# **Volume of a Cylinder**

Technology (T)

✓ Visualization (V)

Specific Outcomes

right prisms and right cylinders.

Category Essential (minimum questions

to cover the outcomes)

Extension/Enrichment

**Planning Notes** 

learned in previous sections.

Typical

**SS4** Develop and apply formulas for determining the volume of

Have students complete the warm-up questions on

BLM 7-3 Chapter 7 Warm-Up to reinforce material

**Question Numbers** 

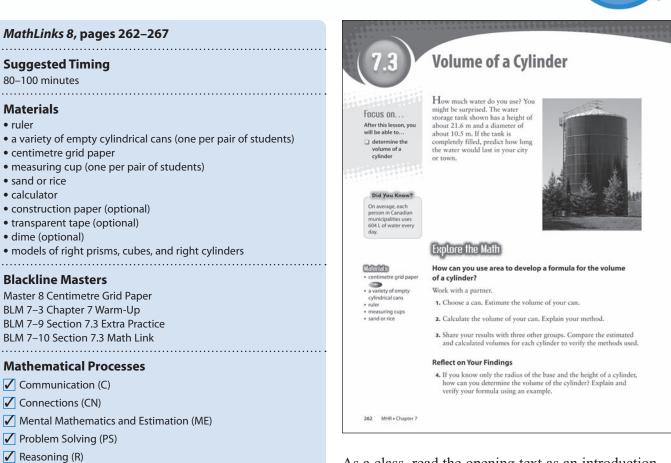
1, 3, 4a), b), 5a), d), 6a), b), 7a),

1, 3, 4a), b), 5a), d), 6a), b), 7a),

1, 3, 11, 12, 15-18, Math Link

b), 8, 10, 12–14, Math Link

b), 8, Math Link



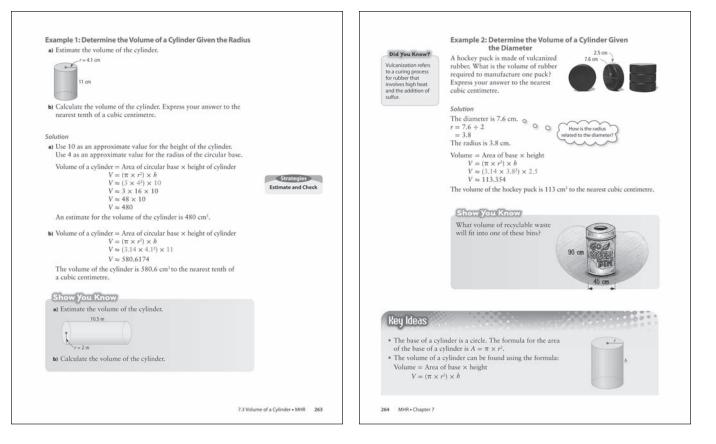
As a class, read the opening text as an introduction to the Explore the Math. Have students brainstorm how to calculate the volume of the water storage tank after identifying its shape (cylinder). After completing Example 2, you might have students do the calculation.

# Explore the Math

In advance, collect empty cans in several different shapes and sizes. Make sure the cans have no sharp edges.

In this exploration, students use a cylindrical can to explore the relationship between the area of the base, the height, and the volume of a cylinder in order to develop a formula for volume. Remind students that they did a similar exploration in section 7.1 using a prism.

Most students will have measured volume in science class using a measuring cup or graduated cylinder. This exploration demonstrates that volumes estimated



by measurement are virtually the same as volumes calculated using the dimensions of the cylinder. In step 6, ensure that students understand that this is true no matter the size or shape of a cylinder.

Have students work in pairs to do the activity. Have different types of measuring devices available, but not visible to students. Before starting the Explore, brainstorm with students what they might need to measure the volume of a cylinder (note that this could be done the previous day). Ask:

- What different methods have you already used to measure the volume of a cylinder? (If students do not make a connection to science or cooking, hold up a container of milk and ask how they could measure out a volume of milk.)
- What equipment do you need to measure volume in this way?
- Where can you get it?
- What is the conversion rate for mL to cm<sup>3</sup>? (1 mL = 1 cm<sup>3</sup>)

Have students collect one can and the other materials they decided they needed to complete this Explore. Make **Master 8 Centimetre Grid Paper** available for students who may find it useful to trace and count the number of  $cm^3$  in the base of their can.

As students work, circulate and observe what they are doing. If necessary, encourage them to extend their thinking from section 7.1 by asking questions such as the following:

- You have now measured the volume of this cylinder. What is the volume?
- If 1 mL is the same as 1 cm<sup>3</sup>, what is another way of reporting the volume of this cylinder?
- How could you use what you learned in section 7.1 to help you calculate the volume of this cylinder?
- How can you use your knowledge of the area of a circle to help you solve this problem?
- What measurements do you need? How can you get them?
- How can you use these measurements?
- How can you calculate the area of the base of this cylinder?
- How can you use that area to calculate the volume of this cylinder?
- Does your calculation agree with the measured volume? Explain why or why not.

Have groups compare the results from the different cylinders they have used. For each cylinder, have them check whether the calculated volume is the same as the measured volume. Discuss why this might be so. Challenge students to generalize what they are observing by developing one or more formulas for calculating the volume of a cylinder. Discuss these formulas in groups and then as a class. Ask:

- How does this formula work?
- Is using the formula the only way to solve this problem? Explain.
- What can you do if you forget the formula? What other strategy(ies) can you use to solve this type of problem?

### **Example 1**

This example illustrates using a formula to estimate and calculate the volume of a cylinder given the radius. Work through the solution as a class. If students understand the shared property that makes calculating the volume of prisms and cylinders similar, then they will understand that a cylinder is a specific case of a general property. Prompt students to explain the benefit of estimating before calculating.

Ask students how else the problem in the Example could be solved. Discuss these strategies, including how else the formula could be stated.

Have students estimate the answer to the Show You Know. In pairs, have them solve the Show You Know using a strategy different from the one in Example 1, and then check their answer using a formula of their choice.

### Example 2

Have students read the problem in Example 2 and ask them how the information provided differs from the information in Example 1. (They are given the diameter instead of the radius of the cylinder.) Ask them to discuss with a partner how this will affect their calculations and then develop a strategy for solving this problem. Discuss the strategies developed.

Encourage students to make a habit of checking whether they are given a measurement for radius or diameter when working with cylinders. Highlight the importance of reading questions carefully. Discuss how estimating an answer might help them identify an error made in calculation.

## **Meeting Student Needs**

- Some students may struggle with radius squared (r<sup>2</sup>) when calculating the volume of a cylinder. Consider having them multiply the radius by the radius (r × r) initially, until they gain understanding. At some point, have students revisit evaluating exponents by hand and with a calculator, and have them discontinue multiplying the radius by the radius.
- Provide students with a similar problem to solve using a familiar context. Or, students might calculate the volume of a cylinder from home.

#### ELL

• Ensure that English language learners understand the following terms: *water storage tanks*, *municipalities*, *radius*, *diameter*, *hockey puck*, *vulcanized rubber*, and *manufacture*. Have students add any new terms to their dictionary.

#### **Gifted and Enrichment**

• Challenge students to research the size of the water storage tank in the school community. Have them use the information from the Did You Know? on page 262 to estimate how long the water in the storage tank would last if it was not replenished.

#### **Common Errors**

- Some students may round numbers too early in their calculations.
- $R_x$  Encourage students to use as many decimal points of accuracy as their calculator provides, and wait until the final answer before rounding. Unless otherwise specified, you may wish to have them round to one decimal place.

#### Answers

#### **Explore the Math**

- **1.-3.** Answers will vary depending on the dimensions of the cans.
- **4.** Answers may vary. Example: Use the formula for the area of a circular base and multiply by the height of a cylinder to determine its volume. Students should verify the formula using an example they develop.

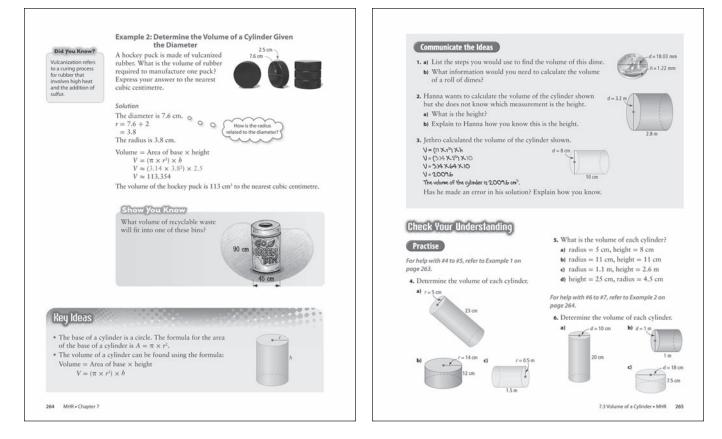
#### Show You Know: Example 1

- a) Answers may vary. Example: Estimate of volume of cylinder: 120 m<sup>3</sup>
- **b)** Volume of cylinder: 131.88 m<sup>3</sup>

#### Show You Know: Example 2

Volume of recyclable waste: 143 066.25 cm<sup>3</sup>

Assessment	Supporting Learning	
Assessment as Learning		
<b>Reflect on Your Findings</b> Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the formula for the volume of a cylinder.	<ul> <li>Encourage students to refer to section 7.2 to help them make the connection between the area and volume of a prism and the area and volume of a cylinder.</li> <li>Make the connections between sections 7.2 and 7.3 explicit for students who might limit themselves to memorizing the formula without understanding it.</li> <li>Check that students understand how to find the area of a circle and use 3.14 as a value for pi.</li> </ul>	
Assessment for Learning		
Example 1 Have students do the Show You Know related to Example 1.	<ul> <li>Encourage students to verbalize their thinking.</li> <li>You may wish to have students work with a partner.</li> <li>Some students may benefit from reactivating their skills with exponents and squaring numbers.</li> <li>Some students may benefit from drawing the base of the cylinder and calculating the area of the base first, before multiplying by the height.</li> <li>Encourage students to avoid using a calculator until the final step of the solution. This should minimize the number of potential errors, and also makes it easier to note where an error is made.</li> <li>Remind students to use estimation to check for the reasonableness of answers.</li> </ul>	
<b>Example 2</b> Have students do the Show You Know related to Example 2.	<ul> <li>Encourage students to verbalize their thinking.</li> <li>You may wish to have students work with a partner.</li> <li>As the first step to solve a problem, have students develop a habit of writing: The radius is Doing this will reduce the chance of using the value of the diameter instead of the radius in applying the formula.</li> <li>Some students may benefit from drawing the base of the cylinder and calculating the area of the base first, before multiplying by the height. Labelling the radius and diameter may help them avoid using the diameter to calculate the area of a circle.</li> </ul>	



# Key Ideas

The Key Ideas summarize the formula for the volume of a right cylinder. Consider having students create a model of a cylinder using either a net or construction paper and tape. Make **Master 8 Centimetre Grid Paper** available for this purpose. Have students label the model with the radius and area of the base at one end, and the height. Students can use the model as a visual reminder of what they learned. Alternatively, have students use an index card to draw and label a diagram of the model and record the formula for volume. Have them store the card in their chapter Foldable.

## **Communicate the Ideas**

Use #1 to gain insight into students' understanding of applying the formula for volume of a cylinder. Some students may be confused about how to respond to a question that does not necessarily refer to measurements. It might be worth noting that many situations in life require measurement before calculating. Have students do #1 individually, and then compare their steps with a partner. For #2, encourage students to make a connection with their work on prisms when identifying the height. If students work in pairs, they should be able to correct any misconceptions about the height of a cylinder. For #3, students are required to identify the error in using the diameter instead of the radius in the formula. Have students do #3 individually.

## **Meeting Student Needs**

• Encourage students to estimate the volume of each cylinder in #1 to #3 by using the volume of a rectangular prism with a square base that completely contains each cylinder. This will give them an upper limit of the volume, and should help catch errors such as using the diameter instead of the radius.

## ELL

• Give English language learners the opportunity to answer the questions in their own language, and then try to explain their thinking in English. Alternatively, allow students to answer the questions using numbers and diagrams.

### **Common Errors**

- Some students may make errors such as using diameter instead of radius, forgetting to square the radius, or confusing squaring with doubling.
- **R**<sub>x</sub> Remind students that estimating before calculating is a helpful way to note errors.

#### **Answers**

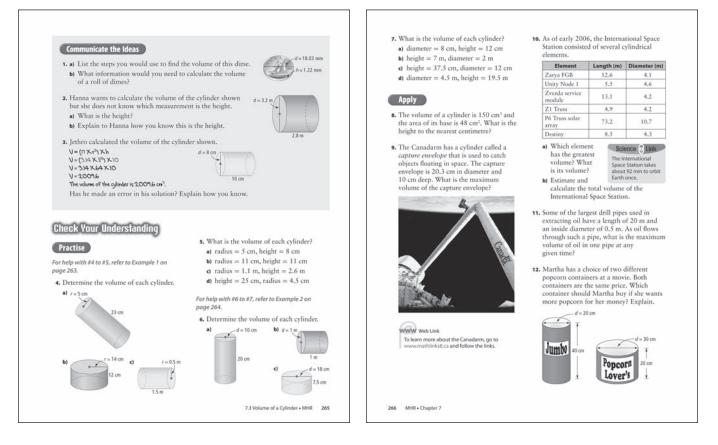
#### **Communicate the Ideas**

- a) Answers may vary. Example: Divide the diameter by 2 to obtain the length of the radius. Square the radius. Multiply by 3.14. Then, multiply the result by the height. The volume of the dime is 311.3 mm<sup>3</sup>.
  - **b)** Answers may vary. Example: You need to know how many dimes are in a roll of dimes.

#### 2. a) Height: 2.8 m

- **b)** Answers may vary. Example: The height is the perpendicular distance between the circular bases.
- **3.** Answers may vary. Example: Jethro has made an error in his solution. He did not use the radius to find the volume; he used the diameter. The correct volume is 502.4 cm<sup>3</sup>.

Assessment	Supporting Learning
Assessment as Learning	
<b>Communicate the Ideas</b> Have all students complete #1 and #3.	<ul> <li>For #1, some students may benefit from handling a dime and then preparing their own labelled diagram.</li> <li>Coach students who struggle with recording the steps for #1 to verbalize the steps and then record the steps.</li> <li>Students who do not identify the error in #3 may benefit from setting up a table with two columns. Have them use column 1 to record the solution using the radius of the cylinder, and column 2 to record the solution using the diameter of the cylinder. This will help students locate the error.</li> <li>To avoid the error in #3, prompt students to write the following statement as the first step in determining the volume of a cylinder: The radius is Alternatively, they can sketch and label each cylinder.</li> </ul>

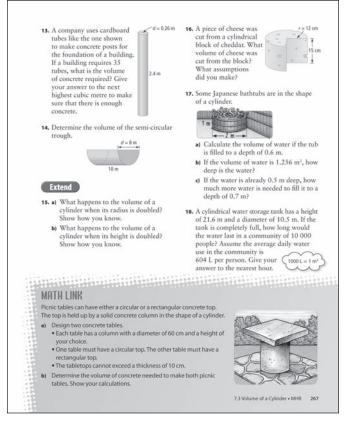


# Check Your Understanding

#### **Practise**

These questions provide students with ample practice in determining the volume of a cylinder, given the radius or diameter, and the height. Encourage students to locate their own errors when their answers do not match those of their peers or the student resource. The limited number of potential numerical errors provides an excellent opportunity for students to develop their error analysis skills.

Consider having students estimate and then calculate the volumes for #4 to #7. Encourage students to draw and label diagrams for #5 and #7.



## Apply

These questions provide a range of contexts for students to apply their skills in solving problems involving volume of cylinders. Encourage students to use their estimation and error analysis skills. Consider giving students some choice in the questions they do.

## Extend

For #15, students are required to consider abstractly how volume changes as the radius or height changes. The remaining questions involve using volume calculations to solve more complex problems.

## Math Link

The Math Link provides an opportunity for students to apply their knowledge of the volume of a cylinder and a rectangular prism. Invite artistic students to draw several examples of tables that fit the criteria. This will help clarify the expectations for the activity, as well as provide a cue to students who have difficulty choosing their own measurements.

## **Meeting Student Needs**

- Providing a template for solving the volume of a cylinder with a place for students to write the radius and height should help them avoid using the diameter instead of the radius in the formula. Walk through the first few problems by providing the formula and the first step, with a place for students to record the values. Move toward having students provide the steps.
- For #10, encourage students to draw and label a diagram as a first step to solving the problem.
- Some students may struggle with the text-dense questions in the Apply and Extend sections. Help students extract the information they need to answer each question.
- Consider allowing students to work in pairs. They might work on one question together and then work individually on the next one. Ensure that students complete a number of questions individually.
- Provide **BLM 7–9 Section 7.3 Extra Practice** to students who would benefit from more practice.

## ELL

- Allow newer English language learners to focus their work on the Practise questions and #9 in Apply as the balance of the questions in the Apply and Extend sections are text dense. The Practise questions allow English language learners to demonstrate what they know. As they develop language skills, have students try some word problems.
- English language learners may not be familiar with the following terms: *Canadarm*, *capture envelope*, *International Space Station*, *cylindrical elements*, *extracting oil*, *cardboard tubes*, *concrete posts*, and *semi-circular trough*. Use the visuals in the student resource, visuals from other sources, and descriptions from other students to help describe each of these terms.

## Answers

#### **Math Link**

- a) Designs will vary. Ensure that students choose reasonable dimensions for a table with a circular top and one with a rectangular top.
- **b)** The volume of concrete will vary depending on dimensions chosen. Check that students have calculated the volume of the top and the volume of the base to determine the total volume of concrete for each table design.

Assessment	Supporting Learning	
Assessment <i>for</i> Learning		
Practise Have students do #4a) and b), #5a) and d), #6a) and b), #7a) and b). Students who have no problems with these questions can go on to the Apply questions.	<ul> <li>Note that #4 and #5 are similar questions that deal with determining the volume of a cylinder given the radius.</li> <li>Provide additional coaching with Example 1 to students who need help with #4a) and b). Coach students through #4a) and b), and then have them try part c) on their own.</li> <li>For #5a) and d), some students may benefit from drawing and labelling a sketch before determining the volume. Suggest that they use a diagram in #4 as a model. Have students who have difficulty with #5a) and d) verbalize the process, coach them through corrections, and then have them try parts b) and c) on their own.</li> <li>Note that #6 and #7 are similar questions that deal with determining the volume of a cylinder given the diameter.</li> <li>Provide additional coaching with Example 2 to students who need help with #6a) and b). Coach students through #6a) and b), and then have them try part c) on their own.</li> <li>For #7a) and b), some students may benefit from drawing and labelling a sketch before determining the volume. Suggest that they use a diagram in #6 as a model. Have students who have difficulty with #7a) and b) verbalize the process, coach them through corrections, and then have them try parts c) and d) on their own.</li> </ul>	
Math Link The Math Link on page 267 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 279.	<ul> <li>This is a multi-step problem. Encourage students to break the problem into individual steps.</li> <li>Draw and label a table with a circular top and a cylindrical column.</li> <li>Draw and label a table with a rectangular top and a cylindrical column.</li> <li>Find the total of the volume of the table with the circular top and the volume of the cylindrical column.</li> <li>Find the total of the volume of the table with the rectangular top and the volume of the cylindrical column.</li> <li>Find the total of the volume of the table with the rectangular top and the volume of the cylindrical column.</li> <li>Find the total of the volume of the table with the rectangular top and the volume of the cylindrical column.</li> <li>Encourage students to draw and label diagrams for each table.</li> <li>Encourage students to write out the formula for each calculation.</li> <li>Make sure that students understand the meaning of <i>exceed</i> in the context of the problem.</li> <li>To help them get started, some students may benefit from using <b>BLM 7–10 Section 7.3 Math Link</b>, which provides scaffolding for this activity.</li> </ul>	
Assessment <i>as</i> Learning		
<ul> <li>Math Learning Log</li> <li>Have students answer the following question:</li> <li>A right cylinder is not a right prism, yet it shares the same general property of volume: volume = area of base × height. What physical property do cylinders and prisms share that makes this so?</li> </ul>	<ul> <li>Make a variety of right prisms, cubes, and cylinders available for students to help them make the connection that cylinders and triangular prisms each have two faces that can be considered a base, while rectangular prisms and cubes have six faces that can be considered a base.</li> <li>Having students verbally describe a formula for finding the volume of a 3-D object and then linking their explanation to the different shapes of cylinders and prisms may help them in answering the question.</li> <li>Ensure that students understand that all 3-D objects (including prisms and cylinders) have volume.</li> <li>Ask students to describe what makes each shape different and how that is reflected in the calculation of its volume.</li> </ul>	