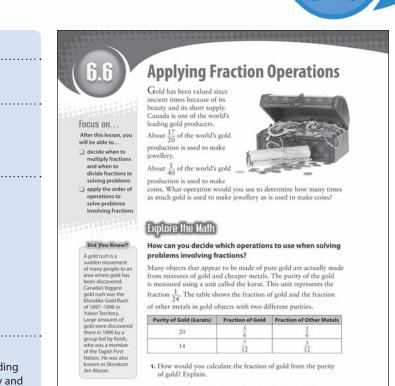
Applying Fraction Operations



 How would you calculate the fraction of other metals from the fraction of gold? Explain.

Because this section involves addition and subtraction as well as multiplication and division, you may wish to have students answer such questions as the following: What total fraction of the world's gold production is used to make jewellery and coins? How much greater is the fraction of the world's gold used to make jewellery than the fraction used to make coins?

Explore the Math

230 MHR + Chapter 6

In this exploration, students decide for themselves which operation(s) to use to solve problems. Make sure students understand that the karat represents the fraction $\frac{1}{24}$ and that this fraction is used in calculations that involve the karat.

Method 1 Have students complete the exploration with a partner or in small groups and then discuss their results with the class. Encourage students to use mental reasoning to guide their decisions. For example, with respect to #1 and looking at the first

MathLinks 8, pages 230-235

Suggested Timing

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up BLM 6–19 Section 6.6 Extra Practice BLM 6–20 Section 6.6 Math Link

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- Reasoning (R)
- Technology (T)
- Visualization (V)

Specific Outcomes

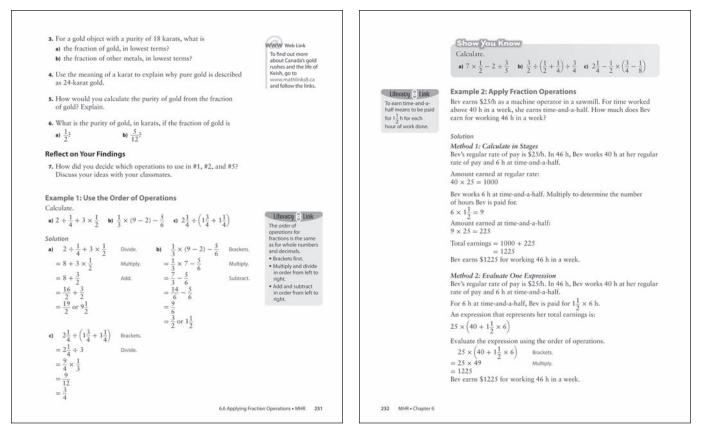
N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 4, 6, 11, Math Link
Typical	1, 2, 4, 6, 11–13, Math Link
Extension/Enrichment	1-3, 14-17

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce material learned in previous sections.

Have students discuss the introductory question and decide which operation is necessary. To some students, the expression *how many times as much* may suggest multiplication rather than division $(\frac{17}{20} \div \frac{1}{40})$. If so, you might ask a question with whole numbers: "If 12 is 3 times as much as a number, what operation would you use to find the number?" If you wish, have students carry out the division $\frac{17}{20} \div \frac{1}{40}$ to determine a numerical answer.



row of the table, you might ask students how they know from the given value of the fraction of gold that the calculation could not involve $20 \div \frac{1}{24}$ or $\frac{1}{24} \div 20$. Because $\frac{1}{24}$ is a proper fraction, the quotient $20 \div \frac{1}{24}$ would be greater than 20. Because 20 is a whole number greater than 1, the quotient $\frac{1}{24} \div 20$ would be less than $\frac{1}{24}$.

Encourage students to think about alternative ways of determining the same answer. For example, in #1, some students may realize that they can divide the purity by 24 to get the fraction of gold. If so, ask why dividing by 24 gives the same answer as multiplying by $\frac{1}{24}$.

Method 2 Have students complete the exploration individually. Then, poll the class to determine which operations students used to determine the answers to #1, #2, and #5 and to check the calculated answers in #3 and #6. If there are differences of opinion, have class or group discussions to decide on the correct answers.

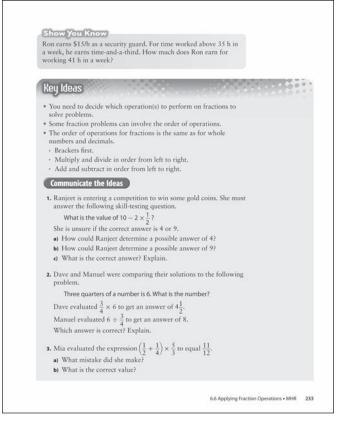
Example 1

This example shows the use of the order of operations in calculations involving fractions and mixed numbers. Remind students that they already know the order of operations from their work with whole numbers and decimals.

If you wish to relate this example to the data and calculations in Explore the Math, you might ask students how they could use the order of operations to calculate the fraction of other metals from the purity of gold.

Guide students by suggesting that they are trying to incorporate the steps used in Explore the Math #1 to #3 into a single expression. For example, #3b) in Explore the Math could be completed, without first completing #3a), by evaluating $1 - 18 \times \frac{1}{24}$. Have students use the order of operations to describe why this expression gives the correct value.

You might then ask students how they could use the order of operations to calculate the purity of gold from the fraction of other metals.



To give an example from the first row of the table in Explore the Math, if the fraction of other metals is $\frac{1}{6}$, the purity of gold is given by $\left(1 - \frac{1}{6}\right) \div \frac{1}{24}$. Have students use the order of operations to explain why the brackets are necessary to give the correct value.

Literacy Link Draw students' attention to the Literacy Link beside Example 1, which explains that the order of operations is the same for fractions as for whole numbers and decimals. Discuss with students what memory device they could use to remember this order. For one suggestion, see the Common Errors on this page.

Example 2

This example illustrates an application of fraction operations. The solution includes two methods, only one of which involves the order of operations. You might ask students which method they prefer and why. Emphasize that students should use whichever method they are more comfortable with.

Literacy Link Draw students' attention to the Literacy Link beside Example 2, which explains the term *time-and-a-half* and its representation as a fraction $\left(1\frac{1}{2}\right)$. You may wish to have students check their calculations using decimals.

Meeting Student Needs

ELL

• Ensure that students understand the following terms by orally explaining them in context: *gold*, *valued*, *ancient times*, *short supply*, *producers*, *jewellery*, *mixtures*, *cheaper metals*, *purity*, and *time-anda-half*. Have students add any new terms to their personal dictionary.

Common Errors

- Some students may not recall that the sum of all the fractions that make up a whole is 1.
- R_x Help students recall this concept by using manipulatives, such as pattern blocks or diagrams, or by discussing an applied example. For example, suppose three people share a six-slice pizza. The first person has three slices, the second person has two slices, and the third person has one slice. What fraction of the pizza does each person have? What is the sum of the fractions? Why?
- Some students may have difficulty in remembering the order of operations.
- **R**_x Encourage the use of a mnemonic, such as BEDMAS.

WWW Web Link

For more information about Canada's gold rushes and the life of Keish, go to www.mathlinks8.ca and follow the links.

Answers

Explore the Math

- **1.** Answers will vary. Example: Divide the purity in karats by 24.
- **2.** Answers will vary. Example: (24 the purity in karats) divided by 24.

3. a)
$$\frac{3}{4}$$
 b) $\frac{1}{4}$

4. Answers will vary. Example: A karat represents $\frac{1}{24}$ purity,

 $\frac{1}{24} \times 24 = 1$, and 1 = 100%. Pure gold is 100% gold, so 24-karat gold is pure gold.

5. Answers will vary. Example: Find an equivalent fraction with a denominator of 24. The numerator represents the purity in karats.

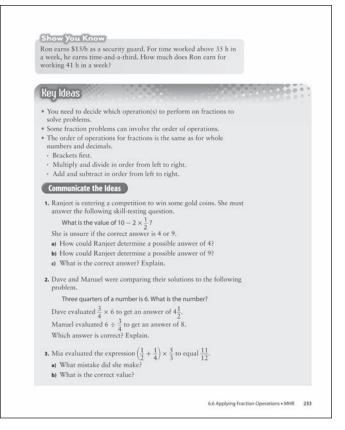
- **6.** a) 12 karats b) 10 karats
- **7.** Answers will vary. Example: I decided to divide by 24 because the purity of gold is based on a karat, which is $\frac{1}{24}$ pure gold.

Show You Know: Example 1

a)
$$\frac{1}{6}$$
 b) $2\frac{2}{3}$ **c)** $1\frac{15}{16}$

Show You Know: Example 2 \$645

Assessment	Supporting Learning	
Assessment <i>as</i> Learning		
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.	 Have all students to discuss the response to #7 as a class. The process is key to understanding the subsequent material presented. It may be beneficial to help students recall the role of the word <i>of</i> in these problems. 	
Assessment for Learning		
Example 1 Have students do the Show You Know related to Example 1.	 It may be a good idea to have students who need assistance verbalize the steps and then complete them one at a time. You may wish to have students work with a partner. For some students, an acronym may help in remembering the order of operations. Some students may need to be coached through the basics of adding fractions and the rules for multiplying and changing mixed to improper fractions. 	
Example 2 Have students do the Show You Know related to Example 2.	 It may be beneficial to have students verbalize the steps and then complete them one at a time. You may wish to have students work with a partner.	



Key Ideas

The Key Ideas summarize the section content and reinforce the order of operations. Stress that the order of operations for fractions is the same as for whole numbers and decimals.

Communicate the Ideas

These questions allow students to consider a choice of operations when problem solving and to apply the order of operations to fractions. In #1, students see how, in the absence of the order of operations, the value of a numerical expression may be ambiguous. In #2, students consider a disagreement over the use of multiplication or division to solve a problem and decide which operation is appropriate. Encourage students to use mental reasoning. Multiplying a

number by the proper fraction $\frac{3}{4}$ must give a product that is less than the original number. Therefore, 6 is less than the original number, so the answer must be greater than 6.

In #3, students analyse and correct an error in applying the order of operations.

Common Errors

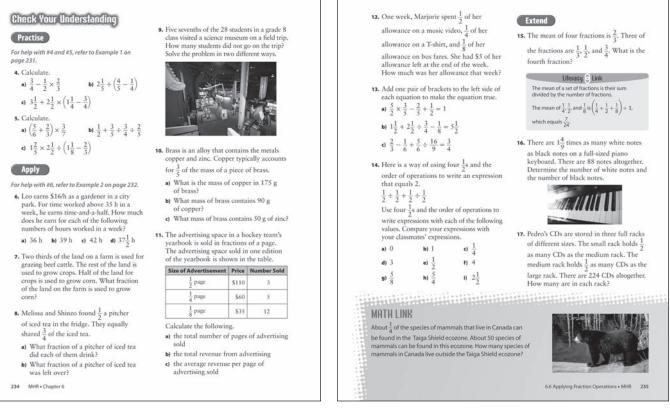
- Some students may still be having difficulties with the basic operations of addition, subtraction, multiplication, and division of fractions while they are solving problems that involve the order of operations.
- R_x Encourage students to continue using concrete and semi-concrete methods for completing individual operations included in the order of operations.

Answers

Communicate the Ideas

- **1.** a) She performed the subtraction before multiplying.
 - **b)** She followed the order of operations by performing the multiplication before the subtraction.
 - c) 9. When following the order of operations, you will obtain the correct answer.
- **2.** 8. Check the answer by taking $\frac{3}{4}$ of 8. The result is 6.
- **3.** a) She multiplied $\frac{1}{4}$ by $\frac{5}{3}$ to obtain $\frac{5}{12}$; then, she added $\frac{1}{2}$ to obtain her answer of $\frac{11}{12}$.
 - **b**) $1\frac{1}{4}$

Assessment	Supporting Learning
Assessment as Learning	
Communicate the Ideas Have all students complete #1 and #2.	 Some students who are developing their understanding of the order of operations may find it difficult to think of the wrong approach. If you have students who seem uncertain of their approach, ask them to answer #1b) only. Students who need assistance with #2 may benefit from verbalizing the solution and solving it using their preferred method first before identifying errors. They can then be coached to look back at the problem and decide where errors have occurred.



Check Your Understanding

Practise

You might have students work with a partner to complete #4 and #5. Partners can then check each other's answers and identify any errors in the application of the order of operations.

Apply

Question 6 is very similar to Example 2. You might have students consider which method they prefer in Example 2 and apply that method in #6.

Stress the importance of reading problems carefully and interpreting the given information correctly. In #7, for example, it is important to keep a clear picture of the given information while reading each sentence. Drawing a diagram of the farm and showing the area for each use may be helpful in this case.

In #9, students are asked to solve the problem in two different ways. One way is to calculate $\frac{5}{7} \times 28$ and subtract the result from 28, or to write the single expression $28 - \frac{5}{7} \times 28$ and evaluate it using the order of operations. Another way is to determine

 $1 - \frac{5}{7}$ and multiply the result by 28, or to write the single expression $28 \times \left(1 - \frac{5}{7}\right)$ and evaluate it using the order of operations. You might take this opportunity to reinforce that there is often more than one way to solve a problem. You might then ask students to describe two ways to solve #10c).

Encourage students to share their answers in #13 and #14 and to check each other's answers when there are disagreements. You might challenge students to look for alternative answers for #14. The question provides an expression that equals 2. You might point out that the given example could equally well be

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \text{ or } \frac{1}{2} + \frac{1}{2} \div \frac{1}{2} + \frac{1}{2} \text{ or}$$
$$\frac{1}{2} \div \left(\frac{1}{2} - \frac{1}{2} \times \frac{1}{2}\right).$$

Extend

You may wish to discuss alternative methods for solving #15 to #17. All three problems can be completed by a Guess and Check method as one possibility. Other methods also exist. For example, #15 can be solved by multiplying the mean by 4 to determine the sum of the fractions, and then using subtraction. In #16, the ratio of white notes to black notes is $1\frac{4}{9}$: 1 or 13:9. The numbers of white notes and black notes are, therefore, given by $\frac{13}{22} \times 88$ and $\frac{9}{22} \times 88$, respectively.

In #17, students who are comfortable with algebraic reasoning may let the number of CDs in the smallest rack be x and solve x + 2x + 4x = 224. Alternatively, if x represents the number of CDs in the largest rack, then $x + \frac{1}{2}x + \frac{1}{4}x = 224$. You may wish to take the opportunity to show that these two algebraic solutions give the same final answer.

Literacy Link Draw students' attention to the Literacy Link, which may help them solve #15. It explains that the mean of a set of fractions is their sum divided by the number of fractions.

Math Link

This Math Link allows students to apply more than one operation to data concerning Canada's ecozones. Encourage students to devise, discuss, and justify alternative solutions. Two possible approaches involve

- evaluating $50 \div \frac{1}{4}$, then subtracting 50
- determining $1 \frac{1}{4}$, then dividing by $\frac{1}{4}$, and then multiplying by 50.

Students can think of the calculations in steps, as described above, or they can write expressions that are evaluated using the order of operations. Examples of these expressions include

$$50 \div \frac{1}{4} - 50 \text{ and } \left(1 - \frac{1}{4}\right) \div \frac{1}{4} \times 50.$$

Meeting Student Needs

• Provide **BLM 6–19 Section 6.6 Extra Practice** to students who would benefit from more practice.

Gifted and Enrichment

• Challenge students to investigate one or more of the following topics: expected value; fraction operations with rational numbers; the differences between rational numbers and irrational numbers; and how to express simple repeating decimals as fractions. • Challenge students to think creatively and have them consider the following problem: Super Mart had a sale. The first \$200 Sara's father spent was discounted by $\frac{1}{10}$, and the amount he spent over \$200 was discounted by $\frac{1}{5}$. Sara's father bought \$275 worth of groceries. Write an expression for the total cost and calculate. [Answers will vary. Example:

$$\left(\frac{9}{10} \times 200\right) + \left(\frac{4}{5} \times 75\right) = 180 + 60 = \$240$$

Common Errors

- Some students may ignore the order of operations and complete all calculations by applying operations in order from left to right.
- $\mathbf{R}_{\mathbf{x}}$ Before students begin each calculation that involves more than one operation, tell them to consult the order of operations. Have students decide which parts of the order of operations apply to the calculation and which part should be done first.
- Because students have been working with multiplication and division in this chapter, they may confuse the rules and make errors in addition and subtraction of fractions (e.g., forgetting to use a common denominator, adding the numerators and adding the denominators).
- $\mathbf{R}_{\mathbf{x}}$ Help reactivate students' skills with addition and subtraction. Have them summarize and highlight the differences between the rules used to carry out the four operations symbolically.
- Some students may make mistakes in word problems because they do not check that they understand the wording of the problem.
- $\mathbf{R}_{\mathbf{x}}$ Encourage students to read the problem carefully and to record the given and required information in their own way (e.g., writing a verbal summary, drawing a diagram). To emphasize the importance of reading problems carefully, you might have students compare the solutions to the following two problems, which differ in only one word:
 - Mei can usually drive home at an average speed of 60 km/h. One day, a winter storm reduced her speed *to* two-thirds of her usual speed. What was her average speed on her drive home that day?
 - Mei can usually drive home at an average speed of 60 km/h. One day, a winter storm reduced her speed by two-thirds of her usual speed. What was her average speed on her drive home that day?

Math Link

150

Assessment	Supporting Learning	
Assessment <i>for</i> Learning		
Practise and Apply Have students do #4, #6, and #11. Students who have no problems with these questions can go on to the rest of the Apply questions.	 Provide additional coaching with Example 1 to students who need help with #4. Coach students through the order of operations. Some students may benefit from having a copy of the order on their desk or readily available in their Foldable. In some cases, teaching them an acronym for the order may be helpful. Assign parts of #5 for additional practice. Provide additional coaching with Example 2 to students who need help with #6 and #11. Some students may benefit from completing multi-step problems in parts. Have them verbalize their thinking and record their steps. 	
Math Link The Math Link on page 235 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 239.	 Make sure that most students do this Math Link, since they will use these basic skills when they design and solve their own questions related to the ecozones in the Wrap It Up! Students who need help getting started could use BLM 6–20 Section 6.6 Math Link, which provides scaffolding. 	
Assessment <i>as</i> Learning		
 Math Learning Log Have students complete the following statements: To solve a question using the order of operations, you When solving problems with fraction operations, I find it difficult to because 	 If students need assistance with the second statement, encourage them to identify the method they feel most comfortable with and why. Then, ask, "What is it about the other method that is difficult?" Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with. 	