

Volume of a Prism

MathLinks 8, pages 254–261

Suggested Timing

80-100 minutes

Materials

- models of right rectangular prisms and right triangular prisms
- calculator (optional)
- centimetre cubes (optional)

Blackline Masters

Master 6 Square Dot Paper Master 7 Isometric Dot Paper Master 8 Centimetre Grid Paper BLM 7–3 Chapter 7 Warm-Up BLM 7–7 Section 7.2 Extra Practice BLM 7–8 Section 7.2 Math Link

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- ✓ Problem Solving (PS)
- 🖌 Reasoning (R)
- 🖌 Technology (T)
- ✓ Visualization (V)

Specific Outcomes

SS4 Develop and apply formulas for determining the volume of right prisms and right cylinders.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 5a), b), 7a), b), 8a), b), 9, Math Link
Typical	1–4, 5a), b), 7a), b), 8a), b), 9, 11–15, Math Link
Extension/Enrichment	1-3, 11, 12, 15-24, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.

As a class, read and discuss the information about recycling bins as a lead in to the Explore the Math. Help students realize that the term *volume* is often used to refer to the amount of material that a container can hold. Students may say that they would need to know the dimensions of the bin in order to determine how much paper was recycled.



Explore the Math

Students explore the relationship between base and height, and volume of right prisms, to develop a formula for volume. Prompt students to recall that in the case of a triangular prism, they need to determine the area of a triangular end of the prism, and then multiply this area by the height of the prism.

Have students work in pairs and use models of right rectangular prisms and right triangular prisms to help them answer the questions. Encourage students to extend their thinking from the last section by asking questions such as the following:

- How can you determine the volume of a prism if you know the area of the base?
- If you don't know the area of the base, but you know the dimensions of the base and the height, how can you determine the volume of a prism?
- Will your answer vary depending on the shape of the base of the prism?
- How will it vary? Use examples to explain.
- When working with volume, does the shape of the base of the prism matter? Explain.





- What information do you need to know about the shape of the base to determine the volume of a prism?
- What dimensions do you need for each type of prism in order to determine the volume?

Challenge students to generalize their suggestions by developing formulas for the two types of prisms shown. Discuss these formulas in groups. During group discussions, encourage students to identify the similarities and differences between the formulas and the reasons for any differences.

Discuss the formulas as a class. Ask:

- What formula is used for this shape? (holding up a sample prism)
- Why is this formula used for this shape?
- How does this formula work?
- Is using the formula the only way to calculate the volume of this shape? Explain.
- What can you do if you forget the formula? What other strategy(ies) can you use to solve this type of problem?

Example 1

This example models using a formula to determine the volume of a right rectangular prism. Before students see this solution, ask them to solve the question using a strategy of their choice. Ask:

- What strategy did you use? Why?
- What other strategy could be used here?
- Does it matter what strategy you use? Explain.

Challenge students to solve the Show You Know using more than one strategy and compare their answers.

This is the first opportunity for students to determine volume by using the formula $l \times w \times h$. Some students may wonder why they are using these three measurements. Connect the formulas to section 7.1 by explaining that area can be unpacked into length and width. Reinforce the continued importance of identifying the dimensions of prisms.

Example 2

This example models using a formula to determine the volume of a right triangular prism. Again, have students solve this problem using a strategy of their choice. Some will prefer to calculate the area of the base, and then multiply that area times the height of the prism in order to find the volume.



Have students use two different methods to solve the Show You Know. Students can work in teams, with one using the formula and the other using a second strategy, such as calculating the area of the triangular base and then using that information to calculate the volume. Have students compare their answers and then review each other's calculations. Are both methods equally useful?

Encourage students to realize that the volume of a right triangular prism is an example of the general property of the volume of prisms (i.e., area of base \times height). Otherwise, they may memorize the formula rather than understand it.

Using the correct dimensions to calculate the volume of a triangular prism is crucial. Ensure that students understand the difference between the height of a triangle, which is used to calculate the area of a triangle, and the height of a prism. You might use a model to help clarify the meaning of these two terms.

Example 3

This example illustrates using volume to solve a problem. To solve the problem, students revisit earlier skills involving fractions. As a class, walk through the example. Some students may think that they should determine $\frac{5}{6}$ of each dimension and then multiply to determine the volume. Explain that it is the contents of the box that are smaller, not the box itself.

Challenge students to use their knowledge of fractions to develop other strategies for solving this problem. For example:

- Calculate $\frac{5}{6}$ of the volume of the box and then subtract that amount from the full volume to identify the amount of empty space.
- Since the empty space is at the top of the box, do this problem in one step by multiplying 5 cm (that is one sixth of the height) by 18 cm by 8 cm.

Have students work with a partner to solve the Show You Know using strategies of their choice. Challenge them to use their knowledge of fractions and to solve the problem in two different ways.

Meeting Student Needs

• Some students may not understand the connection between how volume is determined in section 7.1 and how volume is determined in section 7.2. Consider using the diagram for Example 1 part a), and modelling how to determine volume using both methods. Have students use a table divided into two columns and show multiplying area of the base × height in column 1 and multiplying length × width × height in column 2. Help students observe the connection between the two methods of solving the problem, which should reinforce understanding of the new formulas.

ELL

• Ensure that English language learners understand the following terms: *recycling*, *recyclables*, *recycled paper*, *related*, *formula*, *cereal box*, *contents settle*, and *building blocks*. Have students add any new terms to their dictionary.

Common Errors

- Some students who may memorize the formulas without understanding the generalization from section 7.1 may use the wrong formulas from time to time.
- R_x Coach students to highlight the formula for area on a copy of the formulas for volume of a rectangular prism and a triangular prism.

Answers

Explore the Math

- **1.** Answers may vary. Example: The volume of a prism equals the area of the base multiplied by the height.
- **2.** Answers may vary. Example: Multiply the length of the base by the width of the base. Area of base $= 9 \text{ cm} \times 5 \text{ cm} = 45 \text{ cm}^2$. Volume of prism = area of base \times height; $V = 45 \times 4 = 180$. The volume of the rectangular prism is 180 cm³.
- **3.** Answers may vary. Example: Multiply the length of the triangular base by the height of the triangular base and then divide the result by two. Area of base $=\frac{8 \times 5}{2} = 20$. The area of the base is 20 cm². Volume of prism = area of base × height; $V = 20 \times 4 = 80$. The volume of the triangular prism is 80 cm³.
- 4. a) Answers may vary. Example: Use the formula for the volume of a right rectangular prism, which is V = l × w × h.
 - **b)** Answers may vary. Example: Use the formula for the volume of a right triangular prism, which is $\frac{l \times w}{2} \times h$.

For each of a) and b), look for an example that tests the formula.

- c) Answers may vary. Look for one difference and one similarity. Example:
 - The formulas differ in the way that the area of the base of each shape is calculated. To calculate the area of the base of the right rectangular prism, multiply the length of the base by the height of the base. To calculate the area of the base of the right triangular prism, multiply the length of the triangular base by the height of the triangular base and then divide the result by two.
 - The formulas are similar in that the volumes are calculated by multiplying the area of the base by the height.

Show You Know: Example 1

a) 170 cm³ **b)** 27 m³

Show You Know: Example 2

 180 mm^3

Show You Know: Example 3

Answers may vary. Example: Each child will receive 75 blocks, each with a volume of 4800 cm^3 .

Assessment	Supporting Learning			
Assessment as Learning				
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize their conclusions about the formulas for the volume of a rectangular prism and a triangular prism.	 Some students may express the formulas in terms such as <i>multiply all three numbers</i>. Coach students to use correct math vocabulary to express the formulas. Allow students who struggle with expressing a written formula to orally express their understanding of how to determine volume. Coach them to use correct math vocabulary, and then translate their oral understanding into a written formula. For #4a) and b), some students may benefit from using diagrams labelled with the area of the base. Have students verbalize how the given base area was determined. Have them identify and label the dimensions of each prism, and then recalculate the volume. For #4c), discussing as a class the differences and similarities between the formulas and the reasons why will help some students develop a deeper understanding. Some students may indicate that the difference between the formulas for rectangular prisms and triangular prisms is the ¹/₂ in the formula for the triangular prism. Prompt students to understand that the difference is related to the shape of the base. Visual learners may benefit from drawing and labelling a diagram of the base of a rectangular prism and a triangular prism with the dimensions and area formula to help make this connection. 			
Assessment <i>for</i> Learning				
Example 1 Have students do the Show You Know related to Example 1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Encourage students to refer to the example and follow the steps. Have them record each step. Some students may benefit from sketching the base and labelling its dimensions, before determining the area of the base, and then multiplying by the height of the prism. Have students explain how this process might be the same as multiplying <i>l</i> × <i>w</i> × <i>h</i>. 			
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Ensure that students can verbalize the measurements as they replace each variable with a value. Using the correct vocabulary is key. Some students may benefit from sketching and labelling the base before determining its area, and then multiplying by the height of the prism. Have students explain how this process is similar to calculating (b × h ÷ 2) × h. 			
Example 3 Have students do the Show You Know related to Example 3.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Consider coaching students through multiplying fractions and a whole number. Students who struggle with multiplication facts may benefit from using a calculator. Some students may benefit from using centimetre cubes to model the problem. 			



Key Ideas

The Key Ideas summarize the formulas for calculating the volume of a right rectangular prism, a cube, and a triangular prism. Consider having students build models of each prism using nets, write the formula for the area of the base on an appropriate face, write the formula for the volume on another face, and display their figures around the classroom. You may wish to make **Master 8 Centimetre Grid Paper** available for this purpose. Alternatively, they can prepare their own illustrated formula for each prism on index cards and store these in the appropriate section of their chapter Foldable.

Communicate the Ideas

These questions allow students to communicate their understanding of calculating the volume of right prisms. In #1, students explain the formula for volume of a cube. In #2, students identify the measurements needed to determine the volume of a triangular prism. In #3, students identify an error in calculating the volume of a rectangular prism. Have students work in small groups to discuss the answers before discussing the answers as a whole class. For #1, a class discussion about whether a cube is a unique 3-D shape or a specific type of prism (in which length, width, and height are all equal) may help student understanding.

Meeting Student Needs

- Allow enough time for students to discuss the questions. Using models of each prism and discussing the relationships between the formulas for volume of each prism may help reveal misconceptions and/or unique perspectives that may help other students' understanding.
- For #2, some students may benefit from using a model to show the measurements needed to determine the volume of the ramp (e.g., base and height of triangle, height of prism). Students could label these measurements on a sketch of the ramp.

ELL

- For #2, use the visual of the ramp to explain the term *concrete ramp*.
- For #3, explain that *show an alternative way* means to show another way.
- Allow English language learners to rely on mathematical expressions rather than words to show their understanding. For example, you might tell a student who does not understand what #3 is asking, that it is asking them to show what is wrong with the math and fix it.

Answers

Communicate the Ideas

- **1.** Answers may vary. Example: Yes, the formula $l \times w \times h$ can be used to calculate the volume of a cube. The formula $V = s^3$ can also be used.
- **2.** Answers may vary. Example: Kwan needs to know the height of his back door above the ground, the width of the door, and the length he wants to make the ramp.
- **3.** a) Answers may vary. Example: Jack calculated the volume of frozen yogurt that was left in the carton after his family ate instead of the volume of frozen yogurt that his family ate.
 - **b)** Answers may vary. Example: $V = 1944 \times \frac{1}{4} = 486$. The volume of frozen yogurt that Jack's family ate is 486 cm³.
 - c) Answers may vary. Example: V = 1944 1458 = 486. The volume of frozen yogurt that Jack's family ate is 486 cm³.

Assessment	Supporting Learning
Assessment as Learning	
Communicate the Ideas Have all students complete #1 to #3.	 Have students discuss their understandings in small groups before having them present the shared understanding of their group to the class. Some students may benefit from using a diagram to help explain #1. Some students may not make the connection that the volume of a cube is one of its sides cubed. Encourage students to continue using the formula: V = l × w × h. Ask students who need help with #3 the following question: If after eating a product, there was ³/₄ left, would you say that most of it was eaten? This may help them to make a connection to the question. Some students may benefit from using the class responses as a springboard for their own answers.





Practise

These questions require students to calculate the volume of rectangular prisms, cubes, and triangular prisms with and without diagrams. Encourage students to draw and label diagrams for #5 and #8. For #9 and #10, students are required to use multiplication of fractions to determine the volume of the contents of each prism.

Apply

A variety of problems are presented in #13 to #22 involving real-world contexts. Encourage students to draw and label diagrams for questions in which diagrams are not provided. For #13, draw students' attention to the thought bubble that clarifies the meaning of *deep*. Encourage students to check their answers for reasonableness by referring back to the context of each problem.

Literacy Link For #17, direct students to the term *capacity*. Capacity refers to the volume of material that a container such as a dump truck box will hold.

Extend

In addition to some more challenging applications of volume, these questions provide an opportunity for students to extend their thinking about volume. The calculations in #24 involve rate.

Math Link

The Math Link provides an opportunity for students to apply their understanding of volume using the context of recycling.

Meeting Student Needs

6. Determine the volume of each cube.

7. Determine the volume of each right

8. What is the volume of each right

iangular prism?

a) base of triangle = 3 mheight of triangle = 7 mheight of prism = 8 m

b) base of triangle = 15 cm

height of triangle = 8 cm

height of prism = 20 cm

height of triangle = 9.1 mmheight of prism = 11.3 mm

c) base of triangle = 10 mm

8.1 m

triangular prism.

a) 7 cm For help with #9 and #10, refer to Example 3 of

9. Determine the volume of the contents

12 cm

Determine the volume of the empty

space in each object.

b) $\frac{3}{4}$ full of milk

c) $\frac{1}{6}$ full of water

12 cm

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a) $\frac{4}{5}$ full of facial tissues

b) $\frac{3}{8}$ full

pages 256-257.

c) $\frac{3}{4}$ full

of each right prism. a) $\frac{1}{3}$ full

- Make Master 6 Square Dot Paper and Master 7 Isometric Dot Paper available to students for drawing their diagrams of the 3-D shapes in #5 and #8. It is not necessary for students to draw scale diagrams. Encourage them to make a rough sketch of each shape, however, and then label it.
- Provide BLM 7–7 Section 7.2 Extra Practice to students who would benefit from more practice.

Apply

 Copy and complete the following table. Right Rectangular Prism

Length (cm)	Width (cm)	Height (cm)	Volume (cm ³)
a) 7	2		70
b) 12	1	10	1080
c)	15	5	1200

 Copy and complete the following table. Right Triangular Prism

B. (0	ase :m)	Height of Triangle (cm)	Prism (cm)	Volume (cm ³)
a)	7	2		70
b)	18		10	1080
c)		14	5	700

13. A landscaper has 0.5 m³ of gravel to use as the base of a patio. If the gravel hase must be 10 cm deep and the patio is 2.6 m wide and 2.8 m long, does she have enough gravel? How much extra gravel does she have, or how much more will she need?
14. A glass vase in the shape of a right triangular prism is filled with coloured sand as a decoration in a living room. What is the volume of the vase?

 Calculate the volume of concrete used to make a sidewalk 1.5 m wide and 12.0 m long. The concrete is poured 0.1 m thick.
 Cindy's aquarium stands 75 cm tall and

this pagaritum treasures 1.2 m × 80 cm. At one point during the initial filling, the aquarium has a 12-cm depth of water in it. Cindy needs to fill it to 15 cm from the top before she adds the fish. Draw a diagram and label the dimensions of the aquarium. Determine how much more water Cindy must add before she puts in





ELL

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- For #5, ensure that English language learners understand that *l* represents length, *w* represents width, and *h* represents height of a rectangular prism.
- For #9 and #10, the wording and the fractions might confuse English language learners. Walk through #9a) with students: Draw a sketch that shows ¹/₃ of the prism filled. Highlight the dimensions of the prism needed to calculate the volume of the prism. Calculate the volume of the prism. Then, ask

students what to do next (calculate the volume of the contents). Calculate $\frac{1}{2}$ of the volume.

- For #13, ensure that English language learners understand the following terms: *landscaper*, *gravel*, *patio*, *gravel base*, *deep*, *enough*, and *extra gravel*. If the text is overwhelming, either omit the question or work through the problem orally with students and use visuals to help explain the scenario.
- Clarify the following terms for English language learners: *facial tissues*, *vase*, *decoration*, and *guppy*.
- For #16, use the picture to help clarify the scenario.
- For #17, consider using a picture or drawing a diagram to help explain the scenario and the following terms: *contractor*, *excavating*, *pour the foundation*, *dump truck*, and *soil*.

Gifted and Enrichment

• After completing the Math Link, challenge students to estimate the volume of the school gym. Have them use the calculation of the volume of waste recycled each week in the park (from the Math Link) and then determine how many weeks of recycling would fill the school gym to the ceiling. Have students present their calculations to the class. Have them use this graphic illustration to help students visualize just how much waste is diverted from landfills when it is recycled.

Common Errors

- For #9 and #10, some students may calculate the fraction of each dimension before multiplying to find the volume of the prism.
- R_x It may be helpful to explain that when students eat the contents of a box, the box itself does not change (decrease). Only the *contents* of the box changes (i.e., decreases).

Answers

Math Link

The volume of waste recycled each week is 7682688 cm³ or 7.68 m³.

Assessment	Supporting Learning
Assessment for Learning	
Practise Have students do #4, #5a) and b), #7a) and b), #8a) and b), and #9. Students who have no problems with these questions can go on to the Apply questions.	 Provide additional coaching with Example 1 to students who need help with #4. Students may benefit from identifying the base first, determining its area, and then multiplying by the height. Have students verbalize the process for solving #4a). Work with them to correct the rest of #4, and then have students try #6 on their own. For #5, some students may benefit from drawing and labelling a sketch before determining the volume. Suggest that they use any diagram in #4 as a model. Students may benefit from identifying the base first, determining its area, and then multiplying by the height. Have students verbalize the process for solving #5a). Work with them to correct #5a) and b) before trying part c). Provide additional coaching with Example 2 to students who need help with #7a) and b). Have students verbalize the process for finding the area of a triangle before prompting them to identify the triangular base, determine its area, and then multiply by the height. Work with them to correct #7a) and b), and then have students try part c) on their own. For #8, some students may benefit from drawing and labelling a sketch before determining the volume. Suggest that they use any diagram in #7 as a model. Have students verbalize the process for solving as a model. Have students verbalize the process for solving and labelling a sketch before determining the volume. Suggest that they use any diagram in #7 as a model. Have students verbalize the process for solving #8a). Work with them to correct #8a) and b) before completing part c). Provide additional coaching with Example 3 to students who need help with #9. Students may benefit from recalling how to multiply a whole number by a fraction. Coach students through #9, and then assign #10. Some students may benefit from using a calculator to help determine volumes.
Math Link The Math Link on page 261 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 279.	 Explain the term <i>filled to the brim</i>, if necessary. This is a multi-step problem, which may overwhelm some students. Encourage students to break the problem into individual steps. Find the volume of one recycling bin. Determine the volume of the number of recycling bins that will be emptied. Determine the total volume that will be recycled each week. Concrete and kinesthetic learners may benefit from modelling the situation by measuring an empty box, calculating its volume, filling it with paper and emptying it twice, and then considering what volume of paper they emptied out of the box. To help them get started, some students may benefit from using BLM 7–8 Section 7.2 Math Link, which provides scaffolding for this activity.
Assessment <i>as</i> Learning	
 Math Learning Log Have students answer the following question: Your friend was away from school and missed the class discussion about volume. She is finding it difficult to remember the three different formulas for volume of a prism. How could you explain using one general formula for cubes, right rectangular prisms, and right triangular prisms? Explain your thinking. 	 Some students may benefit from revisiting the ideas in section 7.1 to help them recall that all prisms have volumes equal to the area of the base multiplied by the height of the prism. Encourage students who cannot generalize the formulas to sketch each individual prism and explain the formula for each. What is similar about each formula? What is different? What do the differences have in common? Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.