Understanding Volume

MathLinks 8, pages 246-253

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Suggested Timing 80–100 minutes

80–100 minute

Materials

- models of right rectangular prisms, right triangular prisms, and right cylinders
- ruler
- centimetre cubes
- calculator (optional)

Blackline Masters

Master 2 Two Stars and One Wish Master 8 Centimetre Grid Paper BLM 7–3 Chapter 7 Warm-Up BLM 7–5 Section 7.1 Extra Practice BLM 7–6 Section 7.1 Math Link

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- Technology (T)
- Visualization (V)

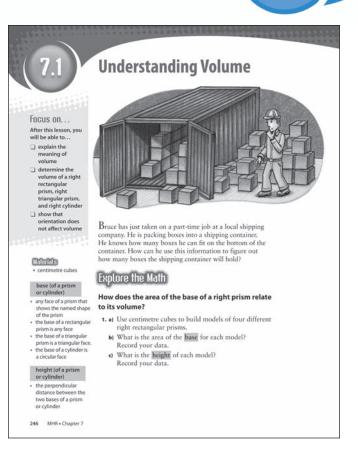
Specific Outcomes

SS4 Develop and apply formulas for determining the volume of right prisms and right cylinders.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 3a), b), 4a), b), 5, 7, 8, Math Link
Typical	1, 2, 3a), b), 4a), b), 5, 7–13, Math Link
Extension/Enrichment	1, 2, 13–18, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.



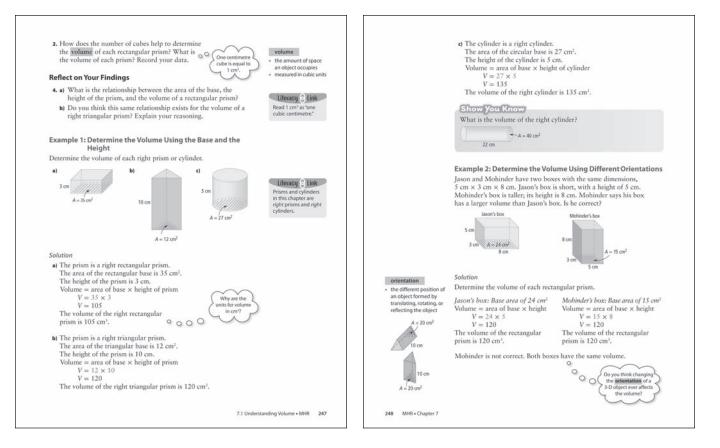
If you do not have 3-D shapes in your classroom, you may wish to prepare models of right rectangular prisms, right triangular prisms, and right cylinders of different lengths, widths, and heights before beginning this chapter.

- For a right rectangular prism, cut a block of wood such that all angles are right angles. Sand the edges smooth, and paint each face a different colour.
- For a right triangular prism, cut a block of wood into a right triangular prism. Ensure that some prisms have right triangle bases and others have non-right triangle bases. For an isosceles triangular prism, you might use a Toblerone® box. Paint each base a different colour.

Note: Toblerone[®] contains nuts. Check your school's policy regarding nut products. Do not use these food boxes if any of your students have nut allergies.

• For a cylinder, cut a dowel. Paint each base a different colour.

Use the opening text as an introduction to the Explore the Math. Ask students to describe situations in which they have packed boxes.



Explore the Math

In this exploration, students use centimetre cubes to explore how the area of the base of a right prism relates to its volume.

Have students work in pairs to build the models. As students build the prisms, encourage them to touch the base and run their finger up the height. Then, have students take the model apart and count the individual cubes that make up the volume.

Circulate as they do this and note the way they are recording their data. If the data is not organized, you may wish to ask the following:

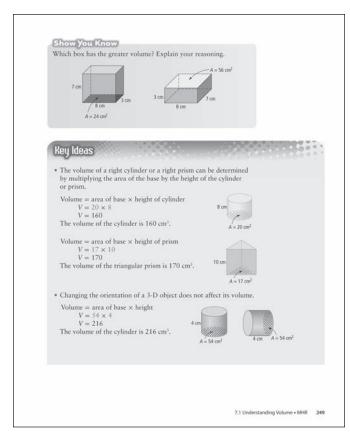
- How can you record your data so that you can compare each model?
- What data do you need to record?
- How might you organize it?
- If you developed a table, what might you use for the column headings in the table? (For some students, you may need to suggest column heads such as Base Dimensions, Area of Base, Height of Prism, Volume or Total Number of Cubes Used.)
- What units are used for measuring volume?

• How can the structures you are building help you to remember the units for volume? (They are working with cubes and volume is measured in cubic units.)

Have students discuss #4 in pairs, then in large groups, then as a class. Challenge them to use the data they have collected to verify their generalizations for #4a). As they discuss #4b), ask:

- How do you think that the volume of a triangular prism might be related to the volume of a triangular prism?
- You may wish to discuss what would happen if they cut a rectangular prism in two diagonally. What would be the volume of each resultant triangular prism? (half the volume of the rectangular prism)
- Challenge students to use another strategy to verify the relationship between triangular and rectangular prisms.

Literacy Link Direct students to the Literacy Link on page 247 that explains how to read the symbol for cubic units. Have students read the cubic units for other measurements, such as mm or mL.



Example 1

The work in this example proceeds directly from what students did during the Explore. Have them consider these answers as a way to verify their generalizations in #4.

Before looking at the solutions, have students consider the visuals for parts a) to c). You may need to focus some students' attention on the way the visuals represent structures similar to those built during the Explore. Ask these students:

- Which visual closely resembles the structures you built during the Explore?
- What is the base and height of this prism?
- How could you determine the volume?
- How do the other shapes differ?
- What information do you have about these shapes?
- How can you use that information to determine the volume?

You may wish to have students build the prism in part a) to verify their solution.

Ask students to do the Show You Know using a strategy of their choice and to discuss their thinking with a partner.

Ask them how their answers might change if they had a rectangular prism or a triangular prism with a base of 40 cm^2 and a height of 22 cm. (The answer

would not change. If students are not clear on this, challenge them to build a rectangular prism this size. What is the volume?)

Literacy Link Direct students to the Literacy Link on page 247 that explains that prisms and cylinders in this chapter are right prisms and cylinders.

Example 2

This example illustrates how changing the orientation of a 3-D object does not affect its volume.

Before having students consider the solution, you may wish to focus student attention on the differences between Jason's and Mohinder's box by asking:

- What are the dimensions of Jason's box?
- What are the dimensions of Mohinder's box.
- How do they differ?
- How are they similar?
- Predict what the volume of these two boxes will be. Explain your thinking.

Then have students use their own strategies to solve this problem. Discuss:

- What strategies did you use?
- Why did you use these strategies?
- What did you find?
- Does this answer seem reasonable? Explain.

Challenge students to find a third orientation that gives the same volume, and then ask if there are any other possible orientations. For right rectangular prisms, any of the six faces can be considered as the base. However, right triangular prisms and cylinders have only two bases each: the triangles and circles at each end. Ensure that students understand that the rectangular faces of a right triangular prism are not bases.

Challenge students to consider whether the orientation of a shape will ever change its volume. You may wish to hold some shapes in the air and change their orientation as you ask:

- Will turning this shape upside down change its volume?
- How about if I put it on its side?
- At an angle?

Have students complete the Show You Know in pairs. Each pair might predict what they will find. Then one individual can calculate the volume of the first prism and the other can calculate the volume of the second prism to test their prediction.

Meeting Student Needs

- Consider using the following scenario as an alternative to the scenario in the introductory text. A museum is returning artifacts to a community. The curator decides to package each artifact in a rectangular box to be placed in a large shipping container. She knows how many boxes fit into the bottom of the container. How can she use this information to figure out how many boxes the shipping container will hold?
- Some students may benefit from additional practice identifying the base and height of prisms and cylinders from diagrams. Once students can consistently identify these measurements correctly, ask them to explain what helped them refine this skill, as this may benefit other students.
- In Example 2, some students may benefit from making and handling models of the two rectangular prisms. Consider having students make and assemble a net for each prism. You may wish to make **Master 8 Centimetre Grid Paper** available for this purpose.

ELL

• Ensure that English language learners understand the following terms: *shipping company, shipping container, area of the base*, and *orientation*. Have students add new terms to their dictionary.

Common Errors

- Some students may place a triangular prism on one of its rectangular sides and use the area of the rectangle as the base area of the prism.
- **R**_x Clarify that triangular prisms and cylinders each have only two bases.
- Some students may record their answers using incorrect units.
- **R**_x Help students recall the correct units for 3-D objects. Remind them that the volume of a 3-D object is equal to the area of the base times its height, or $cm^3 = cm^2 \times cm$. Help them recall calculating area in Chapter 5.

Answers

Explore the Math

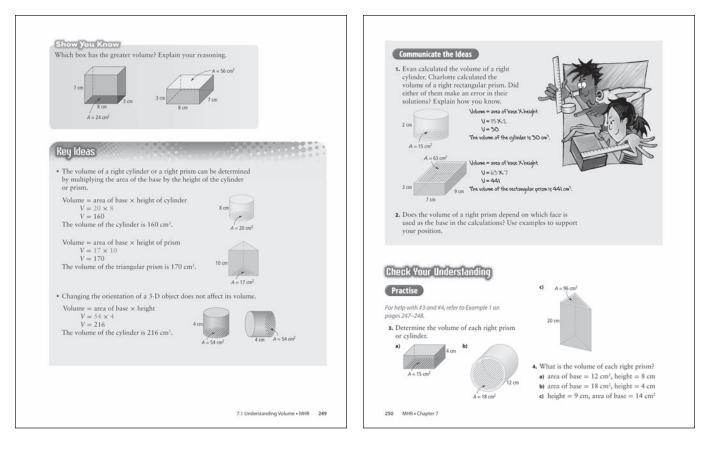
- **1.** a) Models will vary depending on the base dimensions students choose.
 - b), c) Answers will vary depending on the models chosen.
- 2. Answers may vary. Examples:
 - The total number of cubes used represents the volume of each prism.
 - Multiplying the number of cubes in the base by the number of cubes in the height of a prism represents the volume of the prism.
- **3.** Answers will vary depending on the models chosen.
- **4.** a) Answers may vary. Example: The area of the base multiplied by the height equals the volume.
 - b) Answers may vary. Examples:
 - Yes, because the difference between the two kinds of prisms is related to the shape of the base. Multiplying the area of the base of a triangular prism by its height should equal its volume.
 - Yes, the relationship between the area of the base, the height, and volume is the same but a triangular prism has half the volume of a rectangular prism.

Show You Know: Example 1 880 cm³

Show You Know: Example 2

- **a)** 168 cm³ **b)** 168 cm³
- c) Answers may vary. Example: Both rectangular prisms have the same volume. Changing the orientation of a rectangular prism does not affect the volume.

Assessment	Supporting Learning
Assessment as Learning	
Reflect on Your Findings Listen as students discuss what they discover during the Explore the Math. Try to have students generalize the conclusion about their findings.	 Some students may not understand the numeric relationship between the values for area of the base, height of the prism, and volume of their models. Help them to understand the relationships in order to generalize the relationship between area of the base, height, and volume of prisms and cylinders. Encourage students to observe that the volume of 3-D objects changes with changes in height or changes in the area of the base.
Assessment for Learning	
Example 1 Have students do the Show You Know related to Example 1.	 Give students a problem similar to Example 1 to solve before trying the Show You Know on their own. Allow them to work with a partner and talk through their thinking. Make sure that students know why 22 cm is the height of the cylinder, and that they are not simply multiplying 40 × 22 to determine an answer. Some students may benefit from holding a cylinder and moving it to change the orientation of the base, and then explaining how the volume does not change, no matter how they hold the cylinder. Students who need additional experience with this could hold a cylindrical container filled with water or some other substance, and consider whether the volume of the container's contents changes when they change the orientation of the container. Some students will benefit from orally identifying each labelled part of the cylinder. Helping students recall why units are cubed may benefit visual learners who will not always have a diagram to refer to.
Example 2 Have students do the Show You Know related to Example 2.	 Give students a problem similar to Example 2 to solve before trying the Show You Know on their own. Allow them to work with a partner and talk through their thinking. Some students may benefit from visualizing the red base as the face resting on a surface, or imagining that the red face is held in their hand. Encourage students to use what works to help them know the base and height. A model of a prism may benefit some students. Encourage students to turn their books to help them understand the orientation of a 3-D object. Some students may benefit from being coached through a problem that addresses all three orientations. This will help reinforce that orientation does not affect the volume of a 3-D object. Remind students that this problem refers to the volume of a right rectangular prism. For problems involving the volume of right triangular prisms and right cylinders, reinforce that each have only two bases. Consider having students calculate and compare the volume of the two 3-D objects illustrating the term <i>orientation</i> on page 248.



Key Ideas

The Key Ideas summarize how to calculate the volume of a prism and a cylinder and how changing the orientation of a 3-D object does not affect its volume. Have students use index cards to prepare their own summary of the Key Ideas, including an example for each. Have students store their cards in the appropriate section of their chapter Foldable. You might challenge students to integrate the two Key Ideas into one idea, and then add this integrated idea to their chapter Foldable.

Communicate the Ideas

These questions allow students to demonstrate their understanding of volume of cylinders and prisms. In #1, students identify an error in calculating volume. Have students do #1 individually. Students working with others may miss the opportunity of discovering the error for themselves.

In #2, have students work in pairs and use diagrams, words, and possibly 3-D objects, to explain their understanding of how orientation affects the volume of a right prism. Be prepared for one of two possible responses.

- The volume of a right rectangular prism *does not* depend on which face is used as the base.
- The volume of a right triangular prism *does* depend on using either end of the prism as the base.

Have students with different opinions work together, so that they may understand that both answers are correct depending on the prism.

Meeting Student Needs

• For #2, concrete and kinesthetic learners may benefit from a demonstration to show that orientation does not affect volume. For example, fill a plastic container (shaped like a rectangular prism) with water, seal it, and turn it over on its side and top to show that the volume does not change.

Common Errors

- Students who do not understand the importance of identifying the base and the height correctly may be more likely to use measurements incorrectly as Charlotte did in #1.
- R_x Point out that Charlotte may not have looked carefully at the dimensions and the area of the base and chosen the incorrect dimension for height.

Answers

Communicate the Ideas

- **1.** Answers may vary. Example: Evan calculated the volume of the cylinder correctly. He multiplied the area of the base by the height to obtain the volume. Charlotte calculated the volume of the rectangular prism incorrectly. She did not multiply the area of the base by the height. She should have multiplied 63 cm by 3 cm.
- **2.** Answers may vary. Example: The volume of a right prism does not depend on which face is used as the base in the calculations. The volume of the right prism in #1 could have been calculated as follows: Volume = area of base × height = $(9 \times 7) \times 3 = (9 \times 3) \times 7 = (3 \times 7) \times 9 = 189 \text{ cm}^3$.

Assessment	Supporting Learning		
Assessment as Learning			
Have all students complete #1 and #2.	 For #2, some students may benefit from using a 2 × 3 × 4 right rectangular prism and working through the calculations as a group to determine that orientation does not affect volume. For #2, have students use the class responses as a springboard for their own answer. Encourage them to refer to right rectangular prisms and right triangular prisms in their response. Some students may also wish to refer to cylinders. 		
Communicate the Ideas 1. Evan calculated the volume of a right cylinder. Charlotte calculated the volume of a right restangular prism. Did either of them make an error in their solutions? Explain how you know of the cylinder is SO exit. $2 \operatorname{or}_{(-)}$ $4 - 15 \operatorname{or}^{(-)}$ $4 - 5 \operatorname{or}^{(-)}$ $4 - 5 \operatorname{or}^{(-)}$ $3 \operatorname{or}_{(-)}$ $3 \operatorname{or}_{(-)}$ $3 \operatorname{or}_{(-)}$ $3 \operatorname{or}_{(-)}$ $3 \operatorname{or}_{(-)}$ $4 - 5 \operatorname{or}^{(-)}$ $5 \operatorname{or}^{(-)}$ 5	6 cm $A = 48 \text{ cm}^2 = 8 \text{ cm}$	 Apply What is the height of each of the following right rectangular prisms? volume = 32 cm³, area of base = 8 cm² volume = 35 cm³, area of base = 5 cm² volume = 35 cm³, volume = 36 cm³ Nina uses 15 centimetre cubes to make the base of a rectangular prism. Determine the volume if the prism has a total of 5 layers of cubes. Show your thinking. How many ways can you build a rectangular prism from 16 centimetre cubes? Use diagrams or centimetre cubes? Use diagram	
A = 15 cm ² a) area of ba	colume of each right prism? exclume of each right prism? $A = 51 \text{ cm}^2$ 17 cm $A = 51 \text{ cm}^2$ 17 cm B B B B B B B B	 In José is having vegetable soup. The area of the base of the soup can is 10.4 cm², and the height is 10 cm. When José opens the can, he sees that the soup comes up to a height of only 9 cm. What volume of soup is in the can? 	

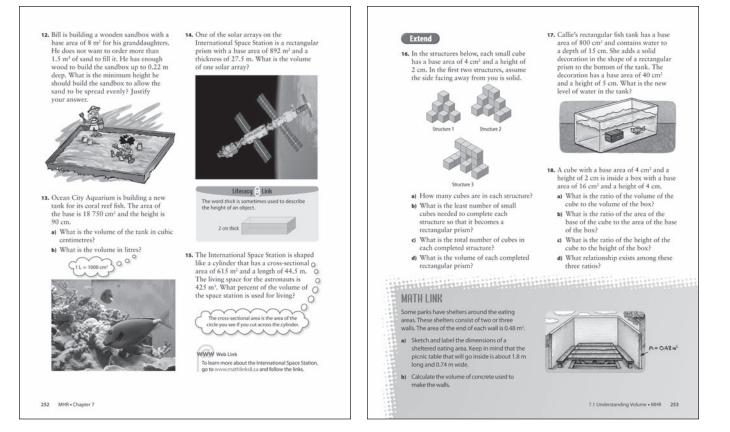
Check Your Understanding

Practise

Have students work in pairs to check each other's numeric answers and units for all questions. Ask students to explain the similarity between the pairs of prisms in #5 and #6 (i.e., they are identical prisms shown in different orientations).

Apply

Students who use incorrect units in #7 may have a weak understanding of measurement concepts. Encourage students to use centimetre cubes to help them answer #8 and #9. Students' answers to #8 and #9 should reveal conceptual understanding. The remaining questions in this section allow students to apply their learning about volume in a variety of contexts.



Literacy Link For #14, direct students to the Literacy Link on page 252 that explains that *thick* can be used to describe the height of an object.

Extend

For #16, students explore the number of possible ways that a task can be completed. Some students may wish to build the structures to prove their conclusions. For #17, students need some understanding of water displacement. For #18, students combine what they have learned about volume with earlier work on ratios.

Math Link

The Math Link allows students to apply their understanding of determining the volume of rectangular prisms. Give students some leeway in how they draw their sketch but emphasize the importance of using measurements that are reasonable.

Meeting Student Needs

• In #7, some students may not use correct units for height. Help students recall the units for height or

length (cm), area (cm²), and volume (cm³). You might use a model of a prism to point out each dimension and the associated units. Some students may benefit from drawing diagrams to illustrate the units that correspond to height, area, and volume and storing them in their chapter Foldable for reference.

- In #8, students may benefit from using centimetre cubes to build the prism. Encourage them to find the area of the base first, and then multiply by 5. Have students verbalize the process and identify the units that the answer will have.
- For the Math Link, encourage students with weak spatial sense to begin by making a rough sketch and then labelling it with the measurements provided in the problem. Through discussion, students should begin to understand the given relative measurements. Some students may prefer to make an initial sketch from the perspective of a bird's-eye view.
- Provide **BLM 7–5 Section 7.1 Extra Practice** to students who would benefit from more practice.

ELL

• Ensure that English language learners understand the following terms: *layers*, *water trough*, *overflows*, *sandbox*, *aquarium*, *solar arrays*, and *astronauts*.

Answers

Math Link

- a) Sketches will vary. Ensure that the sketch is reasonable. Example: If three walls, the long wall should be about 2 m and the short walls should each be at least 0.75 m.
- **b)** Answers will vary depending on dimensions and number of walls. Example:

Wall	Dimensions	Volume
Long wall	$2 \text{ m} \times 0.48 \text{ m}^2$	0.96 m ³
Short wall	$1 \text{ m} \times 0.48 \text{ m}^2$	0.48 m ³

Total volume of concrete needed = $0.96 + (2 \times 0.48)$ = 0.96 + 0.96= 1.92 m^3

It would take 1.92 m³ of concrete to build the walls.

Assessment	Supporting Learning			
Assessment for Learning				
Practise Have students do #3a) and b), #4a) and b), and #5. Students who have no problems with these questions can go on to the Apply questions.	 Provide additional coaching with Example 1 to students who need support for #3a) and b). Work with them to correct these questions, and then have them try part c) on their own. Work with students who have difficulty with #4a) and b) to correct these questions, and then have them try part c) on their own. Encourage visual learners who may be confused in #4 to draw a sketch and label the dimensions of each prism before determining the volume. Since the type of prism is not specified, encourage students to use a design they are comfortable with. Provide additional coaching with Example 2 to students who need support for #5. Work with them to correct #5, and then have students try #6 on their own. If students need additional coaching, walk through #6a) with them and then have students try part b). Students who struggle with basic multiplication facts may benefit from using a calculator or multiplication table. 			
Math Link The Math Link on page 253 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 279.	 Have students verbalize their thinking. Encourage students to draw and label one wall at a time and calculate its volume. Consider providing a model of a sheltered eating area for visual learners. To help them get started, some students may benefit from using BLM 7–6 Section 7.1 Math Link, which provides scaffolding for this activity. 			
Assessment as Learning				
 Math Learning Log Have students answer the following questions: How do you determine the base of a rectangular prism? How are rectangular, triangular, and cylindrical prisms different from each other? How are they similar? 	 Have models of prisms available for students to use. Prompt students to realize that the base of a rectangular prism is the face from which the height is measured. Usually, a rectangular prism rests on its base. As students discuss the different types of prisms, listen for the idea that the shape of the base determines the type of prism. Consider having students exchange their answers with a classmate to check for errors and suggest improvements. You may wish to provide students with Master 2 Two Stars and One Wish for recording their feedback. Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with. 			