Multiplying Improper Fractions and Mixed Numbers

6.4

***MathLinks 8*, pages 216–221**

Suggested Timing
50–60 minutes

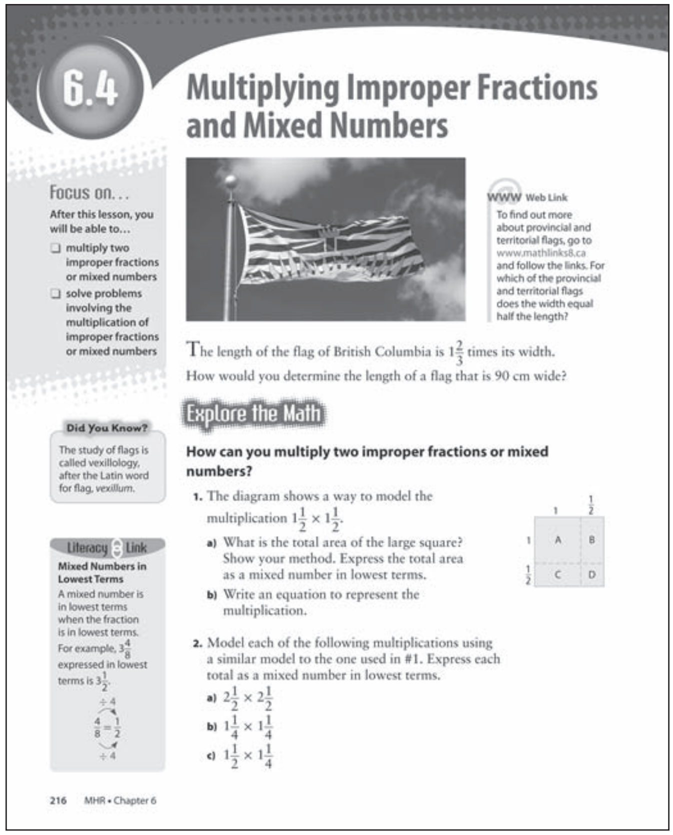
Materials
- ruler
- grid paper (optional)
- fraction strips (optional)
- transparent diagrams or strips (optional)
- dry erase markers (optional)
- coloured pencils (optional)

Blackline Masters
Master 2 Two Stars and One Wish
Master 8 Centimetre Grid Paper
Master 14 Fraction Strips
BLM 6–3 Chapter 6 Warm-Up
BLM 6–8 Rectangles
BLM 6–9 Fraction Number Lines
BLM 6–14 Section 6.4 Extra Practice
BLM 6–15 Section 6.4 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 4a), c), 5b), d), 6c), d), 8b), c), 10, Math Link
Typical	1–3, 4a), c), 5b), d), 6c), d), 8b), c), 10–15, 17, Math Link
Extension/Enrichment	1–3, 15, 16, 18–21, Math Link


6.4
Multiplying Improper Fractions and Mixed Numbers

Focus on...
After this lesson, you will be able to...
- ☐ multiply two improper fractions or mixed numbers
- ☐ solve problems involving the multiplication of improper fractions or mixed numbers

WWW Web Link
To find out more about provincial and territorial flags, go to www.mathlinks8.ca and follow the links. For which of the provincial and territorial flags does the width equal half the length?

The length of the flag of British Columbia is $1\frac{2}{3}$ times its width. How would you determine the length of a flag that is 90 cm wide?

Did You Know?
The study of flags is called vexillology, after the Latin word for flag, *vexillum*.

Literacy Link
Mixed Numbers in Lowest Terms
A mixed number is in lowest terms when the fraction is in lowest terms. For example, $3\frac{4}{8}$ expressed in lowest terms is $3\frac{1}{2}$.
$\frac{4}{8} = \frac{1}{2}$ (÷ 4)

Explore the Math

How can you multiply two improper fractions or mixed numbers?

1. The diagram shows a way to model the multiplication $1\frac{1}{2} \times 1\frac{1}{2}$.
   a) What is the total area of the large square? Show your method. Express the total area as a mixed number in lowest terms.
   b) Write an equation to represent the multiplication.

2. Model each of the following multiplications using a similar model to the one used in #1. Express each total as a mixed number in lowest terms.
   a) $2\frac{1}{2} \times 2\frac{1}{2}$
   b) $1\frac{1}{4} \times 1\frac{1}{4}$
   c) $1\frac{1}{2} \times 1\frac{1}{4}$

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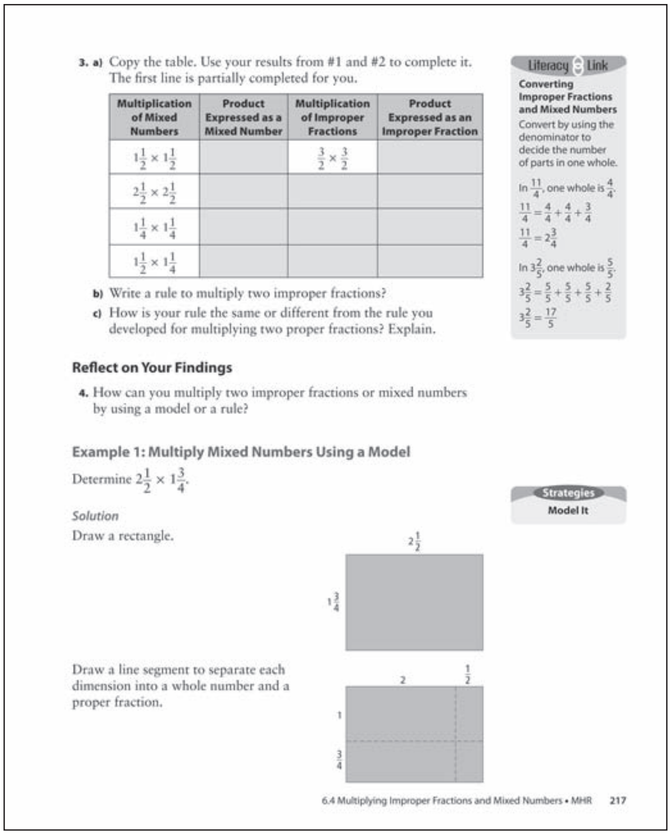
Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce material learned in previous sections.

Students should realize that the length of the flag of British Columbia is determined from the multiplication: $90 \times 1\frac{2}{3}$. If you want students to complete this calculation, they might return to it after they complete Example 2. The multiplication can then be written as $\frac{90}{1} \times \frac{5}{3}$, resulting in a length of 150 cm. You may wish to have students check this calculation using patterning:

$1 = 90$
$\frac{1}{3} = 90 \div 3 = 30$
$1\frac{2}{3} = 90 + 30 + 30 = 150$
The flag is 150 cm long.



3\. a) Copy the table. Use your results from #1 and #2 to complete it. The first line is partially completed for you.

Multiplication of Mixed Numbers	Product Expressed as a Mixed Number	Multiplication of Improper Fractions	Product Expressed as an Improper Fraction
$1\frac{1}{2} \times 1\frac{1}{2}$		$\frac{3}{2} \times \frac{3}{2}$	
$2\frac{1}{2} \times 2\frac{1}{2}$			
$1\frac{1}{4} \times 1\frac{1}{4}$			
$1\frac{1}{2} \times 1\frac{1}{4}$			

b) Write a rule to multiply two improper fractions?

c) How is your rule the same or different from the rule you developed for multiplying two proper fractions? Explain.

Literacy Link

Converting Improper Fractions and Mixed Numbers

Convert by using the denominator to decide the number of parts in one whole.

In $\frac{11}{4}$, one whole is $\frac{4}{4}$.

$\frac{11}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$

$\frac{11}{4} = 2\frac{3}{4}$

In $3\frac{2}{5}$, one whole is $\frac{5}{5}$.

$3\frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5}$

$3\frac{2}{5} = \frac{17}{5}$

Reflect on Your Findings

4\. How can you multiply two improper fractions or mixed numbers by using a model or a rule?

Example 1: Multiply Mixed Numbers Using a Model

Determine $2\frac{1}{2} \times 1\frac{3}{4}$.

Strategies

Model It

Solution

Draw a rectangle.

Draw a line segment to separate each dimension into a whole number and a proper fraction.

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Explore the Math

Students use an area model to develop a rule for multiplying two improper fractions or mixed numbers. Before starting this Explore, challenge students to estimate whether the product of $1\frac{1}{2} \times 1\frac{1}{2}$ is greater or less than $1\frac{1}{2}$. Have them explain their reasoning.

Literacy Link Draw students' attention to the Literacy Links on pages 216 and 217. The first Literacy Link explains that a mixed number is in lowest terms when the fraction is in lowest terms. Have students practise putting mixed numbers, such as $3\frac{6}{8}$, in lowest terms, to show their understanding.

The second Literacy Link explains how to convert improper fractions and mixed numbers by using the denominator to decide the number of parts in one whole. Students learned to convert improper fractions and mixed numbers in Grade 6. However, they may benefit from a reminder.

Have students practise converting improper fractions and mixed numbers, such as $\frac{8}{5}$ and $4\frac{6}{7}$.

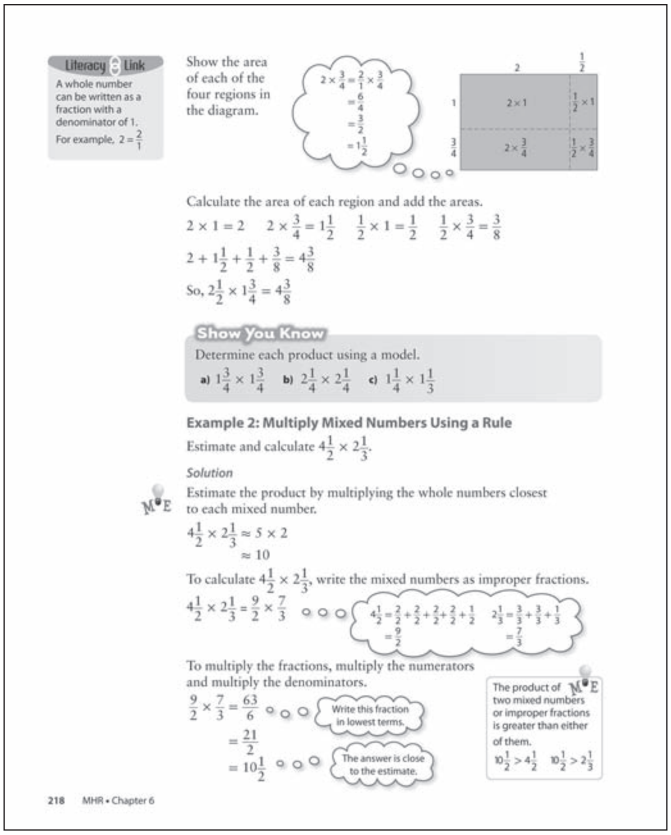
Method 1 Have students work with a partner. Encourage students to use their own strategies for calculating the total area of the diagram. Consider challenging them to find it in more than one way. To do this, you may wish to ask questions such as the following:

- How could you determine the area of each section of the large square?
- What is the connection between the dimensions of each shape and the area of that shape?
- What is the area of each section?
- How could you use the area of each section to determine the total area of the large square?
- How else could you determine the total area of the large square? (For example, students could calculate the area of squares A and B together, since they know how to multiply by 1. They could then separately calculate the areas of squares C and D and add all of the numbers. $1\frac{1}{2} + \frac{1}{2} + \frac{1}{4} = 2\frac{1}{4}$. Alternatively, they could calculate the area of A and C together, and then separately calculate the areas of B and D.)

Also, assist students who are unable to generate their own diagrams in #2.

Encourage students to think about the easiest way to work with sectional areas in #2. For example, you might point out that each side of the square in #2a) could be separated into parts that measure $\frac{1}{2}$ and 2, or into parts that measure 1 and $1\frac{1}{2}$. However, the latter option is problematic because students do not yet know how to multiply mixed numbers. Many other ways of separating parts of the square are possible. For example, each side of the square could be separated into five parts that each measure $\frac{1}{2}$. The result would be 25 sections each with an area of $\frac{1}{4}$. This approach would resemble the use of grid paper, as described in Method 2 below.

Emphasize that the results from #1 and #2 should be used to complete the second column in the table in #3. Have students complete the third column by writing each mixed number from the first column as an improper fraction. The fourth column can be completed by writing each mixed number from the second column as an improper fraction. Students need to examine the numerators and denominators of the improper fractions in the third and fourth columns to answer #3b). Make sure that the multiplication and products recorded in the table are correct before students attempt to make a generalization in #3b).



Literacy Link

A whole number can be written as a fraction with a denominator of 1. For example, $2 = \frac{2}{1}$

Show the area of each of the four regions in the diagram.

$2 \times \frac{3}{4} = \frac{2}{1} \times \frac{3}{4}$
$= \frac{6}{4}$
$= \frac{3}{2}$
$= 1\frac{1}{2}$

	2	$\frac{1}{2}$
1	2×1	$\frac{1}{2} \times 1$
$\frac{3}{4}$	$2 \times \frac{3}{4}$	$\frac{1}{2} \times \frac{3}{4}$

Calculate the area of each region and add the areas.

$2 \times 1 = 2 \quad 2 \times \frac{3}{4} = 1\frac{1}{2} \quad \frac{1}{2} \times 1 = \frac{1}{2} \quad \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$2 + 1\frac{1}{2} + \frac{1}{2} + \frac{3}{8} = 4\frac{3}{8}$

So, $2\frac{1}{2} \times 1\frac{3}{4} = 4\frac{3}{8}$

Show You Know

Determine each product using a model.

a) $1\frac{3}{4} \times 1\frac{3}{4}$ b) $2\frac{1}{4} \times 2\frac{1}{4}$ c) $1\frac{1}{4} \times 1\frac{1}{3}$

Example 2: Multiply Mixed Numbers Using a Rule

Estimate and calculate $4\frac{1}{2} \times 2\frac{1}{3}$.

Solution

ME Estimate the product by multiplying the whole numbers closest to each mixed number.

$4\frac{1}{2} \times 2\frac{1}{3} \approx 5 \times 2$
≈ 10

To calculate $4\frac{1}{2} \times 2\frac{1}{3}$, write the mixed numbers as improper fractions.

$4\frac{1}{2} \times 2\frac{1}{3} = \frac{9}{2} \times \frac{7}{3}$

$4\frac{1}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2} = \frac{9}{2}$ $2\frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$

To multiply the fractions, multiply the numerators and multiply the denominators.

$\frac{9}{2} \times \frac{7}{3} = \frac{63}{6}$ Write this fraction in lowest terms.
$= \frac{21}{2}$
$= 10\frac{1}{2}$ The answer is close to the estimate.

ME The product of two mixed numbers or improper fractions is greater than either of them.
$10\frac{1}{2} > 4\frac{1}{2}$ $10\frac{1}{2} > 2\frac{1}{3}$

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Have students verify the rule they have developed by answering the question in the section opener using multiplication of fractions ($1\frac{2}{3} \times 90 = 150$).

Method 2 Have students model the multiplication using diagrams on grid paper. Provide them with **Master 8 Centimetre Grid Paper**. In the exploration, #1 would be modified as follows. The wording of #2 and #3 would be unaffected.

1. Model the multiplication $1\frac{1}{2} \times 1\frac{1}{2}$ on grid paper.
   a) How many units of length does a side of one small grid square represent? ($\frac{1}{2}$)
   b) Use your answer from part a) to calculate the area of each small grid square. ($\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$)
   c) How many small grid squares are inside the drawn square? (9)
   d) What is the total area of the drawn square? Express your answer as a mixed number. ($2\frac{1}{4}$)
   e) Copy and complete the multiplication statement $1\frac{1}{2} \times 1\frac{1}{2} = \blacksquare$. ($2\frac{1}{4}$)
   f) Model the same multiplication, but use a diagram with a side length of six grid squares. (Note that each small grid square now has a side length of $\frac{1}{4}$ and an area of $\frac{1}{16}$. There are 36 grid squares inside the drawn square, so the total area of the drawn square is $36 \times \frac{1}{16}$, which is $\frac{36}{16}$ or $\frac{9}{4}$ or $2\frac{1}{4}$. Changing the number of grid squares in the model does not affect the value that the model represents.)
   g) Model the same multiplication, but use a diagram with a side length of $1\frac{1}{2}$ grid squares. (Note that each small grid square now has a side length of 1 and an area of 1. The determination of the area of the drawn square is the same as in #1, parts a) to e), in the student resource.)
   h) Use your findings from parts f) and g) to explain why a diagram with a side length of three grid squares is a convenient choice. (Using a diagram with a side length of six grid squares involves more grid squares than necessary and the removal of a common factor to write the product in lowest terms. Using a diagram with a side length of $1\frac{1}{2}$ grid squares involves four separate multiplications, including the multiplication of two fractions, and an addition.)

Method 3 Have students model the multiplication using manipulatives, such as fraction strips. You may wish to hand out **Master 14 Fraction Strips**. The following description assumes that students have already used fraction strips to multiply proper fractions in section 6.3 and are therefore familiar with the use of common denominators in the fraction-strip method. For example, here is a method for determining $2\frac{1}{4} \times 1\frac{1}{2}$.

Use fraction strips to represent $1\frac{1}{2}$.

Now use fraction strips to represent $2\frac{1}{4}$ groups of $1\frac{1}{2}$.

To find the product, add the values represented by the fraction strips:

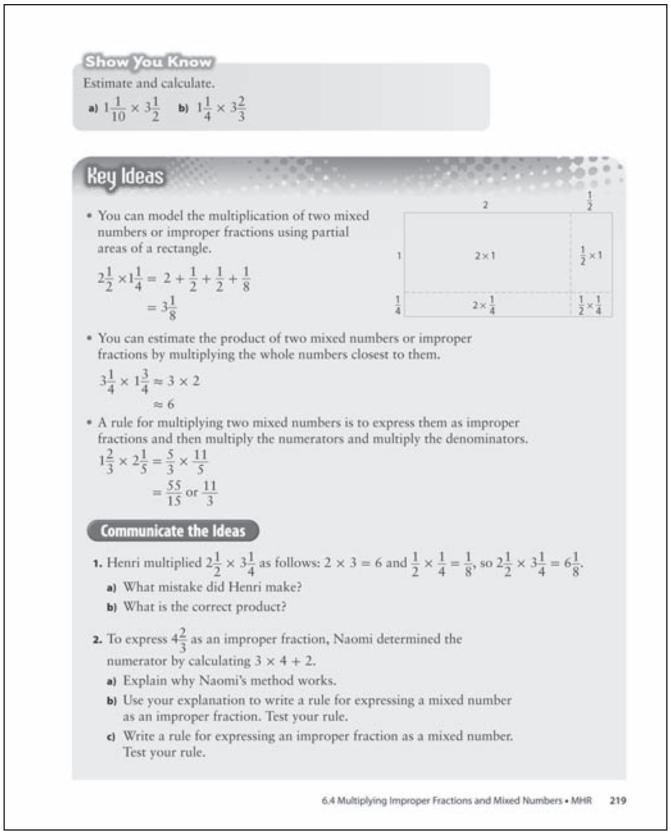
$1\frac{1}{2} + 1\frac{1}{2} + \frac{3}{8} = 3\frac{3}{8}$.

So, $2\frac{1}{4} \times 1\frac{1}{2} = 3\frac{3}{8}$.

Because the order of multiplication does not matter, the same multiplication could be carried out by representing $1\frac{1}{2}$ groups of $2\frac{1}{4}$.

Example 1

This example shows the use of an area model to determine the product of two mixed numbers. Emphasize that the separation of each dimension of the rectangle into a whole number and a proper



Show You Know

Estimate and calculate.

a) $1\frac{1}{10} \times 3\frac{1}{2}$ b) $1\frac{1}{4} \times 3\frac{2}{3}$

Key Ideas

- You can model the multiplication of two mixed numbers or improper fractions using partial areas of a rectangle.

$2\frac{1}{2} \times 1\frac{1}{4} = 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8}$
$= 3\frac{1}{8}$

- You can estimate the product of two mixed numbers or improper fractions by multiplying the whole numbers closest to them.

$3\frac{1}{4} \times 1\frac{3}{4} \approx 3 \times 2$
≈ 6

- A rule for multiplying two mixed numbers is to express them as improper fractions and then multiply the numerators and multiply the denominators.

$1\frac{2}{3} \times 2\frac{1}{5} = \frac{5}{3} \times \frac{11}{5}$
$= \frac{55}{15}$ or $\frac{11}{3}$

Communicate the Ideas

1. Henri multiplied $2\frac{1}{2} \times 3\frac{1}{4}$ as follows: $2 \times 3 = 6$ and $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$, so $2\frac{1}{2} \times 3\frac{1}{4} = 6\frac{1}{8}$.
   a) What mistake did Henri make?
   b) What is the correct product?
2. To express $4\frac{2}{3}$ as an improper fraction, Naomi determined the numerator by calculating $3 \times 4 + 2$.
   a) Explain why Naomi's method works.
   b) Use your explanation to write a rule for expressing a mixed number as an improper fraction. Test your rule.
   c) Write a rule for expressing an improper fraction as a mixed number. Test your rule.

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fraction is for convenience only. Another way might be to separate the length into five parts, each with a length of $\frac{1}{2}$, and to separate the width into seven parts, each with a width of $\frac{1}{4}$. The result would be 35 small rectangles, each with an area of $\frac{1}{8}$, resulting in a total area of $\frac{35}{1} \times \frac{1}{8}$, which is $\frac{35}{8}$ or $4\frac{3}{8}$.

If you wish to include a concrete model of the multiplication in this example, you could use fraction strips to represent $2\frac{1}{2}$ groups of $1\frac{3}{4}$, or $1\frac{3}{4}$ groups of $2\frac{1}{2}$. Refer to the Method 3 discussion in the Explore the Math section above.

Literacy Link Draw students' attention to the Literacy Link on page 218, which explains how a whole number can be written as a fraction with a denominator of 1. Have students show other whole numbers as fractions.

Example 2

This example shows the use of a rule to multiply two mixed numbers and presents a technique for estimating the product. You may wish to discuss that $4\frac{1}{2}$ is approximated as 5, rather than 4, in the estimate. Point out that, because $2\frac{1}{3}$ is approximated by decreasing it to 2, increasing $4\frac{1}{2}$ to 5 will give a better estimate of the product. You might ask students to explain the approximations they would use to estimate $4\frac{1}{2} \times 1\frac{2}{3}$. Stress the importance of comparing the estimate and the calculated product to check that the calculated value is reasonable.

The calculation method used in Example 2 is to multiply before writing the product in lowest terms. You may wish to include the following alternative method, which involves removing common factors before multiplying $\frac{9}{2} \times \frac{7}{3} = \frac{21}{2}$. You may wish to explain this method by factoring 9 to identify the common factor: $\frac{3 \times 3}{2} \times \frac{7}{3} = \frac{\cancel{3} \times 3}{2} \times \frac{7}{\cancel{3}} = \frac{3 \times 7}{2} = \frac{21}{2}$.

Meeting Student Needs

- When working with the area model, some students may benefit from using coloured pencils to outline each size of rectangle. They could use one colour for the rectangles that represent one whole, and separate colours for the rectangles that represent various other sizes of fractions. This would help them better visualize what they are doing as they calculate the area of each rectangle.
- To reactivate students' skills, use visuals and manipulatives to explore the concept of converting improper fractions to mixed numbers and vice versa. Allow students to explore the material, emphasizing the Key Terms used in the section. You may wish to change the order in which students explore the examples.

ELL

- Ensure that students understand the following terms by orally explaining them in context: *improper fractions*, *mixed numbers*, *section*, *refer*, and *laps on a track*. Have students add any new terms to their personal dictionary.
- Demonstrate the term *width* by running a finger along the width of the flag, saying that it is the width. Do the same for *length*. Have students demonstrate their understanding by asking them to point out the width and length of objects in the classroom.

Gifted and Enrichment

- Challenge students to research provincial and territorial flags of Canada. They may find the related Web Link that follows helpful. Consider having students use the information they find to develop and solve a problem involving the multiplication of improper fractions or mixed numbers.

Common Errors

- Some students may have difficulty in converting improper fractions and mixed numbers.

R_x Provide similar problems to those in the Literacy Link on page 217. Pose some questions, such as #4 and #5 in the Practise section. If necessary, have students model the numbers with manipulatives (such as fraction strips or fraction number lines) or diagrams to carry out the conversions. You may wish to hand out **Master 14 Fraction Strips** and/or **BLM 6–9 Fraction Number Lines**.

- Some students may not realize that they can apply the rule for multiplying fractions to multiplication involving a natural number.

R_x Emphasize that any natural number can be written with a denominator of 1.
Therefore, $5 \times \frac{3}{4} = \frac{5}{1} \times \frac{3}{4}$. Multiplying the numerators and multiplying the denominators gives a product of $\frac{15}{4}$ or $3\frac{3}{4}$.

- Some students may not consider whether their answers are reasonable.

R_x Point out the use of mental reasoning beside the solution to Example 2. Ask students to make up some other examples that illustrate this generalization. For example, the product of $1\frac{1}{2} \times 2\frac{1}{4}$ is $3\frac{3}{8}$, which is greater than $1\frac{1}{2}$ or $2\frac{1}{4}$. The product of $\frac{3}{2} \times \frac{5}{4}$ is $\frac{15}{8}$, which is greater than $\frac{3}{2}$ or $\frac{5}{4}$.

WWW Web Link

To find out more about provincial and territorial flags, go to www.mathlinks8.ca and follow the links. Note that the length : width ratio in these links is 4 : 1.

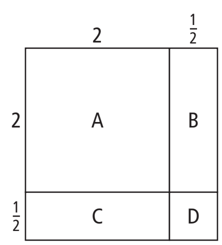
Answers

Explore the Math

1. a) Methods may vary. Example: Multiply the length by the width in each section. A: 1; B: $\frac{1}{2}$; C: $\frac{1}{2}$; D: $\frac{1}{4}$. The total area is the sum of the areas of all sections. $2\frac{1}{4}$

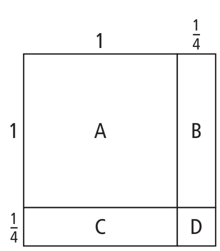
b) $1\frac{1}{2} \times 1\frac{1}{2} = 2\frac{1}{4}$

2. a) $6\frac{1}{4}$



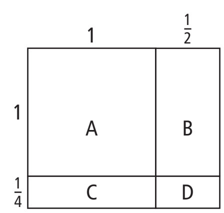
Area of A: 4
Area of B: 1
Area of C: 1
Area of D: $\frac{1}{4}$
Total: $6\frac{1}{4}$

b) $1\frac{9}{16}$



Area of A: 1
Area of B: $\frac{1}{4}$
Area of C: $\frac{1}{4}$
Area of D: $\frac{1}{16}$
Total: $1\frac{9}{16}$

c) $1\frac{7}{8}$



Area of A: 1
Area of B: $\frac{1}{2}$
Area of C: $\frac{1}{4}$
Area of D: $\frac{1}{8}$
Total: $1\frac{7}{8}$

3. a)

Multiplication of Mixed Numbers	Product Expressed as a Mixed Number	Multiplication of Improper Fractions	Product Expressed as an Improper Fraction
$1\frac{1}{2} \times 1\frac{1}{2}$	$2\frac{1}{4}$	$\frac{3}{2} \times \frac{3}{2}$	$\frac{9}{4}$
$2\frac{1}{2} \times 2\frac{1}{2}$	$6\frac{1}{4}$	$\frac{5}{2} \times \frac{5}{2}$	$\frac{25}{4}$
$1\frac{1}{4} \times 1\frac{1}{4}$	$1\frac{9}{16}$	$\frac{5}{4} \times \frac{5}{4}$	$\frac{25}{16}$
$1\frac{1}{2} \times 1\frac{1}{4}$	$1\frac{7}{8}$	$\frac{3}{2} \times \frac{5}{4}$	$\frac{15}{8}$

b) Answers will vary. Example: Multiply the numerators and multiply the denominators.

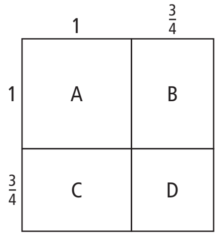
c) The rule is the same because to multiply two proper fractions, you multiply the numerators and multiply the denominators.

4. Answers will vary. Example: To use a diagram, divide a rectangle into sections indicated by each mixed number. Find the area of each section. The sum of the sections is the answer.

Answers

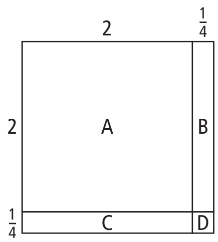
Show You Know: Example 1

a) $3\frac{1}{16}$



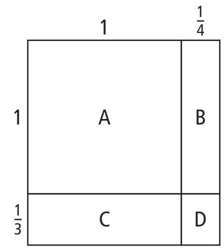
Area of A: 1
Area of B: $\frac{3}{4}$
Area of C: $\frac{3}{4}$
Area of D: $\frac{9}{16}$
Total: $3\frac{1}{16}$

b) $5\frac{1}{16}$



Area of A: 4
Area of B: $\frac{1}{2}$
Area of C: $\frac{1}{2}$
Area of D: $\frac{1}{16}$
Total: $5\frac{1}{16}$

c) $1\frac{2}{3}$



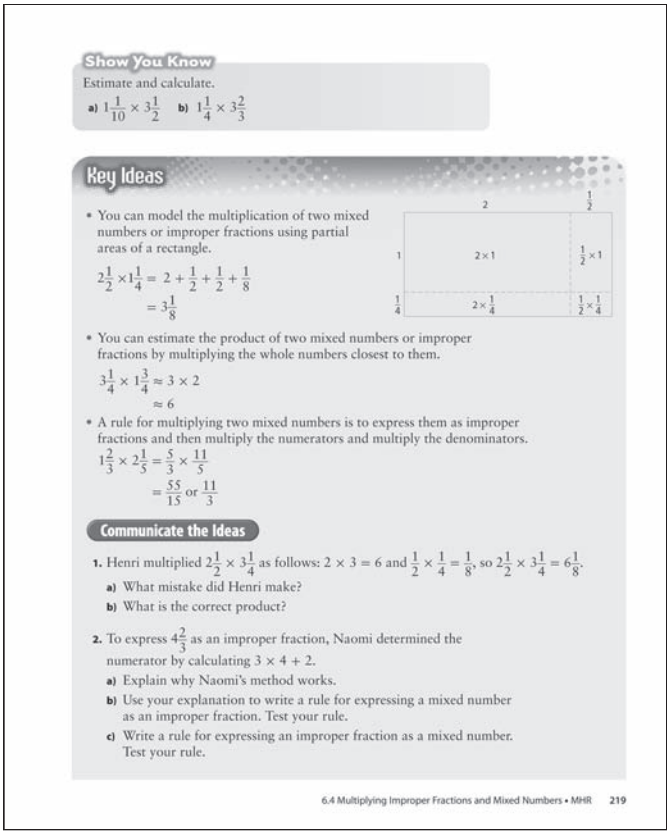
Area of A: 1
Area of B: $\frac{1}{4}$
Area of C: $\frac{1}{3}$
Area of D: $\frac{1}{12}$
Total: $1\frac{2}{3}$

Show You Know: Example 2

a) Estimates will vary. Example: 3; Answer: $3\frac{17}{20}$

b) Estimates will vary. Example: 4; Answer: $4\frac{7}{12}$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize their conclusions.	• Students who need assistance with #4 may find it helpful to refer back to #1. They may find using the same model approach beneficial. They should also refer back to the Literacy Link on page 217, which refers to converting improper fractions and mixed numbers. • Some students may be familiar with Punnett squares from Science and wish to use these for multiplying their fractions.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may need to coach students through the problem. Assist them in setting up the model for part a), and have them explain the multiplication of each part of the square to you. Have them complete parts b) and c) on their own. Check back frequently. • Students may benefit from using **BLM 6–8 Rectangles** to help determine each product.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from being coached through the rule for changing mixed numbers to improper fractions. • Some students may benefit from using a model to solve the problem and obtain an answer first, then using multiplication of improper fractions, and then comparing the results.



Show You Know

Estimate and calculate.

a) $1\frac{1}{10} \times 3\frac{1}{2}$ b) $1\frac{1}{4} \times 3\frac{2}{3}$

Key Ideas

- You can model the multiplication of two mixed numbers or improper fractions using partial areas of a rectangle.

$$2\frac{1}{2} \times 1\frac{1}{4} = 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8}$$
$$= 3\frac{1}{8}$$

	2	$\frac{1}{2}$
1	2×1	$\frac{1}{2} \times 1$
$\frac{1}{4}$	$2 \times \frac{1}{4}$	$\frac{1}{2} \times \frac{1}{4}$

- You can estimate the product of two mixed numbers or improper fractions by multiplying the whole numbers closest to them.

$$3\frac{1}{4} \times 1\frac{3}{4} \approx 3 \times 2$$
$$\approx 6$$

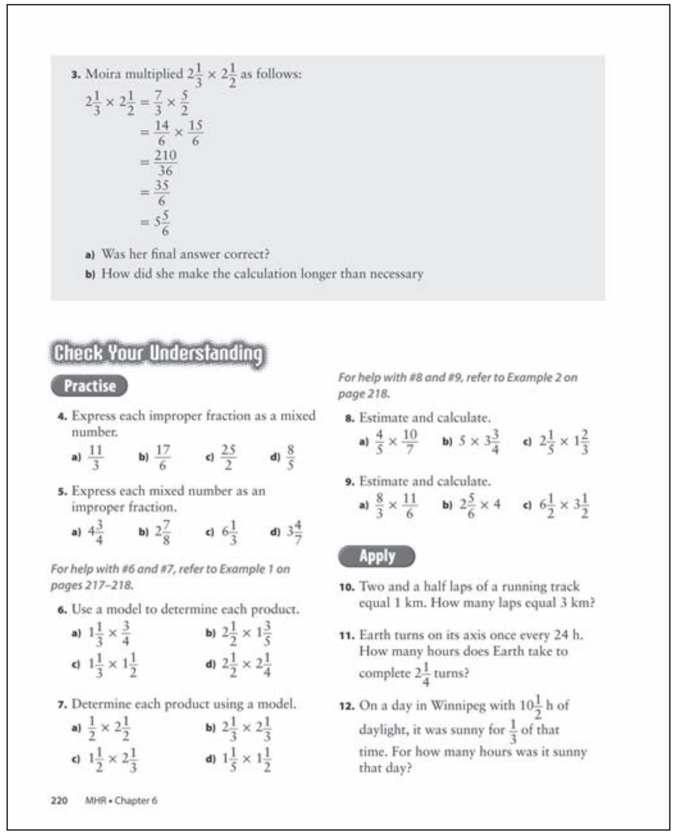
- A rule for multiplying two mixed numbers is to express them as improper fractions and then multiply the numerators and multiply the denominators.

$$1\frac{2}{3} \times 2\frac{1}{5} = \frac{5}{3} \times \frac{11}{5}$$
$$= \frac{55}{15} \text{ or } \frac{11}{3}$$

Communicate the Ideas

1. Henri multiplied $2\frac{1}{2} \times 3\frac{1}{4}$ as follows: $2 \times 3 = 6$ and $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$, so $2\frac{1}{2} \times 3\frac{1}{4} = 6\frac{1}{8}$.
   a) What mistake did Henri make?
   b) What is the correct product?
2. To express $4\frac{2}{3}$ as an improper fraction, Naomi determined the numerator by calculating $3 \times 4 + 2$.
   a) Explain why Naomi's method works.
   b) Use your explanation to write a rule for expressing a mixed number as an improper fraction. Test your rule.
   c) Write a rule for expressing an improper fraction as a mixed number. Test your rule.

6.4 Multiplying Improper Fractions and Mixed Numbers • MHR 219



3. Moira multiplied $2\frac{1}{3} \times 2\frac{1}{2}$ as follows:

$$2\frac{1}{3} \times 2\frac{1}{2} = \frac{7}{3} \times \frac{5}{2}$$
$$= \frac{14}{6} \times \frac{15}{6}$$
$$= \frac{210}{36}$$
$$= \frac{35}{6}$$
$$= 5\frac{5}{6}$$

   a) Was her final answer correct?
   b) How did she make the calculation longer than necessary

Check Your Understanding

Practise

4. Express each improper fraction as a mixed number.
   a) $\frac{11}{3}$ b) $\frac{17}{6}$ c) $\frac{25}{2}$ d) $\frac{8}{5}$
5. Express each mixed number as an improper fraction.
   a) $4\frac{3}{4}$ b) $2\frac{7}{8}$ c) $6\frac{1}{3}$ d) $3\frac{4}{7}$

For help with #6 and #7, refer to Example 1 on pages 217–218.

6. Use a model to determine each product.
   a) $1\frac{1}{3} \times \frac{3}{4}$ b) $2\frac{1}{2} \times 1\frac{3}{5}$
   c) $1\frac{1}{3} \times 1\frac{1}{2}$ d) $2\frac{1}{2} \times 2\frac{1}{4}$
7. Determine each product using a model.
   a) $\frac{1}{2} \times 2\frac{1}{2}$ b) $2\frac{1}{3} \times 2\frac{1}{3}$
   c) $1\frac{1}{2} \times 2\frac{1}{3}$ d) $1\frac{1}{5} \times 1\frac{1}{2}$

For help with #8 and #9, refer to Example 2 on page 218.

8. Estimate and calculate.
   a) $\frac{4}{5} \times \frac{10}{7}$ b) $5 \times 3\frac{3}{4}$ c) $2\frac{1}{5} \times 1\frac{2}{3}$
9. Estimate and calculate.
   a) $\frac{8}{3} \times \frac{11}{6}$ b) $2\frac{5}{6} \times 4$ c) $6\frac{1}{2} \times 3\frac{1}{2}$

Apply

10. Two and a half laps of a running track equal 1 km. How many laps equal 3 km?
11. Earth turns on its axis once every 24 h. How many hours does Earth take to complete $2\frac{1}{4}$ turns?
12. On a day in Winnipeg with $10\frac{1}{2}$ h of daylight, it was sunny for $\frac{1}{3}$ of that time. For how many hours was it sunny that day?

220 MHR • Chapter 6

Key Ideas

The Key Ideas summarize the area and symbolic methods for determining the product of two mixed numbers or improper fractions, and a method for estimating the product. Students could prepare their own list of Key Ideas and include it in their chapter Foldable, especially if they have used other approaches (e.g., modelling with fraction strips or grid paper, and removing common factors before multiplying).

Communicate the Ideas

These questions allow students to explain the multiplication of two mixed numbers. In #1, students consider an error in the application of a rule and then determine the correct product, either diagrammatically or symbolically. You may wish to ask students which method they chose for answering part b), and why they chose it.

In #2, students write rules for expressing a mixed number as an improper fraction and for expressing an improper fraction as a mixed number.

In #3, students consider an unnecessarily long symbolic method for multiplying two mixed numbers. After students have identified the unnecessary steps in the solution provided, you might ask why they think Moira included these steps. (A possible answer is that she was thinking of the steps necessary for addition.)

If you introduced the method of removing common factors before multiplying in Example 2, you might pose the following question: To calculate $\frac{8}{5} \times \frac{10}{7}$, do you prefer to remove common factors before or after you multiply? Explain.

Meeting Student Needs

- Some students may benefit from using **BLM 6–8 Rectangles** to model #1.
- If you included the use of fraction strips and grid paper as alternative methods in this section, you might pose the following questions:
  – Suppose you were asked to determine $1\frac{1}{2} \times 2\frac{1}{3}$ using manipulatives or a diagram. Which method would you choose? Explain why.
  – Avi and Barb both determined $2\frac{1}{2} \times 1\frac{1}{3}$ correctly using fraction strips.
    a) Avi started by using strips to represent $1\frac{1}{3}$. What did he do next?
    b) Barb started by using strips to represent $2\frac{1}{2}$. What did she do next?
    c) Explain why both methods gave the correct product.

ELL

- Students who do not have a lot of facility with the English language might benefit from doing the following for the Communicate the Ideas questions:
  – For #1, have students use visuals and numbers to show the correct product. They can share orally or point out the error in Henri's calculation.
  – For #2, have them test Naomi's rule by using manipulatives to verify that $4\frac{2}{3}$ is indeed $\frac{14}{3}$, and then test that method on other mixed numbers.
  – For #3, have them do the calculation using a method of their choice, compare their answer to Moira's, and then orally discuss or point out the differences.

Common Errors

- Some students may have difficulty in writing rules for expressing a mixed number as an improper fraction and for expressing an improper fraction as a mixed number.

R_x Use manipulatives or diagrams to demonstrate that the rules represent conversions using parts of a whole. If students continue to have difficulty in understanding the rules, or if they make errors in applying them, stress that using the rules can save time but it is not essential. Students can continue to perform conversions by reasoning with parts of a whole and, if necessary, by using concrete or semi-concrete models.

Answers

Communicate the Ideas

1. a) He multiplied the whole numbers and added that product to the product of the fractions, instead of changing the numbers to improper fractions.

b) $8\frac{1}{8}$

2. a) Answers will vary. Example: There are four $\frac{3}{3}$s and one $\frac{2}{3}$ in $4\frac{2}{3}$.

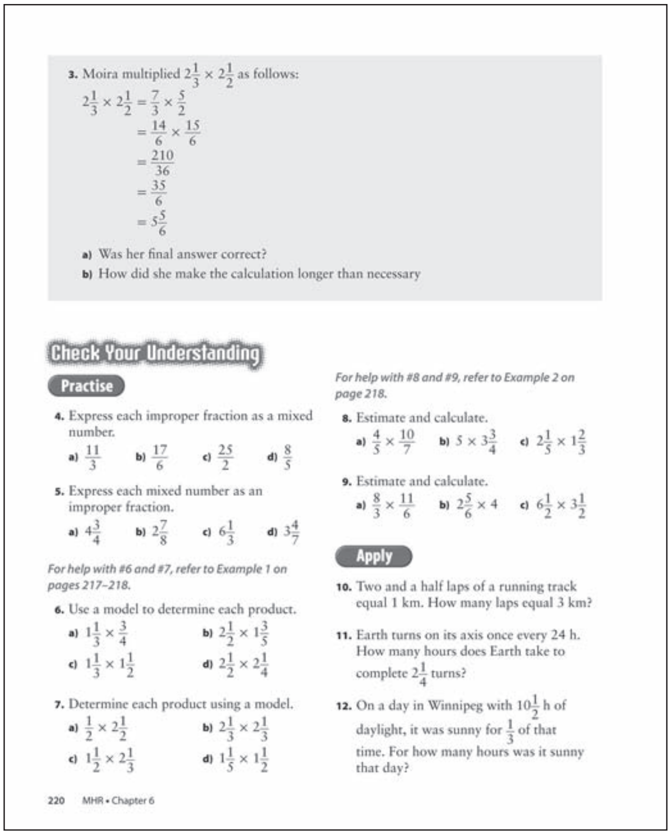
b) A rule is to multiply the denominator by the whole number and add the numerator. Place this number over the denominator.
$7\frac{1}{2} = \frac{(7 \times 2 + 1)}{2} = \frac{15}{2}$

c) A rule is to divide the numerator by the denominator. This quotient is the whole number part of the mixed number. Place the remainder of the quotient over the denominator.
$\frac{17}{3} = 17 \div 3 = 5\frac{2}{3}$

3. a) Yes

b) She did not need to change each fraction into an equivalent form with a common denominator.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to #3.	• Students who need assistance with #1 may benefit from referring back to Example 1 or 2. Encourage students to model the problem and verbalize their thinking. It may be helpful to have them estimate their answer first. • Check each student's response to #2 before discussing the solution as a class, since it is a key question for later understanding. It sets the groundwork for a rule to change mixed fractions to improper fractions. • Consider having students exchange their work in #2 with a classmate to check for errors and suggest improvements. You may wish to provide students with **Master 2 Two Stars and One Wish** for recording their feedback. • The typical approach students take is shown in #3. Making sure that students understand this process is important. It may be necessary to provide similar problems to check for understanding before allowing students to move forward.



3. Moira multiplied $2\frac{1}{3} \times 2\frac{1}{2}$ as follows:

$$\begin{aligned} 2\tfrac{1}{3} \times 2\tfrac{1}{2} &= \tfrac{7}{3} \times \tfrac{5}{2} \\ &= \tfrac{14}{6} \times \tfrac{15}{6} \\ &= \tfrac{210}{36} \\ &= \tfrac{35}{6} \\ &= 5\tfrac{5}{6} \end{aligned}$$

a) Was her final answer correct?
b) How did she make the calculation longer than necessary

Check Your Understanding

Practise

4. Express each improper fraction as a mixed number.
a) $\frac{11}{3}$ b) $\frac{17}{6}$ c) $\frac{25}{2}$ d) $\frac{8}{5}$

5. Express each mixed number as an improper fraction.
a) $4\frac{3}{4}$ b) $2\frac{7}{8}$ c) $6\frac{1}{3}$ d) $3\frac{4}{7}$

For help with #6 and #7, refer to Example 1 on pages 217–218.

6. Use a model to determine each product.
a) $1\frac{1}{3} \times \frac{3}{4}$ b) $2\frac{1}{2} \times 1\frac{3}{5}$
c) $1\frac{1}{3} \times 1\frac{1}{2}$ d) $2\frac{1}{2} \times 2\frac{1}{4}$

7. Determine each product using a model.
a) $\frac{1}{2} \times 2\frac{1}{2}$ b) $2\frac{1}{3} \times 2\frac{1}{3}$
c) $1\frac{1}{2} \times 2\frac{1}{3}$ d) $1\frac{1}{5} \times 1\frac{1}{2}$

For help with #8 and #9, refer to Example 2 on page 218.

8. Estimate and calculate.
a) $\frac{4}{5} \times \frac{10}{7}$ b) $5 \times 3\frac{3}{4}$ c) $2\frac{1}{5} \times 1\frac{2}{3}$

9. Estimate and calculate.
a) $\frac{8}{3} \times \frac{11}{6}$ b) $2\frac{5}{6} \times 4$ c) $6\frac{1}{2} \times 3\frac{1}{2}$

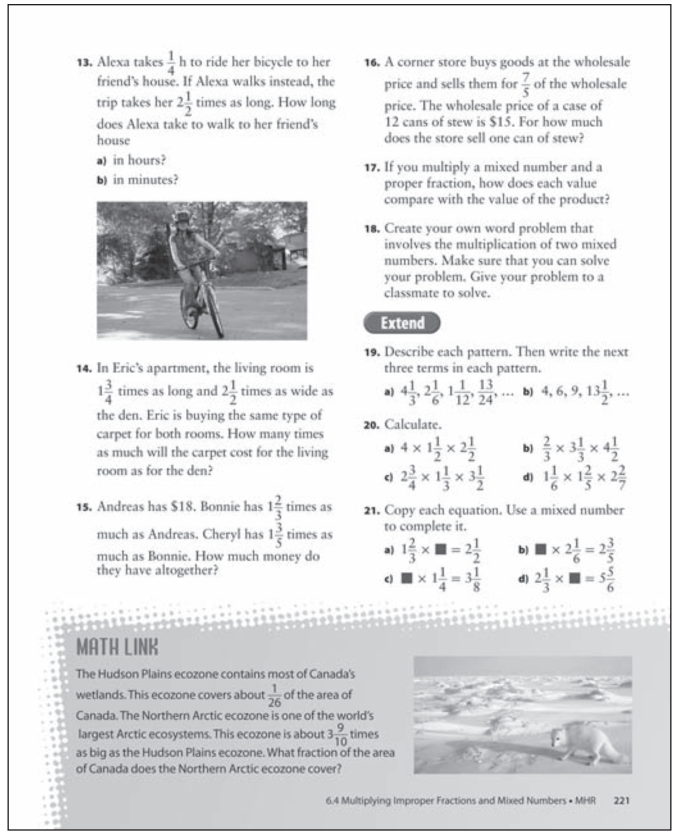
Apply

10. Two and a half laps of a running track equal 1 km. How many laps equal 3 km?

11. Earth turns on its axis once every 24 h. How many hours does Earth take to complete $2\frac{1}{4}$ turns?

12. On a day in Winnipeg with $10\frac{1}{2}$ h of daylight, it was sunny for $\frac{1}{3}$ of that time. For how many hours was it sunny that day?

220 MHR • Chapter 6



13. Alexa takes $\frac{1}{4}$ h to ride her bicycle to her friend's house. If Alexa walks instead, the trip takes her $2\frac{1}{2}$ times as long. How long does Alexa take to walk to her friend's house
a) in hours?
b) in minutes?

14. In Eric's apartment, the living room is $1\frac{3}{4}$ times as long and $2\frac{1}{2}$ times as wide as the den. Eric is buying the same type of carpet for both rooms. How many times as much will the carpet cost for the living room as for the den?

15. Andreas has \$18. Bonnie has $1\frac{2}{3}$ times as much as Andreas. Cheryl has $1\frac{3}{5}$ times as much as Bonnie. How much money do they have altogether?

16. A corner store buys goods at the wholesale price and sells them for $\frac{7}{5}$ of the wholesale price. The wholesale price of a case of 12 cans of stew is \$15. For how much does the store sell one can of stew?

17. If you multiply a mixed number and a proper fraction, how does each value compare with the value of the product?

18. Create your own word problem that involves the multiplication of two mixed numbers. Make sure that you can solve your problem. Give your problem to a classmate to solve.

Extend

19. Describe each pattern. Then write the next three terms in each pattern.
a) $4\frac{1}{3}, 2\frac{1}{6}, 1\frac{1}{12}, \frac{13}{24}, \ldots$ b) $4, 6, 9, 13\frac{1}{2}, \ldots$

20. Calculate.
a) $4 \times 1\frac{1}{2} \times 2\frac{1}{2}$ b) $\frac{2}{3} \times 3\frac{1}{3} \times 4\frac{1}{2}$
c) $2\frac{3}{4} \times 1\frac{1}{3} \times 3\frac{1}{2}$ d) $1\frac{1}{6} \times 1\frac{2}{5} \times 2\frac{2}{7}$

21. Copy each equation. Use a mixed number to complete it.
a) $1\frac{2}{3} \times ■ = 2\frac{1}{2}$ b) $■ \times 2\frac{1}{6} = 2\frac{3}{5}$
c) $■ \times 1\frac{1}{4} = 3\frac{1}{8}$ d) $2\frac{1}{3} \times ■ = 5\frac{5}{6}$

MATH LINK

The Hudson Plains ecozone contains most of Canada's wetlands. This ecozone covers about $\frac{1}{26}$ of the area of Canada. The Northern Arctic ecozone is one of the world's largest Arctic ecosystems. This ecozone is about $3\frac{9}{10}$ times as big as the Hudson Plains ecozone. What fraction of the area of Canada does the Northern Arctic ecozone cover?

6.4 Multiplying Improper Fractions and Mixed Numbers • MHR 221

Check Your Understanding

Practise

If you included the use of fraction strips and/or grid paper as alternative methods in Example 1, they can also be included as alternatives in #6 and #7. In this case, encourage students to compare the methods they used in #6 and #7 and to explain why they chose a particular method.

If you introduced the method of removing common factors before multiplying in Example 2, encourage students to use the method they prefer in #8 and #9 and to explain their preference.

Have students compare their estimates in #8 and #9. Check that they are approximating appropriately. Discuss especially #9c), where better estimates are obtained by approximating the product as 7×3 or 6×4 than as 6×3 or 7×4. Stress that estimation often involves judgments of this type.

Apply

Encourage students to think about the reasonableness of their answers and to use estimation to check them. For example, in #14, the product of $1\frac{3}{4} \times 2\frac{1}{2}$ must be greater than either $1\frac{3}{4}$ or $2\frac{1}{2}$. A reasonable estimate is 2×2, so the product should be close to 4.

In #12, students multiply a proper fraction and a mixed number. The product must be greater than the proper fraction and less than the mixed number (as students will have the opportunity to generalize in #12). In view of the estimation techniques that students have learned in the chapter, a reasonable estimate might be $\frac{1}{2} \times 10 = 5$. If students have developed facility in multiplying fractions and whole numbers and in identifying common factors, you may wish to demonstrate that the use of compatible numbers (e.g., $\frac{1}{3} \times 9$ or $\frac{1}{3} \times 12$) can give a better estimate.

Encourage students to share their solutions to #16, which involves division and multiplication. Some students may determine $\frac{7}{5} \times 15$ first and then divide the result, 21, by 12. Other students may determine

$15 \div 12$ and then multiply the result, $1\frac{1}{4}$, by $\frac{7}{5}$. You might ask students if they find one method easier to understand or to carry out than the other.

For #18, some students may use original contexts for their problems. Other students may adapt the wording of earlier problems (such as #12). You might encourage students to share their problems so that they are exposed to some that show originality.

Extend

For #20, you might encourage students to think about whether they can extend their rule for multiplying two improper fractions or mixed numbers, rather than multiplying two at a time. Question 21 is related to #15 in section 6.3. Refer to the notes for #15 in the previous section of this teacher's resource.

Math Link

This Math Link allows students to apply the multiplication of a mixed number and a proper fraction to data concerning Canada's ecozones.

Meeting Student Needs

- Some students may benefit from using **BLM 6–8 Rectangles** to model questions.
- Provide **BLM 6–14 Section 6.4 Extra Practice** to students who would benefit from more practice.

ELL

- For #14, ensure that students understand the term *times as much*.

Gifted and Enrichment

- If you introduced the method of removing common factors before multiplying in Example 2, you may wish to include the following problem: Determine the product. Explain your method.
  $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4} \times 1\frac{1}{5} \times 1\frac{1}{6} \ldots \times 1\frac{1}{25}$
  (Answers will vary. Examples:
  – The trick here is to write each mixed number as an improper fraction, which then makes the common factors obvious:
    $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \times \frac{7}{6} \times \ldots \times \frac{26}{25}$.
    The first numerator equals the second denominator, and so on. All you are left with after all the cancelling of equal pairs is a large number of 1s, the first denominator, 2, and the final numerator, 26. $26 \div 2 = 13$.
  – Using patterning, students can find that multiplying by each successive fraction increases the product by $\frac{1}{2}$. The initial fraction, $\frac{3}{2}$, plus 13 increments of $\frac{1}{2}$, give a final result of 13.)

Common Errors

- In a symbolic multiplication of a fraction by a natural number, some students may multiply both the numerator and denominator of the fraction by the natural number.

R_x Remind students to rewrite the natural number as a fraction with a denominator of 1 before applying the multiplication rule.

Answers

Math Link

$\frac{3}{20}$

Assessment	Supporting Learning
Assessment *for* Learning	
Practise Have students do #4a), c), #5b), d), #6c), d), #8b), and c). Students who have no problems with these questions can go on to the Apply questions.	• Provide additional coaching for #4 and #5 by referring students to Communicate the Ideas #2. Then, have students complete the remaining parts of each question on their own to check for student understanding. • Provide additional coaching with Example 1 to students who need help with #6c) and d). Coach students through these parts of the question and then assign the remaining parts of #6 or #7. Check back with students several times to make sure that they understand the concepts. • Provide additional coaching with Example 2 to students who need help with #8b) and c). Assign the remaining parts of #8 or #9 to check for student understanding before having them move on.
Math Link The Math Link on page 221 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 239.	• All students should complete this Math Link, since they will use these basic skills when they design and solve their own questions related to the ecozones in the Wrap It Up! • All students would benefit from completing this Math Link since multiplication of mixed numbers is a concept that is often more difficult for students, and the application of problem solving is good practice for all students. • Students who need help getting started could use **BLM 6–15 Section 6.4 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students complete the following statements: • To change a mixed number to an improper fraction, you … • When multiplying mixed numbers together, you …	• Many students may benefit from using a model to complete the statements. Encourage them to record their thinking. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with.