac{1}{2}$

To multiply fractions, multiply the numerators and multiply the denominators.

$\frac{8}{15} \times \frac{5}{6} = \frac{40}{90}$

$= \frac{4}{9}$

The answer seems reasonable, because it is close to the estimate of $\frac{1}{2}$.

Show You Know

Estimate and calculate.

a) $\frac{3}{5} \times \frac{2}{9}$ b) $\frac{5}{6} \times \frac{4}{5}$

Key Ideas

- You can multiply two proper fractions using paper folding.

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

- You can multiply two proper fractions using diagrams.

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

- A rule for multiplying two proper fractions is to multiply the numerators and multiply the denominators.

$\frac{3}{5} \times \frac{5}{6} = \frac{15}{30}$ or $\frac{1}{2}$

- You can estimate the product of two proper fractions by first deciding whether each fraction is closer to 0, $\frac{1}{2}$, or 1.

$\frac{3}{7} \approx \frac{1}{2}$ $\frac{4}{7} \approx \frac{1}{2}$ $\frac{3}{7} \times \frac{4}{7} \approx \frac{1}{4}$

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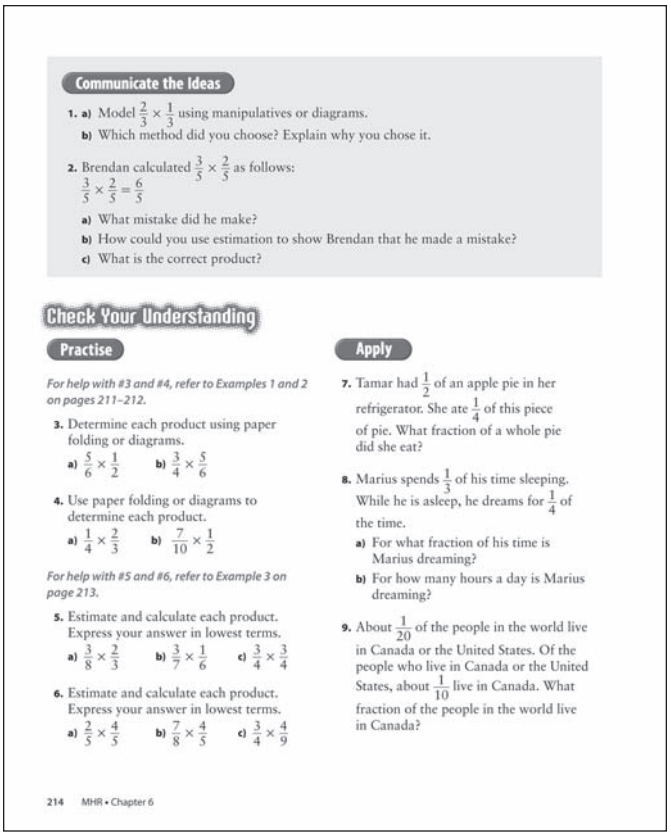

Communicate the Ideas

1. a) Model $\frac{2}{3} \times \frac{1}{3}$ using manipulatives or diagrams.
   b) Which method did you choose? Explain why you chose it.
2. Brendan calculated $\frac{3}{5} \times \frac{2}{5}$ as follows:
   $\frac{3}{5} \times \frac{2}{5} = \frac{6}{5}$
   a) What mistake did he make?
   b) How could you use estimation to show Brendan that he made a mistake?
   c) What is the correct product?

Check Your Understanding

Practise

For help with #3 and #4, refer to Examples 1 and 2 on pages 211–212.

3. Determine each product using paper folding or diagrams.
   a) $\frac{5}{6} \times \frac{1}{2}$ b) $\frac{3}{4} \times \frac{5}{6}$
4. Use paper folding or diagrams to determine each product.
   a) $\frac{1}{4} \times \frac{2}{3}$ b) $\frac{7}{10} \times \frac{1}{2}$

For help with #5 and #6, refer to Example 3 on page 213.

5. Estimate and calculate each product. Express your answer in lowest terms.
   a) $\frac{3}{8} \times \frac{2}{3}$ b) $\frac{3}{7} \times \frac{1}{6}$ c) $\frac{3}{4} \times \frac{3}{4}$
6. Estimate and calculate each product. Express your answer in lowest terms.
   a) $\frac{2}{5} \times \frac{4}{5}$ b) $\frac{7}{8} \times \frac{4}{5}$ c) $\frac{3}{4} \times \frac{4}{9}$

Apply

7. Tamar had $\frac{1}{2}$ of an apple pie in her refrigerator. She ate $\frac{1}{4}$ of this piece of pie. What fraction of a whole pie did she eat?
8. Marius spends $\frac{1}{3}$ of his time sleeping. While he is asleep, he dreams for $\frac{1}{4}$ of the time.
   a) For what fraction of his time is Marius dreaming?
   b) For how many hours a day is Marius dreaming?
9. About $\frac{1}{20}$ of the people in the world live in Canada or the United States. Of the people who live in Canada or the United States, about $\frac{1}{10}$ live in Canada. What fraction of the people in the world live in Canada?

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This section summarizes concrete, semi-concrete, and symbolic methods for multiplying two proper fractions, and a method for estimating the product. Students could prepare their own list of Key Ideas and include it in their chapter Foldable, especially if they have used other approaches (such as modelling with fraction strips and removing common factors before multiplying).

Communicate the Ideas

These questions allow students to explain ways of modelling the multiplication of two proper fractions. In #1, students choose among the methods they have seen and explain their choice. In #2, students consider a possible error in the application of a rule to multiply two proper fractions.

If you introduced the method of removing common factors before multiplying in Example 3, you might pose the following question: To calculate $\frac{9}{20} \times \frac{2}{3}$, do you prefer to remove common factors before or after you multiply? Explain.

Meeting Student Needs

- If you included fraction strips as an alternative method in Example 1, some students might benefit from considering the following questions: Francesca and Aziz both determined $\frac{1}{2} \times \frac{1}{3}$ correctly using fraction strips.
  a) Francesca started by using a strip to represent $\frac{1}{3}$. What did she do next?
  b) Aziz started by using a strip to represent $\frac{1}{2}$. What did he do next?
  c) Explain why both methods gave the correct product.

Common Errors

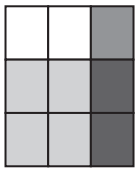
- Some students may confuse the rules for multiplication and addition of two proper fractions, for example, by assuming that fractions with different denominators should be written with a common denominator before they are multiplied, or by adding the numerators and multiplying the denominators.

R_x Help reactivate students' skills in adding two proper fractions, concretely if necessary, and point out the distinctions between addition and multiplication. Encourage students to use reasoning and estimation to check that their calculated products are reasonable.

Answers

Communicate the Ideas

1. a) Answers will vary. Example:



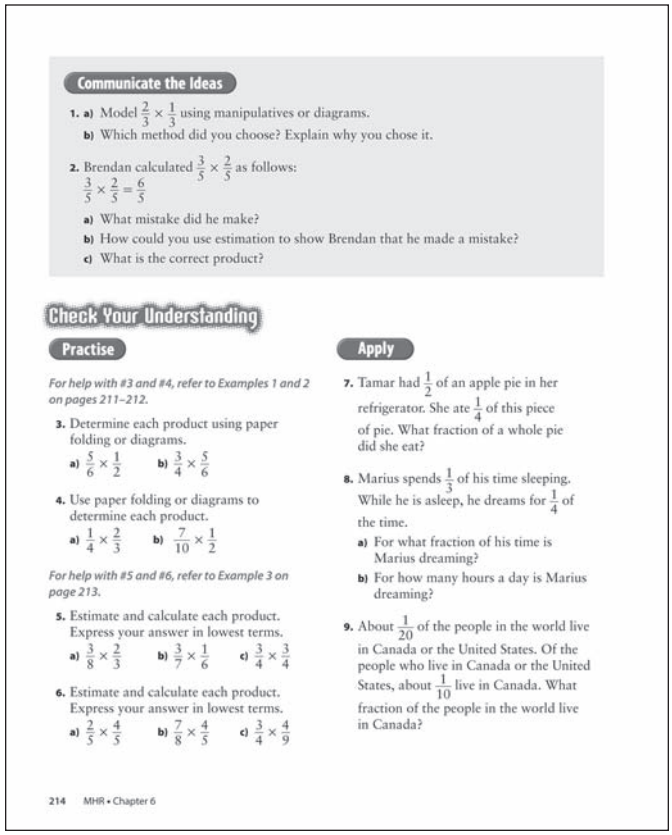
b) Answers will vary. Example: I chose a diagram because it is like paper folding.

2. a) He did not multiply the denominators.

b) Answers will vary. Example: If he estimated the answer as $\frac{1}{4}$, he would have noticed that his answer is incorrect.

c) $\frac{6}{25}$

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and #2.	• Students who need assistance with #1 should be asked to model what they believe $\frac{2}{3}$ and $\frac{1}{3}$ look like. • Have them verbalize this thinking or draw it using a method of their choice. Clarify any misconceptions of the individual fractions before moving onto the process of division. • Ask what it means to multiply by $\frac{1}{3}$. • Some students may benefit from referring back to Examples 1 and 2. • Students who need assistance with #2 should be referred back to Explore the Math #2 to #5, and Example 3. Again, allow students to model their thinking through a process that is easiest for them to think through.



Communicate the Ideas

1. a) Model $\frac{2}{3} \times \frac{1}{3}$ using manipulatives or diagrams.
   b) Which method did you choose? Explain why you chose it.
2. Brendan calculated $\frac{3}{5} \times \frac{2}{5}$ as follows:
   $\frac{3}{5} \times \frac{2}{5} = \frac{6}{5}$
   a) What mistake did he make?
   b) How could you use estimation to show Brendan that he made a mistake?
   c) What is the correct product?

Check Your Understanding

Practise

For help with #3 and #4, refer to Examples 1 and 2 on pages 211–212.

3. Determine each product using paper folding or diagrams.
   a) $\frac{5}{6} \times \frac{1}{2}$ b) $\frac{3}{4} \times \frac{5}{6}$
4. Use paper folding or diagrams to determine each product.
   a) $\frac{1}{4} \times \frac{2}{3}$ b) $\frac{7}{10} \times \frac{1}{2}$

For help with #5 and #6, refer to Example 3 on page 213.

5. Estimate and calculate each product. Express your answer in lowest terms.
   a) $\frac{3}{8} \times \frac{2}{3}$ b) $\frac{3}{7} \times \frac{1}{6}$ c) $\frac{3}{4} \times \frac{3}{4}$
6. Estimate and calculate each product. Express your answer in lowest terms.
   a) $\frac{2}{5} \times \frac{4}{5}$ b) $\frac{7}{8} \times \frac{4}{5}$ c) $\frac{3}{4} \times \frac{4}{9}$

Apply

7. Tamar had $\frac{1}{2}$ of an apple pie in her refrigerator. She ate $\frac{1}{4}$ of this piece of pie. What fraction of a whole pie did she eat?
8. Marius spends $\frac{1}{3}$ of his time sleeping. While he is asleep, he dreams for $\frac{1}{4}$ of the time.
   a) For what fraction of his time is Marius dreaming?
   b) For how many hours a day is Marius dreaming?
9. About $\frac{1}{20}$ of the people in the world live in Canada or the United States. Of the people who live in Canada or the United States, about $\frac{1}{10}$ live in Canada. What fraction of the people in the world live in Canada?

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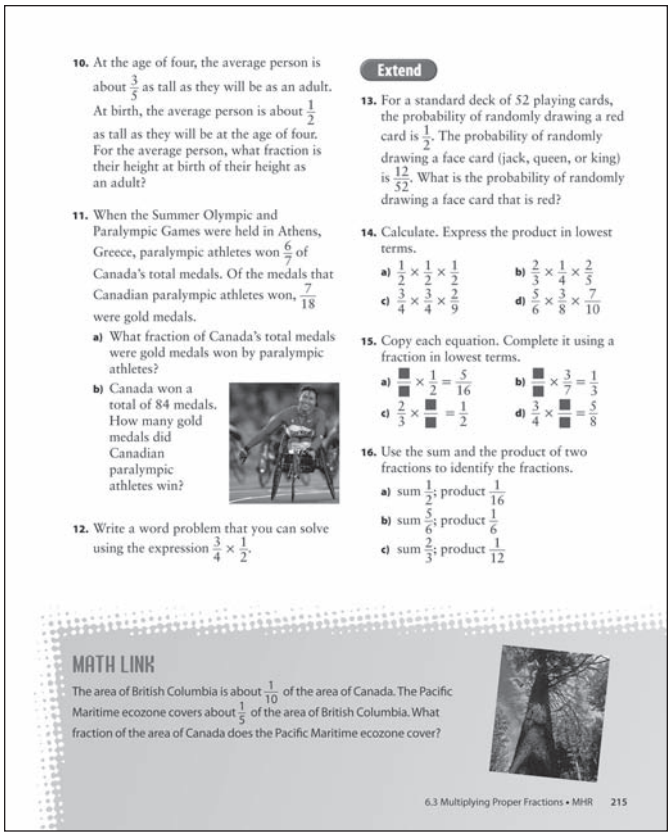
Check Your Understanding

Practise

If you included fraction strips as an alternative method in Example 1, they can also be included as an alternative in #3 and #4. If you introduced the method of removing common factors before multiplying in Example 3, encourage students to use the method they prefer in #5 and #6 and to explain their preference.

Encourage students to compare the methods they used in #3 and #4 and to explain why they chose a particular method. Have students compare their estimates in #5 and #6. Check that they are approximating appropriately. Discuss especially #5c), where the best estimate is obtained by approximating one $\frac{3}{4}$ as $\frac{1}{2}$ and the other $\frac{3}{4}$ as 1. Stress that estimation often involves judgments of this type.



10. At the age of four, the average person is about $\frac{3}{5}$ as tall as they will be as an adult. At birth, the average person is about $\frac{1}{2}$ as tall as they will be at the age of four. For the average person, what fraction is their height at birth of their height as an adult?

11. When the Summer Olympic and Paralympic Games were held in Athens, Greece, paralympic athletes won $\frac{6}{7}$ of Canada's total medals. Of the medals that Canadian paralympic athletes won, $\frac{7}{18}$ were gold medals.
a) What fraction of Canada's total medals were gold medals won by paralympic athletes?
b) Canada won a total of 84 medals. How many gold medals did Canadian paralympic athletes win?

12. Write a word problem that you can solve using the expression $\frac{3}{4} \times \frac{1}{2}$.

Extend

13. For a standard deck of 52 playing cards, the probability of randomly drawing a red card is $\frac{1}{2}$. The probability of randomly drawing a face card (jack, queen, or king) is $\frac{12}{52}$. What is the probability of randomly drawing a face card that is red?

14. Calculate. Express the product in lowest terms.
a) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
b) $\frac{2}{3} \times \frac{1}{4} \times \frac{2}{5}$
c) $\frac{3}{4} \times \frac{3}{4} \times \frac{2}{9}$
d) $\frac{5}{6} \times \frac{3}{8} \times \frac{7}{10}$

15. Copy each equation. Complete it using a fraction in lowest terms.
a) $\blacksquare \times \frac{1}{2} = \frac{5}{16}$
b) $\blacksquare \times \frac{3}{7} = \frac{1}{3}$
c) $\frac{2}{3} \times \blacksquare = \frac{1}{2}$
d) $\frac{3}{4} \times \blacksquare = \frac{5}{8}$

16. Use the sum and the product of two fractions to identify the fractions.
a) sum $\frac{1}{2}$; product $\frac{1}{16}$
b) sum $\frac{5}{6}$; product $\frac{1}{6}$
c) sum $\frac{2}{3}$; product $\frac{1}{12}$

MATH LINK

The area of British Columbia is about $\frac{1}{10}$ of the area of Canada. The Pacific Maritime ecozone covers about $\frac{1}{5}$ of the area of British Columbia. What fraction of the area of Canada does the Pacific Maritime ecozone cover?

6.3 Multiplying Proper Fractions • MHR 215

Apply

Encourage students to think about the reasonableness of their answers and to use estimation to check them.

For #12, some students may use original contexts for their problems. Other students may adapt the wording of earlier problems (e.g., #7). You might point out that earlier problems that include real data (such as #9) cannot be used in this way. You might encourage students to share their problems so that they are exposed to some that show originality.

Extend

Since #13 anticipates a concept to be covered in Chapter 11, some students may benefit from different wording:. If you draw a card randomly many times, you expect $\frac{1}{2}$ of the cards drawn to be red cards and $\frac{3}{13}$ of the cards drawn to be face cards. You might then ask students what fraction of the cards drawn they expect to be red face cards.

For #14, you might encourage students to think about whether they can extend their rule for multiplying two proper fractions to include a greater number of fractions, rather than multiplying two at a time.

For #15, students may begin by applying their rule for multiplying two proper fractions. This approach works well in part a), where it is clear that $5 \times 1 = 5$ in the numerators and $8 \times 2 = 16$ in the denominators. However, the other parts of #15 are less straightforward. To apply the multiplication rule, students will need to experiment by rewriting each product so that it is not expressed in lowest terms.

For #16, students may use a Guess and Check approach. They can test two fractions with the given sum to determine if the product is correct, or they can test two fractions with the given product to determine if the sum is correct.

Math Link

This Math Link allows students to apply the multiplication of two proper fractions to data concerning Canada's ecozones.

Meeting Student Needs

- Some students may benefit from using **BLM 6–8 Rectangles** to model questions.
- Provide **BLM 6–12 Section 6.3 Extra Practice** to students who would benefit from more practice.

ELL

- For #7, draw a picture of a pie on the board, and while reading the question, indicate $\frac{1}{4}$ of the pie.
- Some of the Apply questions are fairly challenging. You may wish to give students only a few questions at a time, so that they can take the time to work through them.

Gifted and Enrichment

- If you introduced the method of removing common factors before multiplying in Example 3, you may wish to include the following problem:
Determine the product. Explain your method.
$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \ldots \times \frac{99}{100}$
(Answers will vary. Example: $\frac{1}{100}$. The first denominator equals the second numerator, the second denominator equals the third numerator, and so on. All you are left with after cancelling the equal pair of numbers is a large number of ones and the final denominator, 100.)

Common Errors

- Some students may have difficulty in estimating the product of two proper fractions.

R_x If students are unable to decide if each fraction is closer to 0, $\frac{1}{2}$, or 1, suggest that they use a point on a number line to represent the value of each fraction, such as those on **BLM 6–9 Fraction Number Lines**. For the method shown in Example 3, you might have students consider all the possible estimates for the product of any two proper fractions. There are only six possibilities, three of which give the same estimated product: $0 \times 0 = 0$, $0 \times \frac{1}{2} = 0$, $0 \times 1 = 0$, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, $\frac{1}{2} \times 1 = \frac{1}{2}$, and $1 \times 1 = 1$. Have students give examples of multiplication statements that result in each of these estimated products.

Answers

Math Link

$\frac{1}{50}$

Assessment	Supporting Learning
Assessment *for* Learning	
Practise Have students do #3 and #5. Students who have no problems with these questions can go on to the Apply questions.	• Provide additional coaching with Examples 1 and 2 to students who need help with #3. Coach students through #3 using a method of their choice. Have students try #4 before going on. • Provide additional coaching with Example 3 to students who need help with #5. Have students try #6 before going on. Encourage students to use a model and have them verbalize their thinking. Check back with them several times to make sure that they understand the concepts. • You may wish to provide **BLM 6–9 Fraction Number Lines** to help students visualize whether a fraction is closer to 0, $\frac{1}{2}$, or 1.
Math Link The Math Link on page 215 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 239.	• Most students should do this Math Link, since they will use these basic skills when they design and solve their own questions related to the ecozones in the Wrap It Up! • Students who need help getting started could use **BLM 6–13 Section 6.3 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students complete the following statements: • When multiplying fractions, I find it easiest to solve using … • The most difficult part of solving $\frac{3}{5} \times \frac{3}{4}$ is … because …	• Encourage students to draw a diagram to help them. • Depending on students' learning style, have them provide oral or written answers. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with.