6.5 Dividing Fractions and Mixed Numbers

***MathLinks 8*, pages 222–229**

Suggested Timing
60–75 minutes

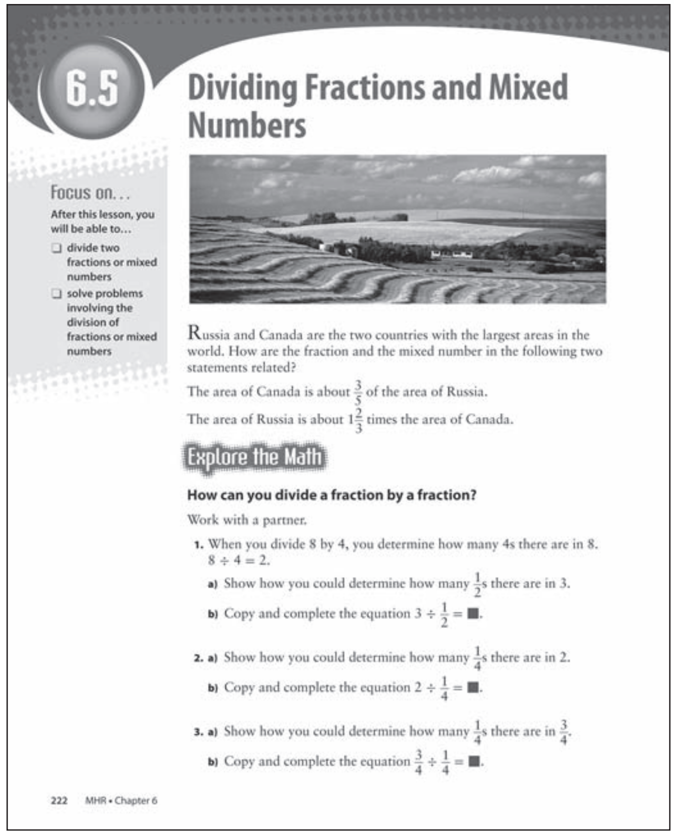
Materials
- ruler
- fraction strips (optional)
- transparent strips or diagrams (optional)
- dry erase markers (optional)
- coloured pencils (optional)

Blackline Masters
Master 14 Fraction Strips
BLM 6–3 Chapter 6 Warm-Up
BLM 6–8 Rectangles
BLM 6–16 Fraction Division Table
BLM 6–17 Section 6.5 Extra Practice
BLM 6–18 Section 6.5 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5a), c), 7a), c), 9, 11, Math Link
Typical	1–3, 5a), c), 7a), c), 9, 11–19, Math Link
Extension/Enrichment	1–4, 19–24


6.5 Dividing Fractions and Mixed Numbers

Focus on...
After this lesson, you will be able to...
- ☐ divide two fractions or mixed numbers
- ☐ solve problems involving the division of fractions or mixed numbers

Russia and Canada are the two countries with the largest areas in the world. How are the fraction and the mixed number in the following two statements related?

The area of Canada is about $\frac{3}{5}$ of the area of Russia.

The area of Russia is about $1\frac{2}{3}$ times the area of Canada.

Explore the Math

How can you divide a fraction by a fraction?

Work with a partner.

1. When you divide 8 by 4, you determine how many 4s there are in 8. $8 \div 4 = 2$.
   a) Show how you could determine how many $\frac{1}{2}$s there are in 3.
   b) Copy and complete the equation $3 \div \frac{1}{2} = ■$.
2. a) Show how you could determine how many $\frac{1}{4}$s there are in 2.
   b) Copy and complete the equation $2 \div \frac{1}{4} = ■$.
3. a) Show how you could determine how many $\frac{1}{4}$s there are in $\frac{3}{4}$.
   b) Copy and complete the equation $\frac{3}{4} \div \frac{1}{4} = ■$.

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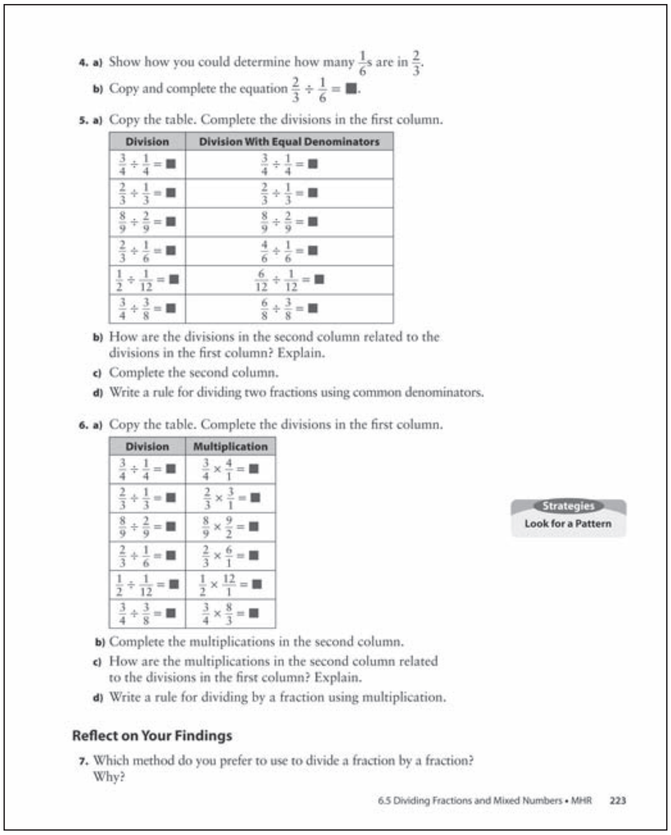
Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce material learned in previous sections.

Have students discuss the opening paragraph. Discuss with students how the two fractions are related. Students may benefit from you suggesting that they express $1\frac{2}{3}$ as an improper fraction. Students' explanations may vary. One possibility is for them to think in terms of ratios. If Canada : Russia is 3 : 5, then Russia : Canada is 5 : 3.

After students encounter the term *reciprocal* in Example 2, you may wish to return to this context and ask how $\frac{3}{5}$ and $1\frac{2}{3}$ are related ($1\frac{2}{3}$ or $\frac{5}{3}$ is the reciprocal of $\frac{3}{5}$).



4. a) Show how you could determine how many $\frac{1}{6}$s are in $\frac{2}{3}$.

b) Copy and complete the equation $\frac{2}{3} \div \frac{1}{6} = \blacksquare$.

5. a) Copy the table. Complete the divisions in the first column.

Division	Division With Equal Denominators
$\frac{3}{4} \div \frac{1}{4} = \blacksquare$	$\frac{3}{4} \div \frac{1}{4} = \blacksquare$
$\frac{2}{3} \div \frac{1}{3} = \blacksquare$	$\frac{2}{3} \div \frac{1}{3} = \blacksquare$
$\frac{8}{9} \div \frac{2}{9} = \blacksquare$	$\frac{8}{9} \div \frac{2}{9} = \blacksquare$
$\frac{2}{3} \div \frac{1}{6} = \blacksquare$	$\frac{4}{6} \div \frac{1}{6} = \blacksquare$
$\frac{1}{2} \div \frac{1}{12} = \blacksquare$	$\frac{6}{12} \div \frac{1}{12} = \blacksquare$
$\frac{3}{4} \div \frac{3}{8} = \blacksquare$	$\frac{6}{8} \div \frac{3}{8} = \blacksquare$

b) How are the divisions in the second column related to the divisions in the first column? Explain.

c) Complete the second column.

d) Write a rule for dividing two fractions using common denominators.

6. a) Copy the table. Complete the divisions in the first column.

Division	Multiplication
$\frac{3}{4} \div \frac{1}{4} = \blacksquare$	$\frac{3}{4} \times \frac{4}{1} = \blacksquare$
$\frac{2}{3} \div \frac{1}{3} = \blacksquare$	$\frac{2}{3} \times \frac{3}{1} = \blacksquare$
$\frac{8}{9} \div \frac{2}{9} = \blacksquare$	$\frac{8}{9} \times \frac{9}{2} = \blacksquare$
$\frac{2}{3} \div \frac{1}{6} = \blacksquare$	$\frac{2}{3} \times \frac{6}{1} = \blacksquare$
$\frac{1}{2} \div \frac{1}{12} = \blacksquare$	$\frac{1}{2} \times \frac{12}{1} = \blacksquare$
$\frac{3}{4} \div \frac{3}{8} = \blacksquare$	$\frac{3}{4} \times \frac{8}{3} = \blacksquare$

Strategies
Look for a Pattern

b) Complete the multiplications in the second column.

c) How are the multiplications in the second column related to the divisions in the first column? Explain.

d) Write a rule for dividing by a fraction using multiplication.

Reflect on Your Findings

7. Which method do you prefer to use to divide a fraction by a fraction? Why?

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Explore the Math

Students use diagrams to develop rules for dividing a fraction by a fraction.

Method 1 Have students work with a partner. Challenge them to develop diagrams or manipulatives to model the division statements in #1.

In #5, students are organizing data that will help them identify patterns. Some students may benefit from using a copy of the table in #5a), which is provided on **BLM 6–16 Fraction Division Table**. Make sure that the results recorded in the table are correct before students attempt to make generalizations.

When working on #5b), you may wish to draw students' attention to the last three rows of the table, where the division questions have been restated using common denominators. This was not necessary in the first three rows because the fractions already had common denominators. You may need to help some students analyse the difference by asking such questions as the following:

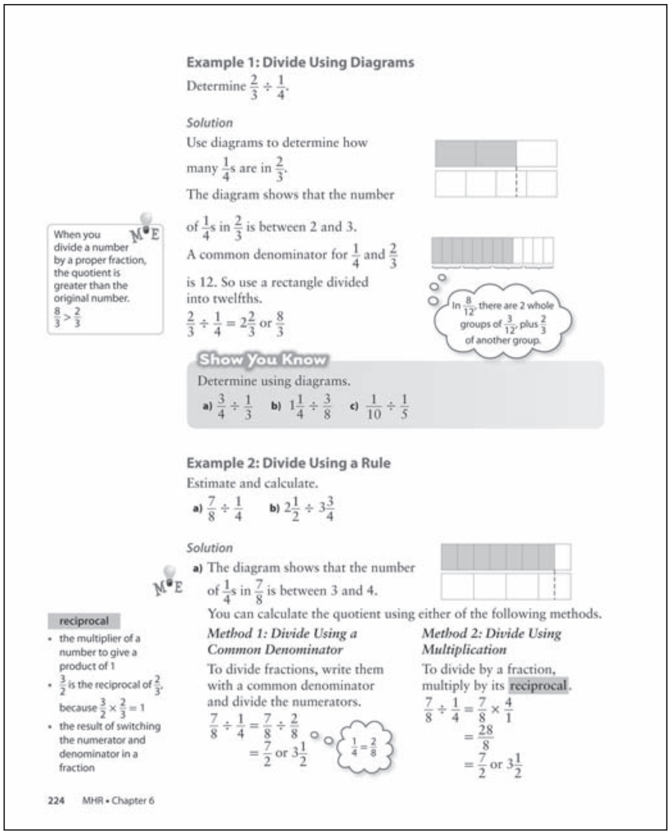
- Let's compare the fractions here row by row. How do the division statements in the first row compare? the second row? the third row? the fourth row?
- What is different about the division questions in the fourth row?
- What might be the same?
- How can you test that?
- What do you notice about the division statements in the fifth and sixth rows?
- Why do you think this change was made?
- How might such a change help you to divide these fractions?
- If two fractions have the same denominator, how can you divide them?
- How can you test your idea?
- What method will you use to verify the answers from your test?
- How many times do you think you need to test your idea before you decide that it works?
- What does your test suggest about your idea?
- Does your idea need revising? If so, how can you revise it?

The rule that emerges more obviously from the use of diagrams is not the common rule involving multiplication by the reciprocal, but rather a rule based on the use of common denominators. The latter rule is therefore established first (in #5). Stress that the divisions in the second column of the table are the same as in the first column. The final three divisions have been rewritten in the second column so that the dividend and the divisor have the same denominator. After students have completed the first column, they can record the same quotients in the second column. The divisions in the second column can be used to establish that if the two denominators are the same, the quotient of two fractions is the quotient of their numerators.

In #6, students are again organizing data that will help them identify patterns. Some students may benefit from using a copy of the table in #6a), which is provided on **BLM 6–16 Fraction Division Table**. Again, make sure that the results recorded in the table are correct before students attempt to make generalizations.

When students are doing the analysis for #6c), you may wish to ask questions such as the following:

- Look at the question in the first column of row 1. Now, look at the question in the second column. What is similar about each question?
- What is different?
- Do these same similarities and differences follow for the other rows in this table?
- How is the second fraction in each question in column 1 related to the second fraction in each question in column 2? (Students may not know the term *reciprocal*, but should notice that the fractions



Example 1: Divide Using Diagrams

Determine $\frac{2}{3} \div \frac{1}{4}$.

Solution

Use diagrams to determine how many $\frac{1}{4}$s are in $\frac{2}{3}$.

The diagram shows that the number of $\frac{1}{4}$s in $\frac{2}{3}$ is between 2 and 3.

A common denominator for $\frac{1}{4}$ and $\frac{2}{3}$ is 12. So use a rectangle divided into twelfths.

$\frac{2}{3} \div \frac{1}{4} = 2\frac{2}{3}$ or $\frac{8}{3}$

When you divide a number by a proper fraction, the quotient is greater than the original number. $\frac{8}{3} > \frac{2}{3}$

In $\frac{8}{12}$, there are 2 whole groups of $\frac{3}{12}$, plus $\frac{2}{3}$ of another group.

Show You Know

Determine using diagrams.

a) $\frac{3}{4} \div \frac{1}{3}$ b) $1\frac{1}{4} \div \frac{3}{8}$ c) $\frac{1}{10} \div \frac{1}{5}$

Example 2: Divide Using a Rule

Estimate and calculate.

a) $\frac{7}{8} \div \frac{1}{4}$ b) $2\frac{1}{2} \div 3\frac{3}{4}$

Solution

a) The diagram shows that the number of $\frac{1}{4}$s in $\frac{7}{8}$ is between 3 and 4.

You can calculate the quotient using either of the following methods.

Method 1: Divide Using a Common Denominator

To divide fractions, write them with a common denominator and divide the numerators.

$\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \div \frac{2}{8} = \frac{7}{2}$ or $3\frac{1}{2}$

$\frac{1}{4} = \frac{2}{8}$

Method 2: Divide Using Multiplication

To divide by a fraction, multiply by its reciprocal.

$\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \times \frac{4}{1} = \frac{28}{8} = \frac{7}{2}$ or $3\frac{1}{2}$

reciprocal
- the multiplier of a number to give a product of 1
- $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$, because $\frac{3}{2} \times \frac{2}{3} = 1$
- the result of switching the numerator and denominator in a fraction

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use the same numerals, except in opposite parts of the fraction. For example, the numeral that is the numerator in column 1 is the denominator in column 2.)

- How are the answers in the first column related to the answers in the second column?
- Explain how your observations about the contents of each column might assist you in dividing two fractions.
- How could you test your idea?
- Can you think of a way to generalize your idea to develop a strategy for dividing two fractions?
- Share your idea with a friend. Can the friend use your idea to divide two fractions?
- What method will you use to verify the answers from your test?

To complete #6, students must multiply the fractions in the second column of the table by applying their knowledge from section 6.4. The fact that the products in the second column equal the quotients in the first column can be used to establish the rule involving multiplication by the reciprocal.

By the end of the Explore the Math, students will have developed and tested two methods for dividing fractions. Discuss as a class which method individual students prefer, and why.

Method 2 Have students model the divisions using manipulatives, such as fraction strips. You may wish to hand out **Master 14 Fraction Strips**. If students have already used fraction strips to multiply fractions and mixed numbers, they will be familiar with the use of common denominators when working with fraction strips. The fraction-strip method for dividing fractions looks similar to the use of diagrams.

For example, here is a method for modelling $\frac{2}{3} \div \frac{1}{6}$ in #4a): Use a fraction strip to represent $\frac{2}{3}$. To divide by $\frac{1}{6}$, determine how many $\frac{1}{6}$s there are in $\frac{4}{6}$. The answer is 4. So, $\frac{2}{3} \div \frac{1}{6} = 4$.

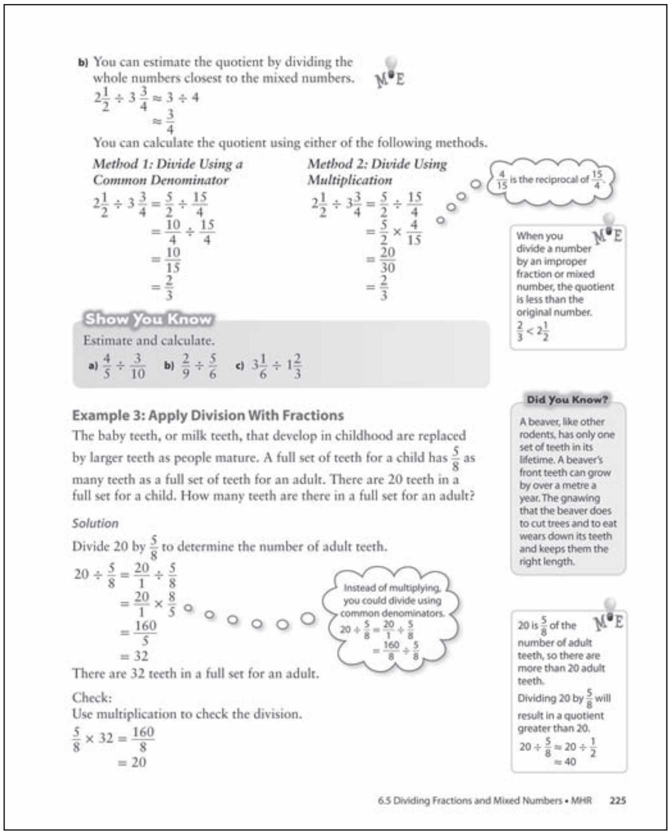
Example 1

This example shows the use of diagrams to divide two proper fractions. Before students determine a common denominator to complete the division, they can use the diagrams of a unit divided into thirds and a unit divided into quarters to estimate visually the quotient in relation to whole-number benchmarks. The use of brace brackets in the final diagram shows how many $\frac{3}{12}$s are in $\frac{8}{12}$ (i.e., how many $\frac{1}{4}$s are in $\frac{2}{3}$). Stress the importance of comparing the result to the estimate to check that the result is reasonable.

You may wish to have students model the division in this example using fraction strips. As pointed out in Method 2 for Explore the Math, above, the use of fraction strips resembles the use of diagrams.

Example 2

This example shows the use of two rules to divide two proper fractions and two mixed numbers and presents techniques for estimating the quotients. The estimation method in part a) is the same as in Example 1. The estimation method in part b) involves approximating mixed numbers to the nearest whole numbers, as for the multiplication of mixed numbers in section 6.4. You may wish to discuss why $2\frac{1}{2}$ is approximated as 3, rather than 2, in the estimate. Point out that because $3\frac{3}{4}$ is approximated by increasing it to 4, increasing $2\frac{1}{2}$ to 3 will give a better estimate of the quotient. You might ask students to explain the approximations they would use to estimate $2\frac{1}{2} \div 3\frac{1}{4}$. Stress the importance of comparing the estimate and the calculated product to check that the calculated value is reasonable.



b) You can estimate the quotient by dividing the whole numbers closest to the mixed numbers.

$$2\frac{1}{2} \div 3\frac{3}{4} \approx 3 \div 4$$
$$\approx \frac{3}{4}$$

You can calculate the quotient using either of the following methods.

Method 1: Divide Using a Common Denominator

$$2\frac{1}{2} \div 3\frac{3}{4} = \frac{5}{2} \div \frac{15}{4}$$
$$= \frac{10}{4} \div \frac{15}{4}$$
$$= \frac{10}{15}$$
$$= \frac{2}{3}$$

Method 2: Divide Using Multiplication

$$2\frac{1}{2} \div 3\frac{3}{4} = \frac{5}{2} \div \frac{15}{4}$$
$$= \frac{5}{2} \times \frac{4}{15}$$
$$= \frac{20}{30}$$
$$= \frac{2}{3}$$

$\frac{4}{15}$ is the reciprocal of $\frac{15}{4}$.

M E

When you divide a number by an improper fraction or mixed number, the quotient is less than the original number.
$\frac{2}{3} < 2\frac{1}{2}$

Show You Know

Estimate and calculate.

a) $\frac{4}{5} \div \frac{3}{10}$ **b)** $\frac{2}{9} \div \frac{5}{6}$ **c)** $3\frac{1}{6} \div 1\frac{2}{3}$

Example 3: Apply Division With Fractions

The baby teeth, or milk teeth, that develop in childhood are replaced by larger teeth as people mature. A full set of teeth for a child has $\frac{5}{8}$ as many teeth as a full set of teeth for an adult. There are 20 teeth in a full set for a child. How many teeth are there in a full set for an adult?

Solution

Divide 20 by $\frac{5}{8}$ to determine the number of adult teeth.

$$20 \div \frac{5}{8} = \frac{20}{1} \div \frac{5}{8}$$
$$= \frac{20}{1} \times \frac{8}{5}$$
$$= \frac{160}{5}$$
$$= 32$$

Instead of multiplying, you could divide using common denominators.
$20 \div \frac{5}{8} = \frac{20}{1} \div \frac{5}{8}$
$= \frac{160}{8} \div \frac{5}{8}$

There are 32 teeth in a full set for an adult.

Check:
Use multiplication to check the division.

$$\frac{5}{8} \times 32 = \frac{160}{8}$$
$$= 20$$

Did You Know?

A beaver, like other rodents, has only one set of teeth in its lifetime. A beaver's front teeth can grow by over a metre a year. The gnawing that the beaver does to cut trees and to eat wears down its teeth and keeps them the right length.

M E

20 is $\frac{5}{8}$ of the number of adult teeth, so there are more than 20 adult teeth.
Dividing 20 by $\frac{5}{8}$ will result in a quotient greater than 20.
$20 \div \frac{5}{8} \approx 20 \div \frac{1}{2}$
≈ 40

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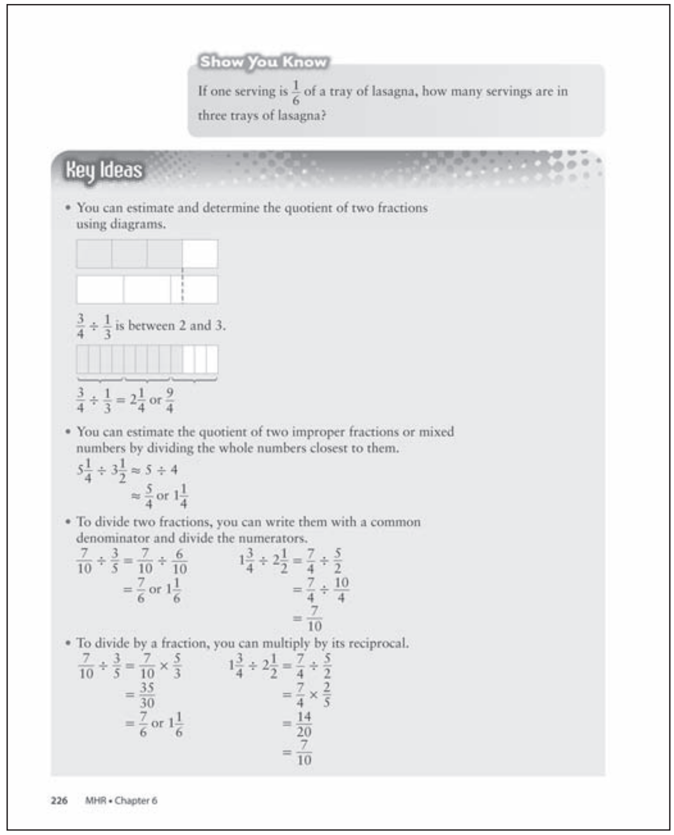

Show You Know

If one serving is $\frac{1}{6}$ of a tray of lasagna, how many servings are in three trays of lasagna?

Key Ideas

- You can estimate and determine the quotient of two fractions using diagrams.

$\frac{3}{4} \div \frac{1}{3}$ is between 2 and 3.

$\frac{3}{4} \div \frac{1}{3} = 2\frac{1}{4}$ or $\frac{9}{4}$

- You can estimate the quotient of two improper fractions or mixed numbers by dividing the whole numbers closest to them.

$$5\frac{1}{4} \div 3\frac{1}{2} \approx 5 \div 4$$
$$\approx \frac{5}{4} \text{ or } 1\frac{1}{4}$$

- To divide two fractions, you can write them with a common denominator and divide the numerators.

$$\frac{7}{10} \div \frac{3}{5} = \frac{7}{10} \div \frac{6}{10}$$
$$= \frac{7}{6} \text{ or } 1\frac{1}{6}$$

$$1\frac{3}{4} \div 2\frac{1}{2} = \frac{7}{4} \div \frac{5}{2}$$
$$= \frac{7}{4} \div \frac{10}{4}$$
$$= \frac{7}{10}$$

- To divide by a fraction, you can multiply by its reciprocal.

$$\frac{7}{10} \div \frac{3}{5} = \frac{7}{10} \times \frac{5}{3}$$
$$= \frac{35}{30}$$
$$= \frac{7}{6} \text{ or } 1\frac{1}{6}$$

$$1\frac{3}{4} \div 2\frac{1}{2} = \frac{7}{4} \div \frac{5}{2}$$
$$= \frac{7}{4} \times \frac{2}{5}$$
$$= \frac{14}{20}$$
$$= \frac{7}{10}$$

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The calculation method used in Method 1 of each part of Example 2 relies on the use of common denominators. Stress that when students use this method, no inversion of the divisor is involved. The calculation method used in Method 2 of each part of Example 2 relies on the use of multiplication. Stress that when students use this method, the divisor is inverted to become its reciprocal. You may wish to explain why this method works. For example,

for Method 2 of part a), $\frac{7}{8} \div \frac{1}{4} = \dfrac{\frac{7}{8}}{\frac{1}{4}}$

$$= \frac{\frac{7}{8} \times \frac{4}{1}}{\frac{1}{4} \times \frac{4}{1}}$$

$$= \frac{\frac{7}{8} \times \frac{4}{1}}{1}$$

$$= \frac{7}{8} \times \frac{4}{1}$$

Example 3

This example illustrates an application of the division of a whole number by a fraction. The division is completed using one of the rules. The possible use of the other rule is shown in a thought bubble. Remind students that any whole number can be written as an improper fraction with a denominator of 1 (e.g., 20 as $\frac{20}{1}$). You may wish to ask students why it is inconvenient to solve the problem in Example 3 by drawing rectangles instead of using a rule.

Stress the importance of checking the reasonableness of the answer. In addition to mental reasoning and estimation, the example also includes the use of the inverse operation, multiplication, to check the accuracy of the division.

Meeting Student Needs

- Some students may benefit from using a virtual manipulative to explore and practise dividing fractions using fraction number bar lines. See the related Web Link below.

ELL

- Ensure that students understand the following terms: *consistent*, *largest land area*, *baby teeth*, *childhood*, *replaced*, *larger teeth*, and *mature*.

Common Errors

- Some students may not grasp that the order of division of two different fractions affects the value of the quotient (unlike in multiplication, where the order of the terms does not matter).

R_x Have students practise modelling the division of two different fractions in both orders, so that they see the relationship between the two quotients (i.e., they are reciprocals of each other).

- Some students may confuse the two division rules.

R_x Encourage students to decide which rule they prefer and to use that rule exclusively.

- Some students may not consider whether their answers are reasonable.

R_x Point out the use of mental reasoning beside the solutions to Examples 1 and 2. Ask students to make up some other examples that illustrate the generalization for each example. For example, the quotient of $1 \div \frac{1}{2}$ is 2, which is greater than 1. The quotient of $1\frac{1}{4} \div 2\frac{1}{2}$ is $\frac{1}{2}$, which is less than $1\frac{1}{4}$.

WWW Web Link

For an interactive activity that students can use to practise division using fraction number bar lines, go to www.mathlinks8.ca and follow the links.

To find out more about teeth for any discussion related to Example 3 or the Did You Know? on page 225, including how to care for teeth, go to www.mathlinks8.ca and follow the links.

Answers

Explore the Math

1. a) Answer will vary. Example: 

b) $3 \div \frac{1}{2} = 6$

2. a) Answer will vary. Example: 

b) $2 \div \frac{1}{4} = 8$

3. a) Answer will vary. Example: 

b) $\frac{3}{4} \div \frac{1}{4} = 3$

4. a) Answer will vary. Example: 

b) $\frac{2}{3} \div \frac{1}{6} = 4$ = 

5. a)

Division
$\frac{3}{4} \div \frac{1}{4} = 3$
$\frac{2}{3} \div \frac{1}{3} = 2$
$\frac{8}{9} \div \frac{2}{9} = 4$
$\frac{2}{3} \div \frac{1}{6} = 4$
$\frac{1}{2} \div \frac{1}{12} = 6$
$\frac{3}{4} \div \frac{3}{8} = 2$

b) Answers will vary. Example: The divisions are the same, except the fractions in the second column have common denominators.

c)

Division With Equal Denominators
$\frac{3}{4} \div \frac{1}{4} = 3$
$\frac{2}{3} \div \frac{1}{3} = 2$
$\frac{8}{9} \div \frac{2}{9} = 4$
$\frac{4}{6} \div \frac{1}{6} = 4$
$\frac{6}{12} \div \frac{1}{12} = 6$
$\frac{6}{8} \div \frac{3}{8} = 2$

d) A rule is to divide the numerators to get the numerator of the answer. The denominator of the answer is 1.

6. a) See table in #5a).

b)

Multiplication
$\frac{3}{4} \times \frac{4}{1} = 3$
$\frac{2}{3} \times \frac{3}{1} = 2$
$\frac{8}{9} \times \frac{9}{2} = 4$
$\frac{2}{3} \times \frac{6}{1} = 4$
$\frac{1}{2} \times \frac{12}{1} = 6$
$\frac{3}{4} \times \frac{3}{8} = 2$

c) Answers will vary. Example: They are equivalent.

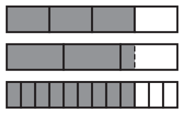
d) A rule is to switch the numerator and denominator of the fraction following the division sign, and then multiply.

7. Answers will vary.

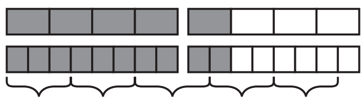
Answers

Show You Know: Example 1

a) $2\frac{1}{4}$

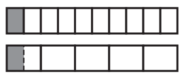


b) $3\frac{1}{3}$



$\frac{5}{4} \div \frac{3}{8} = 3\frac{1}{3}$

c) $\frac{1}{2}$



$\frac{1}{10} \div \frac{1}{5} = \frac{1}{2}$

Show You Know: Example 2

a) Estimates will vary. Example: 2. Answer: $2\frac{2}{3}$

b) Estimates will vary. Example: $\frac{1}{3}$. Answer: $\frac{4}{15}$

c) Estimates will vary. Example: $1\frac{1}{2}$. Answer: $1\frac{9}{10}$

Show You Know: Example 3

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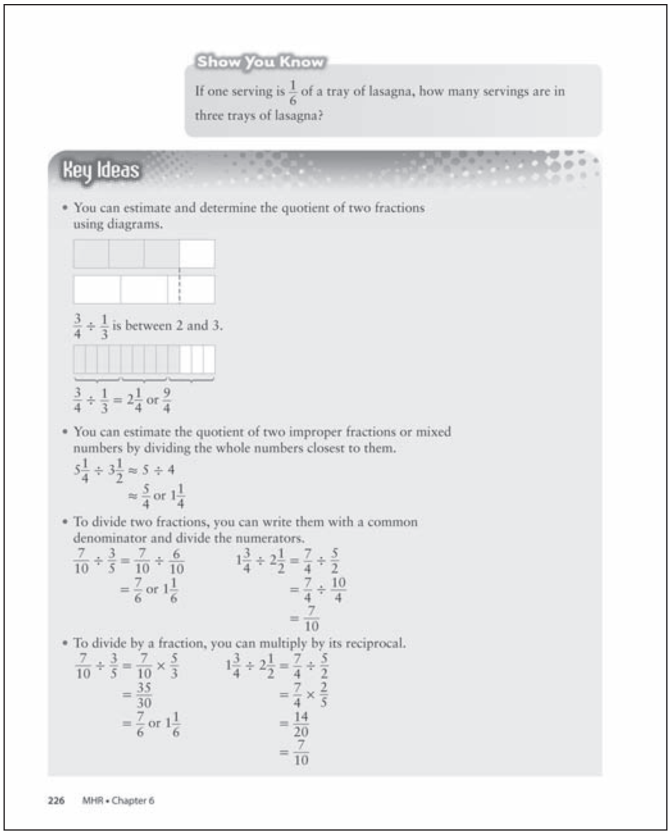
Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.	• Some students may benefit from recalling factors and divisibility rules. Having a chart available on their desk or in their chapter Foldable may be helpful. • Have students identify which method they prefer.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to use a diagram of their own choice. They need not necessarily use rectangles. • Some students may benefit from using **BLM 6–8 Rectangles** to help determine quotients. • Coach students through how many will be in each part. • Have students write out their thinking algebraically.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from using **BLM 6–8 Rectangles** to estimate quotients. • Have students identify which method is easier for them to understand. • Have students verbalize how they determine the total number of parts that will be in the answer (denominator). Clarify any misconceptions.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students identify which method is easier for them to understand. • Have them verbalize how they determine the total number of parts that will be in the answer (denominator). Clarify any misconceptions. • Have students write out their thinking algebraically.



Show You Know

If one serving is $\frac{1}{6}$ of a tray of lasagna, how many servings are in three trays of lasagna?

Key Ideas

- You can estimate and determine the quotient of two fractions using diagrams.

$\frac{3}{4} \div \frac{1}{3}$ is between 2 and 3.

$\frac{3}{4} \div \frac{1}{3} = 2\frac{1}{4}$ or $\frac{9}{4}$

- You can estimate the quotient of two improper fractions or mixed numbers by dividing the whole numbers closest to them.

$$5\frac{1}{4} \div 3\frac{1}{2} \approx 5 \div 4$$
$$\approx \frac{5}{4} \text{ or } 1\frac{1}{4}$$

- To divide two fractions, you can write them with a common denominator and divide the numerators.

$$\frac{7}{10} \div \frac{3}{5} = \frac{7}{10} \div \frac{6}{10} = \frac{7}{6} \text{ or } 1\frac{1}{6}$$
$$1\frac{3}{4} \div 2\frac{1}{2} = \frac{7}{4} \div \frac{5}{2} = \frac{7}{4} \div \frac{10}{4} = \frac{7}{10}$$

- To divide by a fraction, you can multiply by its reciprocal.

$$\frac{7}{10} \div \frac{3}{5} = \frac{7}{10} \times \frac{5}{3} = \frac{35}{30} = \frac{7}{6} \text{ or } 1\frac{1}{6}$$
$$1\frac{3}{4} \div 2\frac{1}{2} = \frac{7}{4} \div \frac{5}{2} = \frac{7}{4} \times \frac{2}{5} = \frac{14}{20} = \frac{7}{10}$$

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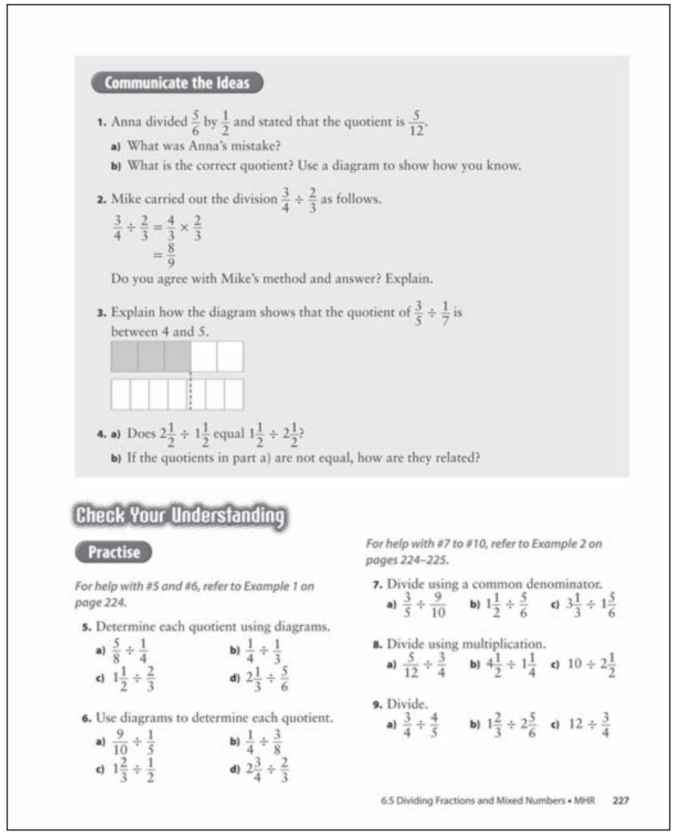


Communicate the Ideas

1. Anna divided $\frac{5}{6}$ by $\frac{1}{2}$ and stated that the quotient is $\frac{5}{12}$.
   a) What was Anna's mistake?
   b) What is the correct quotient? Use a diagram to show how you know.
2. Mike carried out the division $\frac{3}{4} \div \frac{2}{3}$ as follows.
   $$\frac{3}{4} \div \frac{2}{3} = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$
   Do you agree with Mike's method and answer? Explain.
3. Explain how the diagram shows that the quotient of $\frac{3}{5} \div \frac{1}{7}$ is between 4 and 5.
4. a) Does $2\frac{1}{2} \div 1\frac{1}{2}$ equal $1\frac{1}{2} \div 2\frac{1}{2}$?
   b) If the quotients in part a) are not equal, how are they related?

Check Your Understanding

Practise

For help with #5 and #6, refer to Example 1 on page 224.

5. Determine each quotient using diagrams.
   a) $\frac{5}{8} \div \frac{1}{4}$ b) $\frac{1}{4} \div \frac{1}{3}$
   c) $1\frac{1}{2} \div \frac{2}{3}$ d) $2\frac{1}{3} \div \frac{5}{6}$
6. Use diagrams to determine each quotient.
   a) $\frac{9}{10} \div \frac{1}{5}$ b) $\frac{1}{4} \div \frac{3}{8}$
   c) $1\frac{2}{3} \div \frac{1}{2}$ d) $2\frac{3}{4} \div \frac{2}{3}$

For help with #7 to #10, refer to Example 2 on pages 224–225.

7. Divide using a common denominator.
   a) $\frac{3}{5} \div \frac{9}{10}$ b) $1\frac{1}{2} \div \frac{5}{6}$ c) $3\frac{1}{3} \div 1\frac{5}{6}$
8. Divide using multiplication.
   a) $\frac{5}{12} \div \frac{3}{4}$ b) $4\frac{1}{2} \div 1\frac{1}{4}$ c) $10 \div 2\frac{1}{2}$
9. Divide.
   a) $\frac{3}{4} \div \frac{4}{5}$ b) $1\frac{2}{3} \div 2\frac{5}{6}$ c) $12 \div \frac{3}{4}$

6.5 Dividing Fractions and Mixed Numbers • MHR 227

Key Ideas

The Key Ideas summarize the diagrammatic and symbolic methods for determining the quotient of two fractions or mixed numbers, and methods for estimating the quotient. Students could prepare their own list of Key Ideas and include it in their chapter Foldable, especially if they have used other approaches (e.g., modelling with fraction strips). Encourage students to develop their own examples.

Communicate the Ideas

These questions allow students to explain the division of two fractions or mixed numbers. In #1, students consider a case of confusion between division and multiplication, and then determine the correct quotient diagrammatically. You may wish to ask students how estimation alone would indicate that Anna's answer was incorrect.

In #2, students observe and correct an error in the application of a rule for division. You might encourage students to model the division diagrammatically, as well as apply the relevant rule correctly.

In #3, students explain the visual method for estimating the quotient of two fractions. You might ask why the method used for estimating the product of two proper fractions by approximating each one to 0, $\frac{1}{2}$, or 1 is *not* useful for the division in #3. (The fraction $\frac{1}{7}$ is approximately 0, but division by 0 is undefined.)

In #4a), students consider the fact that order matters in division (i.e., division is not commutative). In #4b), they determine that dividing in the wrong order gives the reciprocal of the correct quotient.

Meeting Student Needs

- Some students may benefit from using **BLM 6–8 Rectangles** to verify the quotient in #1.

Common Errors

- Some students may determine quotients incorrectly by multiplying without using the reciprocal.

R_x Encourage students to consider the reasonableness of their answers and to use multiplication to check them.

- Some students may write the reciprocal of a mixed number by inverting only the fractional part (e.g., writing $1\frac{2}{3}$ as $1\frac{3}{2}$ and obtaining a reciprocal of $\frac{5}{2}$ instead of $\frac{3}{5}$).

R_x Emphasize that the entire mixed number should be rewritten as an improper fraction (e.g., $1\frac{2}{3}$ as $\frac{5}{3}$) before students write the reciprocal. Explain that because the product of the mixed number and its reciprocal is 1, the reciprocal must be less than 1.

Answers

Communicate the Ideas

1. a) She multiplied the numerators and the denominators, but did not replace the second fraction with its reciprocal.

b) $1\frac{2}{3}$,

$\frac{5}{6} \div \frac{1}{2} = 1\frac{2}{3}$

2. Mike is incorrect. He should have replaced the second fraction with its reciprocal.

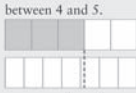
3. Answers will vary. Example: There are between four and five $\frac{1}{7}$s in $\frac{3}{5}$.

4. a) No **b)** They are reciprocals of each other.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to #3.	• Encourage students who need assistance with #1 to use a diagram that they are comfortable with. Have them verbalize their thinking. Encourage them to record their thinking. • Question 2 is linked to the rule for dividing fractions. Ensure students have a good understanding of this problem before allowing them to move on. • It may be beneficial to discuss the response to #3 as a whole class. This question is easier for students to do algebraically, but conceptually the diagram offers the potential for rich discussion with all learners.

Communicate the Ideas

1. Anna divided $\frac{5}{6}$ by $\frac{1}{2}$ and stated that the quotient is $\frac{5}{12}$.
   a) What was Anna's mistake?
   b) What is the correct quotient? Use a diagram to show how you know.
2. Mike carried out the division $\frac{3}{4} \div \frac{2}{3}$ as follows.
   $$\frac{3}{4} \div \frac{2}{3} = \frac{4}{3} \times \frac{2}{3}$$
   $$= \frac{8}{9}$$
   Do you agree with Mike's method and answer? Explain.
3. Explain how the diagram shows that the quotient of $\frac{3}{5} \div \frac{1}{7}$ is between 4 and 5.



4. a) Does $2\frac{1}{2} \div 1\frac{1}{2}$ equal $1\frac{1}{2} \div 2\frac{1}{2}$?
   b) If the quotients in part a) are not equal, how are they related?

Check Your Understanding

Practise

For help with #5 and #6, refer to Example 1 on page 224.

5. Determine each quotient using diagrams.
   a) $\frac{5}{8} \div \frac{1}{4}$ b) $\frac{1}{4} \div \frac{1}{3}$
   c) $1\frac{1}{2} \div \frac{2}{3}$ d) $2\frac{1}{3} \div \frac{5}{6}$
6. Use diagrams to determine each quotient.
   a) $\frac{9}{10} \div \frac{1}{5}$ b) $\frac{1}{4} \div \frac{3}{8}$
   c) $1\frac{2}{3} \div \frac{1}{2}$ d) $2\frac{3}{4} \div \frac{2}{3}$

For help with #7 to #10, refer to Example 2 on pages 224–225.

7. Divide using a common denominator.
   a) $\frac{3}{5} \div \frac{9}{10}$ b) $1\frac{1}{2} \div \frac{5}{6}$ c) $3\frac{1}{3} \div 1\frac{5}{6}$
8. Divide using multiplication.
   a) $\frac{5}{12} \div \frac{3}{4}$ b) $4\frac{1}{2} \div 1\frac{1}{4}$ c) $10 \div 2\frac{1}{2}$
9. Divide.
   a) $\frac{3}{4} \div \frac{4}{5}$ b) $1\frac{2}{3} \div 2\frac{5}{6}$ c) $12 \div \frac{3}{4}$

6.5 Dividing Fractions and Mixed Numbers • MHR 227

10. Divide.
    a) $1\frac{1}{12} \div 2\frac{1}{2}$ b) $\frac{8}{11} \div \frac{4}{5}$ c) $1\frac{3}{8} \div 2\frac{3}{4}$

Apply

For help with #11 to #13, refer to Example 3 on page 225.

11. In a comedy review, each performer has a $\frac{1}{4}$-h slot. How many performers are there in a 2-h show?
12. It takes $2\frac{1}{2}$ scoops of flour to make one cake. How many cakes do 15 scoops of flour make?
13. Three quarters of a can of apple juice fills six glasses. How many glasses will a whole can of apple juice fill?
14. An incandescent light bulb uses about $4\frac{1}{2}$ times as much energy as a compact fluorescent light bulb to produce the same amount of light. What fraction of the energy used by the incandescent bulb does the fluorescent light bulb use?

15. Shana and Zack painted their rooms using paint in cans of the same size. Shana used $1\frac{1}{2}$ cans of paint. Zack used $2\frac{3}{4}$ cans of paint. How many times as much paint did Zack use as Shana?
16. Of all the land on Earth, about $\frac{3}{10}$ is in Asia and about $\frac{3}{25}$ is in South America. How many times as big as South America is Asia?
17. The average wind speed in Calgary is $\frac{4}{5}$ of the average wind speed in Regina. The average wind speed in Calgary is 16 km/h. What is the average wind speed in Regina?
18. Use examples to explain your answer to each of the following.
    a) Can the reciprocal of a proper fraction be a proper fraction?
    b) Can the product of two proper fractions be greater than 1?
    c) Can the quotient of two proper fractions be greater than 1?
19. a) The world's longest river is the Nile in Africa, with a length of 6825 km. This is about $1\frac{5}{8}$ times a long as the Mackenzie River, which is Canada's longest river.
    a) How long is the Mackenzie River?
    b) The Mackenzie River is about $2\frac{1}{10}$ times as long as the Columbia River. How long is the Columbia River?

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Check Your Understanding

Practise

If you included the use of fraction strips as an alternative method in Example 1, it can also be included as an alternative in #5 and #6. In this case, encourage students to compare the methods they used in #5 and #6 and to explain why they chose a particular method.

After students have practised using the two division rules in #7 and #8, encourage them to complete #9 and #10 using their preferred method. You might ask students which rule they prefer and why. Encourage students to consider the reasonableness of their answers in #7 to #10 by using mental reasoning and diagrammatic estimation.

Apply

Encourage students to think about the reasonableness of their answers and to use estimation and multiplication to check them. For example, in #15, the quotient of $2\frac{3}{4} \div 1\frac{1}{2}$ must be less than $2\frac{3}{4}$ because the divisor is a mixed number. A reasonable estimate is $3 \div 2$, so the quotient should be close to $1\frac{1}{2}$. Students can check the calculated quotient, $1\frac{5}{6}$, by calculating that the product of $1\frac{5}{6} \times 1\frac{1}{2}$ gives $2\frac{3}{4}$.

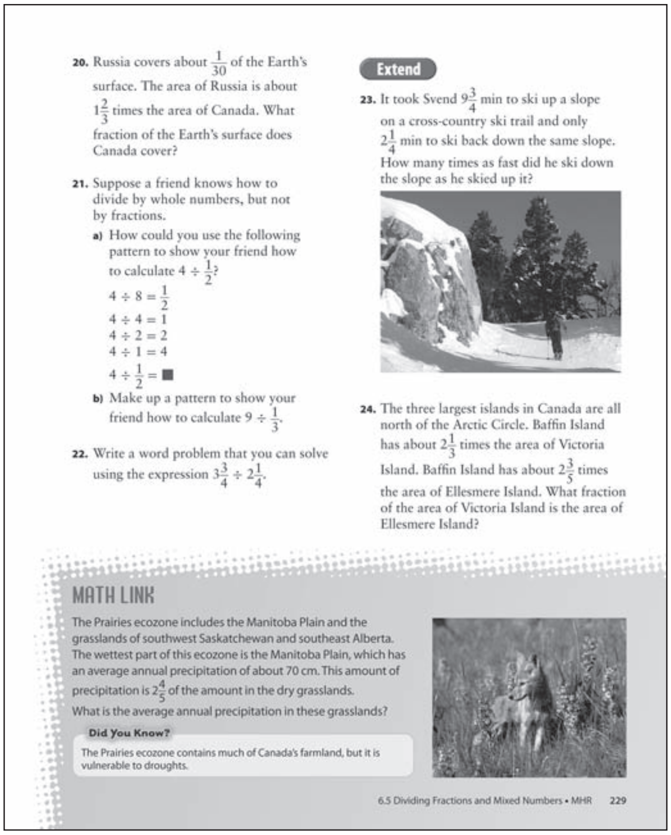
The reasoning required in #14 is similar to that in the first paragraph of this section. For example, using ratios, if incandescent : fluorescent is $4\frac{1}{2}$: 1 or 9 : 2, then fluorescent : incandescent is 2 : 9.

Encourage students to share their answers and examples in #18.

In #20, students return to the context presented in the opening paragraph of this section.

For #21, you may wish to have students work in pairs and act out the question. Partner A could explain the given pattern to Partner B. They could work together to devise a pattern for part b) and then switch roles, so that B explains the new pattern to A or to another student in the class.

For #22, some students may use original contexts for their problems. Other students may adapt the wording of earlier problems (e.g., #15). You might point out that earlier problems that include real data (such as #19) cannot be used in this way. You might encourage students to share their problems so that they are exposed to some that show originality.



20. Russia covers about $\frac{1}{30}$ of the Earth's surface. The area of Russia is about $1\frac{2}{3}$ times the area of Canada. What fraction of the Earth's surface does Canada cover?

21. Suppose a friend knows how to divide by whole numbers, but not by fractions.
    a) How could you use the following pattern to show your friend how to calculate $4 \div \frac{1}{2}$?
    $4 \div 8 = \frac{1}{2}$
    $4 \div 4 = 1$
    $4 \div 2 = 2$
    $4 \div 1 = 4$
    $4 \div \frac{1}{2} = \blacksquare$
    b) Make up a pattern to show your friend how to calculate $9 \div \frac{1}{3}$.

22. Write a word problem that you can solve using the expression $3\frac{3}{4} \div 2\frac{1}{4}$.

Extend

23. It took Svend $9\frac{3}{4}$ min to ski up a slope on a cross-country ski trail and only $2\frac{1}{4}$ min to ski back down the same slope. How many times as fast did he ski down the slope as he skied up it?

24. The three largest islands in Canada are all north of the Arctic Circle. Baffin Island has about $2\frac{1}{3}$ times the area of Victoria Island. Baffin Island has about $2\frac{3}{5}$ times the area of Ellesmere Island. What fraction of the area of Victoria Island is the area of Ellesmere Island?

MATH LINK

The Prairies ecozone includes the Manitoba Plain and the grasslands of southwest Saskatchewan and southeast Alberta. The wettest part of this ecozone is the Manitoba Plain, which has an average annual precipitation of about 70 cm. This amount of precipitation is $2\frac{4}{5}$ of the amount in the dry grasslands. What is the average annual precipitation in these grasslands?

Did You Know?

The Prairies ecozone contains much of Canada's farmland, but it is vulnerable to droughts.

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Extend

In #23, students need to understand that the faster the speed, the shorter the time taken.

Mental reasoning is important in #24. Correctly interpreting the two sentences that include mixed numbers indicates that Ellesmere Island is smaller than Victoria Island, so the expected answer is a proper fraction.

Meeting Student Needs

- Some students may benefit from using **BLM 6–8 Rectangles** to help determine quotients.
- Provide **BLM 6–17 Section 6.5 Extra Practice** to students who would benefit from more practice.

ELL

- For #11, draw a schedule for a two-hour show. Ensure that students understand that *slot* means time period.
- For #14, use the pictures in the student resource to help explain the terms *incandescent* and *fluorescent*.

Math Link

This Math Link allows students to apply the division of a whole number by a mixed number to data concerning Canada's ecozones.

For the Wrap It Up! problem on page 239 in this chapter, students will develop original questions using given data. To help prepare students, you might discuss how the following question could be written from the information in the Math Link and have students determine how the solution in the Math Link would change if the data were presented in this way: The Prairies ecozone includes the Manitoba Plain and the grasslands of southwest Saskatchewan and southeast Alberta. The wettest part of this ecozone is the Manitoba Plain, which has an average annual precipitation of about 70 cm. The amount of precipitation in the dry grasslands is $\frac{5}{14}$ of the amount in the Manitoba Plain. What is the average annual precipitation in these grasslands? (Note how students would multiply here, but get the same answer as for the Math Link: $\frac{5}{14} \times 70 = \frac{350}{14}$ $= 25$ cm.)

Gifted and Enrichment

- If you wish to encourage algebraic reasoning, you may wish to include the following problem:
  If $\frac{1}{x} = \frac{2}{5} \div \frac{5}{2}$, what is the value of x?
  ($\frac{1}{x} = \frac{2}{5} \times \frac{2}{5}$
  $\frac{1}{x} = \frac{4}{25}$
  So, $x = \frac{25}{4}$ or $6\frac{1}{4}$)
- If you introduced the method of removing common factors before multiplying in Example 2 of section 6.4, you may wish to include the following problem: Determine the quotient. Explain your method.
  $1 \div \frac{2}{3} \div \frac{3}{4} \div \frac{4}{5} \div \frac{5}{6} \div \frac{6}{7} \div \frac{7}{8} \div \frac{8}{9} \div \frac{9}{10}$
  ($1 \div \frac{2}{3} \div \frac{3}{4} \div \frac{4}{5} \div \frac{5}{6} \div \frac{6}{7} \div \frac{7}{8} \div \frac{8}{9} \div \frac{9}{10}$
  $= 1 \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \times \frac{7}{6} \times \frac{8}{7} \times \frac{9}{8} \times \frac{10}{9}$
  After the cancelling of equal pairs, you are left with $10 \div 2$, which is 5.)

Common Errors

- Some students may not recognize a natural number as an improper fraction and may have difficulty in applying a division rule when a natural number is involved.

R_x Remind students to rewrite a natural number as a fraction with a denominator of 1 before applying a division rule. For example, $\frac{3}{4} \div 2 = \frac{3}{4} \div \frac{2}{1}$. This division can be completed using a common denominator, $\frac{3}{4} \div \frac{8}{4} = \frac{3}{8}$, or using multiplication, $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.

Answers

Math Link

25 cm

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #5a), c), #7a), c), #9, and #11. Students who have no problems with these questions can go on to the rest of the Apply questions.	• Provide additional coaching with Example 1 to students who need help with #5a) and c). Coach students through #5. Have them identify the diagram or model that is easiest for them to work with. Have them verbalize their thinking and record the steps to determine each quotient. Assign the balance of #5 or #6 to check for understanding. Check back with students several times to make sure that they understand the concepts. • Provide additional coaching with Example 2 to students who need help with #7a), c), and #9. Again students may benefit from starting and solving the questions with a diagram of their choosing first. Assign the balance of #7 or #10 to check for understanding. • Provide additional coaching with Example 3 to students who need help with #11. Encourage students to use diagrams and to write out their thinking mathematically.
Math Link The Math Link on page 229 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 239.	• Make sure that all students do this Math Link, since they will use these basic skills when they design and solve their own questions related to the ecozones in the Wrap It Up! • Division is often challenging for students so it would be beneficial for all students to complete the Math Link. • Students who need help getting started could use **BLM 6–18 Section 6.5 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students complete the following statements: • To solve $3\frac{1}{2} \div 2\frac{1}{4}$, I would … • The most confusing thing about dividing fractions is …	• Some students may not identify any difficulties. Have these students identify their two favourite ways to solve a division problem and explain why they made these choices. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with.