Solving Problems Involving Prisms and Cylinders

7.4

***MathLinks 8*, pages 268–275**

Suggested Timing
80–100 minutes

Materials
- centimetre cubes
- centimetre grid paper
- ruler
- transparent strips (optional)
- calculator
- rolled up newspaper or magazine (optional)
- modelling clay (optional)
- tape measure (optional)

Blackline Masters
Master 8 Centimetre Grid Paper
BLM 7–3 Chapter 7 Warm-Up
BLM 7–11 Section 7.4 Extra Practice
BLM 7–12 Section 7.4 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☐ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

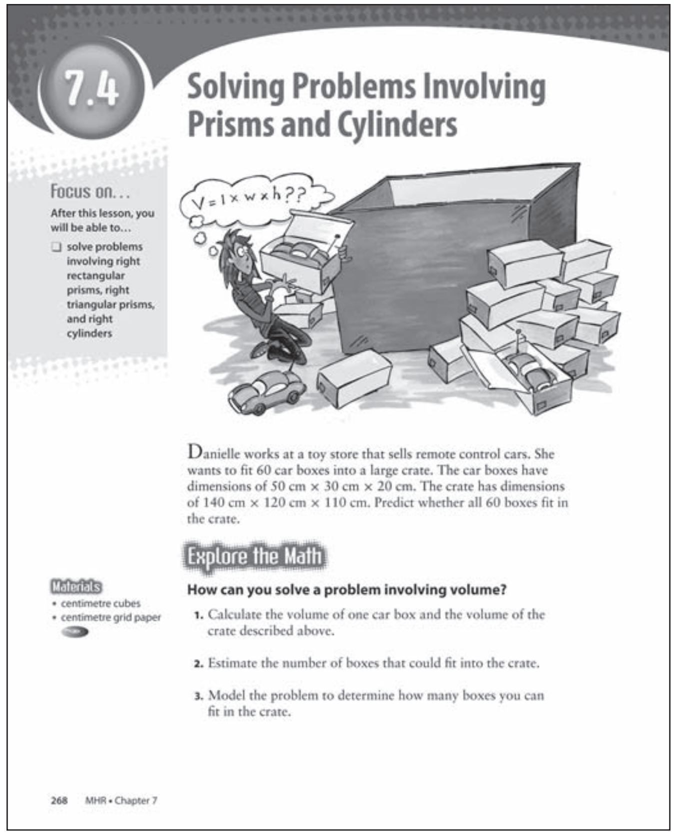
Specific Outcomes
SS4 Develop and apply formulas for determining the volume of right prisms and right cylinders.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–7, Math Link
Typical	1–5, 7, 9–11, 13, 14, Math Link
Extension/Enrichment	1, 2, 14–21, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.

As a class, discuss how people take the volume of prisms and cylinders into account in daily life. Have students use examples from their own life such as packing a cooler for a picnic, loading a laundry hamper, or filling a granary or silo.



7.4 Solving Problems Involving Prisms and Cylinders

Focus on...
After this lesson, you will be able to...
- ☐ solve problems involving right rectangular prisms, right triangular prisms, and right cylinders

Danielle works at a toy store that sells remote control cars. She wants to fit 60 car boxes into a large crate. The car boxes have dimensions of 50 cm × 30 cm × 20 cm. The crate has dimensions of 140 cm × 120 cm × 110 cm. Predict whether all 60 boxes fit in the crate.

Explore the Math

Materials
- centimetre cubes
- centimetre grid paper

How can you solve a problem involving volume?

1. Calculate the volume of one car box and the volume of the crate described above.
2. Estimate the number of boxes that could fit into the crate.
3. Model the problem to determine how many boxes you can fit in the crate.

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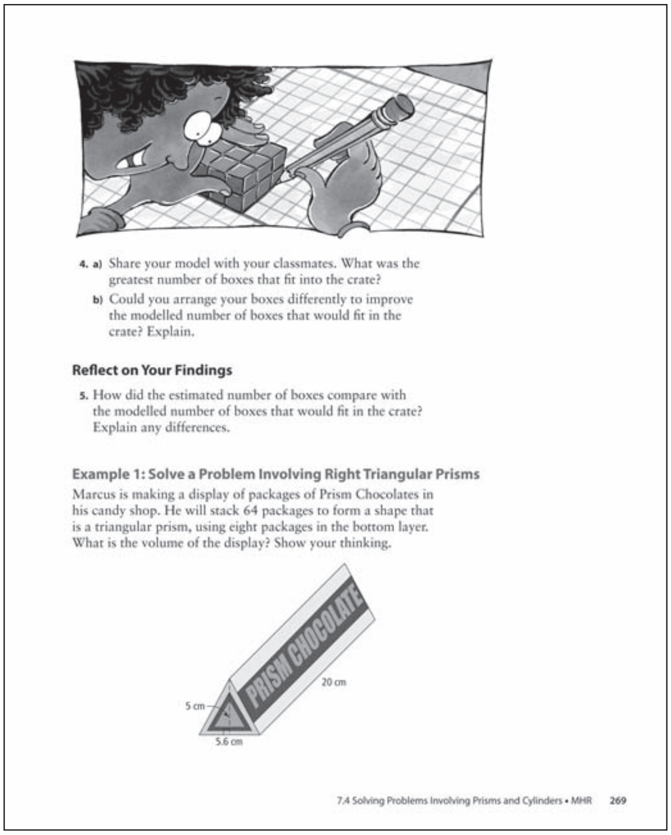
As a class, read the opening text as an introduction to the Explore the Math. Have students make predictions and explain the rationale for their predictions. After they complete the Explore the Math have them compare their predictions to the calculated answer.

Explore the Math

Encourage students to think of this mathematical exploration as similar to a scientific exploration. You may wish to highlight the connection between math and science by saying that mathematicians are often referred to as scientists with no labs.

Have students work in pairs to complete the activity. Provide them with centimetre cubes and **Master 8 Centimetre Grid Paper**. As students work, circulate and ask questions such as the following:
- How can you model the car box?
- How can you model the crate?
- How will you place the car boxes into the crate?
- Is there more than one way to place the crate? Explain.



4. a) Share your model with your classmates. What was the greatest number of boxes that fit into the crate?
   b) Could you arrange your boxes differently to improve the modelled number of boxes that would fit in the crate? Explain.

Reflect on Your Findings

5. How did the estimated number of boxes compare with the modelled number of boxes that would fit in the crate? Explain any differences.

Example 1: Solve a Problem Involving Right Triangular Prisms

Marcus is making a display of packages of Prism Chocolates in his candy shop. He will stack 64 packages to form a shape that is a triangular prism, using eight packages in the bottom layer. What is the volume of the display? Show your thinking.

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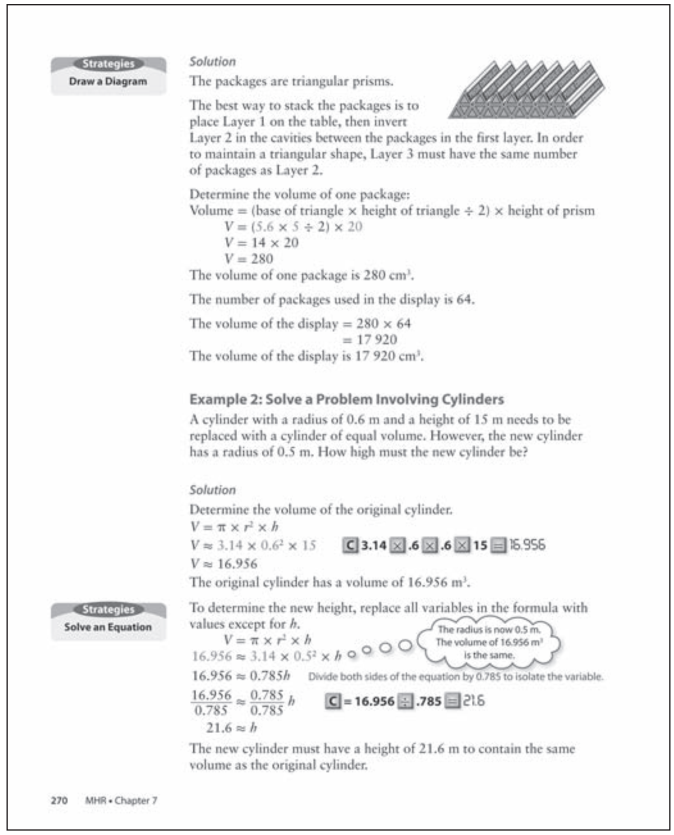

Strategies
Draw a Diagram

Solution

The packages are triangular prisms.

The best way to stack the packages is to place Layer 1 on the table, then invert Layer 2 in the cavities between the packages in the first layer. In order to maintain a triangular shape, Layer 3 must have the same number of packages as Layer 2.

Determine the volume of one package:
Volume = (base of triangle × height of triangle ÷ 2) × height of prism
$V = (5.6 \times 5 \div 2) \times 20$
$V = 14 \times 20$
$V = 280$
The volume of one package is 280 cm^3.

The number of packages used in the display is 64.

The volume of the display = 280 × 64
= 17 920
The volume of the display is 17 920 cm^3.

Example 2: Solve a Problem Involving Cylinders

A cylinder with a radius of 0.6 m and a height of 15 m needs to be replaced with a cylinder of equal volume. However, the new cylinder has a radius of 0.5 m. How high must the new cylinder be?

Solution

Determine the volume of the original cylinder.
$V = \pi \times r^2 \times h$
$V \approx 3.14 \times 0.6^2 \times 15$ C 3.14 × .6 × .6 × 15 = 16.956
$V \approx 16.956$
The original cylinder has a volume of 16.956 m^3.

Strategies
Solve an Equation

To determine the new height, replace all variables in the formula with values except for h.
$V = \pi \times r^2 \times h$
$16.956 \approx 3.14 \times 0.5^2 \times h$ (The radius is now 0.5 m. The volume of 16.956 m^3 is the same.)
$16.956 \approx 0.785h$ Divide both sides of the equation by 0.785 to isolate the variable.
$\frac{16.956}{0.785} \approx \frac{0.785}{0.785}h$ C = 16.956 ÷ .785 = 21.6
$21.6 \approx h$
The new cylinder must have a height of 21.6 m to contain the same volume as the original cylinder.

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- Is there more than one way the boxes will fit? Explain.
- How can you fit the car boxes to get the maximum number into the crate?
- How can you use what you know about volume to help you solve this problem?
- How can you use your knowledge of divisibility rules to help you here?
- How can you use estimation to help you here?

When students share their different models with the class, ask:

- What are the different ways to fit the car boxes into the crate?
- What is the highest number of car boxes that will fit in the crate?
- Is there more than one way to fit in this number of boxes?
- How does your answer compare to the answer you would get if you calculated the volume of one car box and then divided that into the volume of the crate?
- Why is that calculation not a good choice of strategies for solving this problem?

As a class, discuss the various strategies that students used to solve this problem and how they connected this work on volume to other areas of mathematics.

Example 1

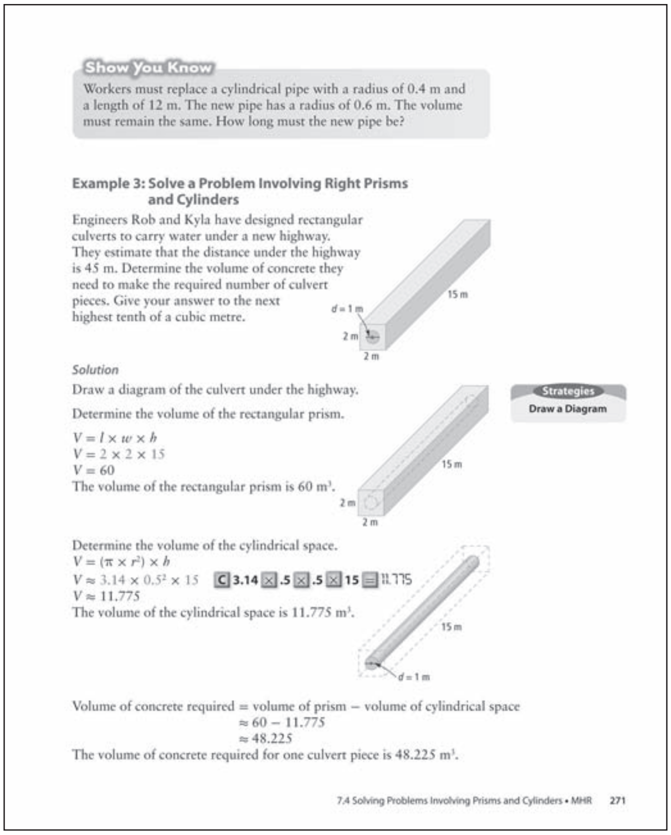
Have students look at the chocolate box on the bottom of page 269. Challenge them to solve this problem before turning the page. Ask:

- How can you fit these prisms together to provide a solid stack? (Encourage them to draw a diagram showing their thinking. Some students may find it helpful to use **Master 7 Isometric Dot Paper** to draw a front view of the stack.)
- Is there more than one way?
- How can you use your knowledge of the area of a triangle and the volume of a triangular prism to solve this problem?
- Is there another way to solve this problem? Explain. (For example, the base of the stack will be 5.6 cm × 8 or 44.8 cm. The display will be 8 layers or 40 cm high. The area of the front of the display is 896 cm^2. The volume of the display is 17 920 cm^3.)

Discuss the different strategies that students have and the advantages and disadvantages of each strategy.

Example 2

In this example, students solve a problem involving right cylinders. Encourage students to predict whether the new height of the cylinder will be longer or shorter than the original.



Show You Know

Workers must replace a cylindrical pipe with a radius of 0.4 m and a length of 12 m. The new pipe has a radius of 0.6 m. The volume must remain the same. How long must the new pipe be?

Example 3: Solve a Problem Involving Right Prisms and Cylinders

Engineers Rob and Kyla have designed rectangular culverts to carry water under a new highway. They estimate that the distance under the highway is 45 m. Determine the volume of concrete they need to make the required number of culvert pieces. Give your answer to the next highest tenth of a cubic metre.

Solution

Draw a diagram of the culvert under the highway.

Strategies
Draw a Diagram

Determine the volume of the rectangular prism.

$V = l \times w \times h$
$V = 2 \times 2 \times 15$
$V = 60$
The volume of the rectangular prism is 60 m^3.

Determine the volume of the cylindrical space.
$V = (\pi \times r^2) \times h$
$V \approx 3.14 \times 0.5^2 \times 15$ C 3.14 × .5 × .5 × 15 = 11.775
$V \approx 11.775$
The volume of the cylindrical space is 11.775 m^3.

Volume of concrete required = volume of prism − volume of cylindrical space
$\approx 60 - 11.775$
≈ 48.225
The volume of concrete required for one culvert piece is 48.225 m^3.

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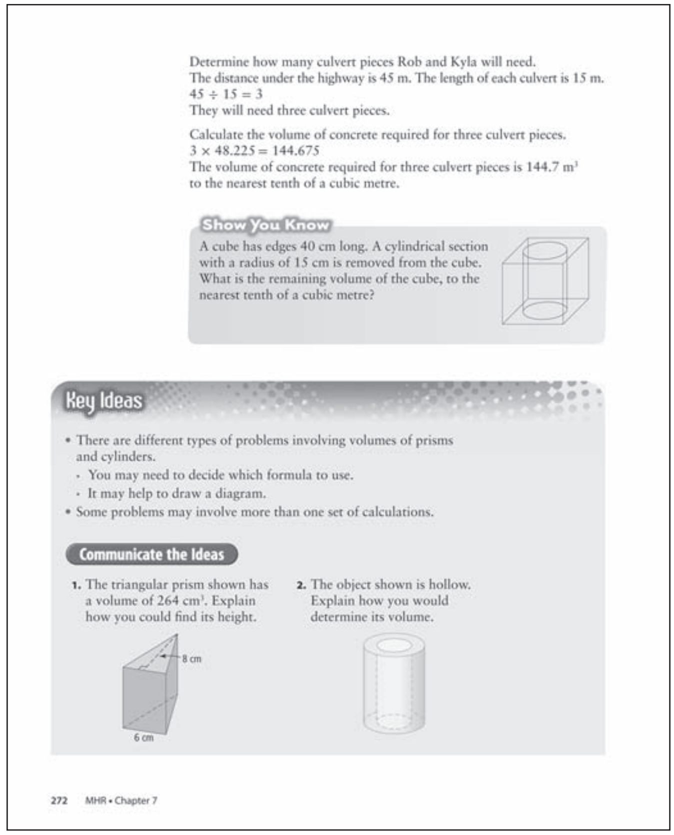

Determine how many culvert pieces Rob and Kyla will need.
The distance under the highway is 45 m. The length of each culvert is 15 m.
$45 \div 15 = 3$
They will need three culvert pieces.

Calculate the volume of concrete required for three culvert pieces.
$3 \times 48.225 = 144.675$
The volume of concrete required for three culvert pieces is 144.7 m^3 to the nearest tenth of a cubic metre.

Show You Know

A cube has edges 40 cm long. A cylindrical section with a radius of 15 cm is removed from the cube. What is the remaining volume of the cube, to the nearest tenth of a cubic metre?

Key Ideas

- There are different types of problems involving volumes of prisms and cylinders.
  - You may need to decide which formula to use.
  - It may help to draw a diagram.
- Some problems may involve more than one set of calculations.

Communicate the Ideas

1. The triangular prism shown has a volume of 264 cm^3. Explain how you could find its height.
2. The object shown is hollow. Explain how you would determine its volume.

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As students consider this problem, ask:

- How can a diagram help in solving problems such as this? (Have students draw a diagram and discuss how it helps them visualize the situation.)
- What do you know about the first cylinder?
- What do you know about the second cylinder?
- What do you need to find out?
- How can you use your knowledge of volume to help you?
- What other skills do you need to solve this problem?
- Are there other ways to solve this problem? Explain.

Have students work in pairs or groups of three and use a strategy of their choice to answer the Show You Know.

Example 3

In this example, students solve a problem involving right prisms and cylinders. Have pairs or small groups of students work through the solution. You may wish to provide coaching to some groups by asking:

- How many different shapes are here? (rectangular prism and cylinder)
- What shape or what part of a shape will the cement form?
- How can you find the volume of that shape?
- What do you know about volume that will help you here?
- How many culverts like this will you need? How do you know?
- How much cement will you need for that number of culverts?

Have groups present their solutions to the class. Discuss what strategies students used.

Ask the same groups to solve the problem in the Show You Know using the strategy they think is the most useful. You may wish to have students consider what the beginning measurements are in and what the answer needs to be in. Discuss as a class when students might wish to convert from cm to m and how the timing of this conversion might make a difference. To connect length, area, and volume measurements, you may wish to have students develop models of 1 cm, 1 cm^2, 1 cm^3, 1 m, 1 m^2, and 1 m^3. Have them connect back to their work on conversion rates in Chapter 2 and discuss conversion rates for cm to m, cm^2 to m^2, and cm^3 to m^3 before solving this Show You Know.

Meeting Student Needs

- Students who have difficulty visualizing how the packages are being stacked in Example 1 may benefit from using 15 identical triangular prisms to build the first two layers of the model with eight prisms in Layer 1 and seven prisms in Layer 2.
- Encourage students who experience difficulties with these problems to write out the question, circle the relevant values and units, and highlight the key words that provide information about what is wanted and what calculations are needed.

ELL

- Ensure that English language learners understand the following terms: *remote control cars*, *crate*, *verify*, *stack*, *invert*, *pipe*, and *culvert*.

Gifted and Enrichment

- Extend the Explore the Math by asking students how large a crate Danielle needs to fit all 60 car boxes. There are several answers. Example: 150 cm × 120 cm × 100 cm, if the car boxes are placed with either the 20 cm × 30 cm side down or 30 cm × 50 cm side down. For exactly 60 boxes with the 20 cm × 50 cm side down, the crate would be 100 cm × 200 cm × 90 cm.
- Challenge students to extend their solution to Example 1 by using ten packages in the first layer. Have them find a general solution for calculating the total number of packages needed for any number of packages used in the bottom layer.

Common Errors

- Some students may try to solve problems without drawing a diagram.

R_x Encourage students to draw and label a diagram for each problem. Not doing so increases the likelihood that students will make errors when choosing the formulas and operations required to solve a problem.

Answers

Explore the Math

1. Volume of one car box: 30 000 cm^3; Volume of the crate: 1 848 000 cm^3
2. Answers may vary. Example: The estimate is that 60 boxes will fit in the crate.
3. There are many possibilities for modelling the problem. Examples:
   - Use a rectangle drawn on grid paper to model the placement of a single layer of car boxes, and then determine how many layers can fit in the model of the crate.
   - Use a carton and equal-sized boxes to model the problem.

   Using the model of their choice, students may check each face of the car box on the bottom of the crate.
4. **a)**, **b)** Answers will vary. Note that there are three ways to fit the car boxes, depending on which face is downward.
5. Answers may vary. Example: The estimate of the number of boxes was lower than the modelled number of boxes. As a class, discuss any differences between the estimates and the actual models.

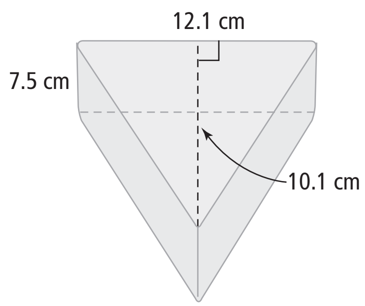
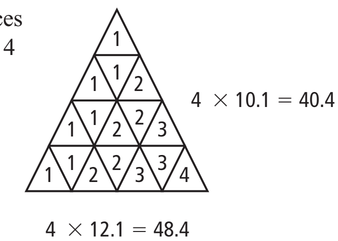
Show You Know: Example 2

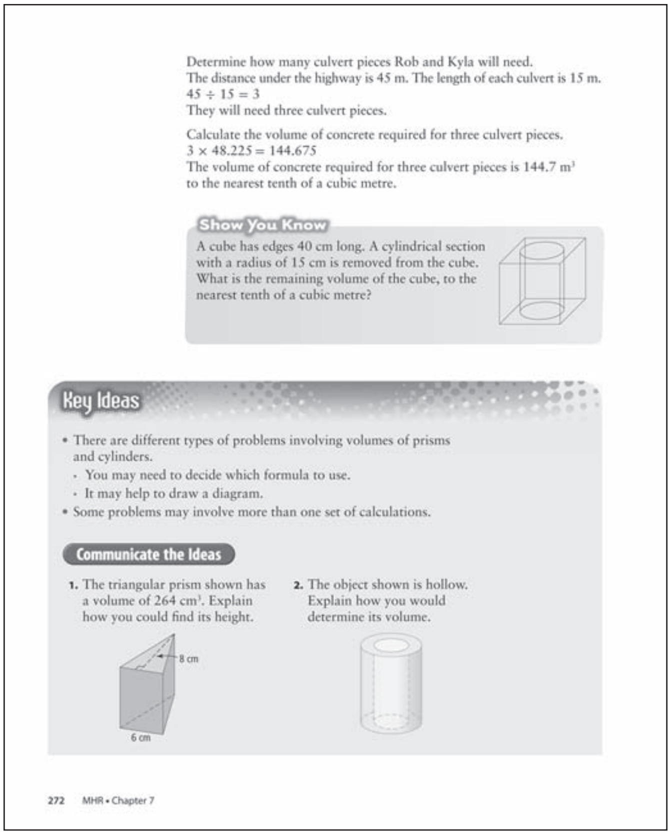
5.3 m

Show You Know: Example 3

35 740 cm^3

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings. Help students make a connection with the idea that doing mathematics is often the same as doing science. Both involve stating a hypothesis and carrying out an experiment to test the hypothesis. Ask students to identify the hypothesis in their exploration.	• Students who have difficulty with modelling the problem may not understand why the dimensions of the car box change. They may not be able to complete the rest of the exploration. Have students consider the two dimensions of area instead by fitting as many car boxes as possible on the bottom of the crate. Students can relate this to their understanding of proportional diagrams, which should then allow them to extend to three dimensions. • As a class, discuss fitting small boxes into a crate. Ask if this problem can be solved by dividing the crate's volume by the volume of an individual car box. Will that always work out evenly? If the boxes do not fit exactly, how would students handle that? Would they leave empty space in the crate? Would they cut some of the boxes to fit? These questions serve as important "think" questions for students when solving a real-world problem. • Extend the discussion by asking students to substitute a water tank for the crate and fill it with water using a pail. Assume that the water tank and the pail are the same size respectively as the crate and the car box. In this case, 60 pails full of water will fit inside the water tank.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 There is no Show You Know related to Example 1. Consider providing the following problem: A cheese factory has 16 triangular prisms that contain cheese. The manager wants to build a large triangular prism using 4 pieces of cheese for the base. The prism should have a base of 48.4 cm and a height of 40.4 cm. **a)** Are there enough cheeses to make this large prism? Show how you know. **b)** Calculate the volume of the display. Verify your calculation. 	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students review the visual in Example 1 before starting to sketch how the cheeses will be put together in the display. Also have them consider which part of the cheese should be used as the base of the large prism. Then, ask students to draw and label a diagram showing what is planned. Have them calculate the number of prisms needed, the size of the base of the structure, and how high the resultant structure will be. • Once students have drawn a suitable structure, have them calculate the total volume of the structure. They can check their work by calculating the volume of one cheese prism and multiplying by 16. • It may benefit some students to be given the picture of the structure and asked the volume of the structure given the total base and height and the base and height of one piece of cheese. Answer: **a)** Yes, there are enough cheeses. The diagram shows 4 pieces in layer 1, 3 in each of layers 2 and 3, 2 in each of layers 4 and 5, and 1 in each of layers 6 and 7. $4 + 3 + 3 + 2 + 2 + 1 + 1 = 16$  **b)** Volume of large prism $= (48.4 \times 40.4 \div 2) \times 7.5$ $= 977.68 \times 7.5$ $= 7332.6$ The volume of the display is 7332.6 cm^3. Volume of one cheese $= (12.1 \times 10.1 \div 2) \times 7.5$ $= 61.105 \times 7.5$ $= 458.2875$ Volume of 16 cheeses $= 458.2875 \times 16$ $= 7332.6$ The volume of 16 cheeses is 7332.6 cm^3. Both answers are the same. The volume is correct.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to draw and label a diagram. • Model the situation to help students understand that the length of the new pipe must be shorter in order for the volume to remain the same. Roll up a newspaper or magazine, and then roll it less tightly, resulting in a greater radius. Students should observe that volume increases when radius increases and length (of the rolled newspaper) stays the same. Therefore, if the volume must remain the same and the radius increases, the length (height) must be shorter. • Some students may benefit from a visual approach to help understand the problem. If modelling clay is available, have students roll out a long tube and measure its dimensions and calculate its volume. Then, using the same amount of modelling clay, have students make a shorter, fatter roll. Have them measure its dimensions and calculate its volume. Both rolls have the same volume. Using the same piece of clay will show students that volume does not change, even though radius and height may change.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to draw and label a diagram and use shading to indicate the volume being asked for. • Some students may benefit from a visual approach, such as placing a cylindrical can into a box, to help understand the problem. They might use the representation to sketch and label a diagram.



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Show You Know

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Key Ideas

- There are different types of problems involving volumes of prisms and cylinders.
  - You may need to decide which formula to use.
  - It may help to draw a diagram.
- Some problems may involve more than one set of calculations.

Communicate the Ideas

1. The triangular prism shown has a volume of 264 cm^3. Explain how you could find its height.
2. The object shown is hollow. Explain how you would determine its volume.

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Key Ideas

The Key Ideas summarize the main points about solving problems involving volumes of prisms and cylinders. Have students use index cards to sketch a cube, a right rectangular prism, a cylinder, and a right triangular prism with the dimensions labelled using variables rather than numbers. Have them write the formula for volume beside each 3-D shape.

Communicate the Ideas

These questions allow students to demonstrate their understanding of solving problems involving the volume of prisms and cylinders. Use #1 to help you identify students who are having difficulty with the dimensions needed to determine volume of a triangular prism. Consider having students work with a partner and use a sage and scribe technique, with the sage talking through the steps and the scribe recording the steps. Have students switch roles for #2, which asks students to explain the steps to solve the problem without doing any calculations.

Meeting Student Needs

- Students may benefit from using a template to help them organize information provided in a problem. Encourage students to use a highlighter and identify key information or write down key information before beginning any calculations.

ELL

- Explain the term *hollow*.

Common Errors

- Some students may be challenged by problems involving multiple steps, multiple objects, or multiple measurements.

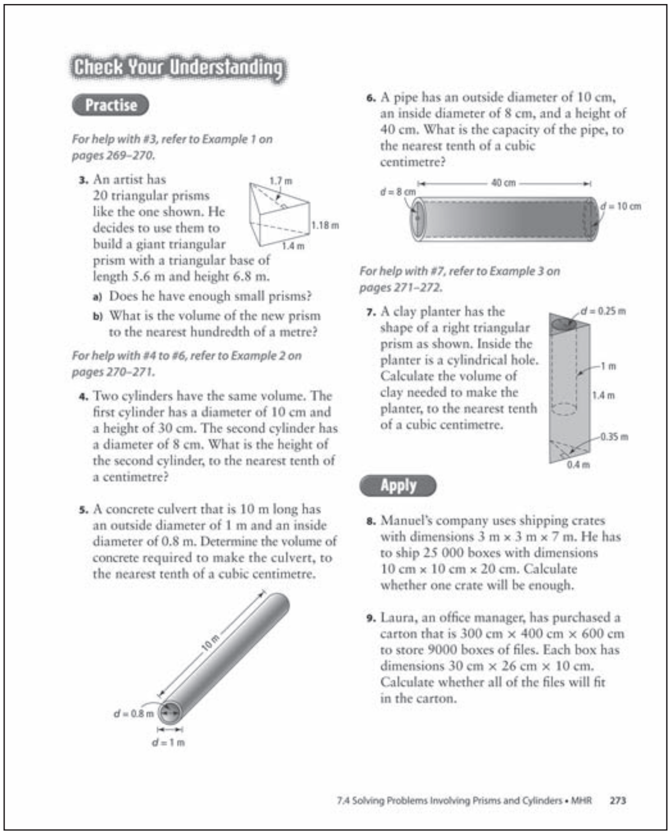
R_x Encourage students to try each problem. Help them identify key information and the steps needed to solve the problem. Have them draw and label a diagram, if appropriate.

Answers

Communicate the Ideas

1. Answers may vary. Example: Divide the volume by the area of the base.
2. Answers may vary. Example: Subtract the volume of the smaller cylinder from the volume of the larger cylinder.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and #2.	• Allow students to present the steps to solve #1 and #2 either in written form or orally. Some students may benefit if you coach them through talking out the steps and record the steps for them. • Some students may benefit from approaching #1 in reverse order, in other words, by explaining how to determine the volume and then isolating the height. • Help students unable to articulate the steps for solving #2 by providing the first step (determine the volume of the larger cylinder). With coaching, students should be able to list the remaining steps. They may benefit from referring to Example 3. • Talking through the steps with a partner allows students to explain how they would go about solving a problem, which may vary from another student's approach.



Check Your Understanding

Practise

For help with #3, refer to Example 1 on pages 269–270.

3. An artist has 20 triangular prisms like the one shown. He decides to use them to build a giant triangular prism with a triangular base of length 5.6 m and height 6.8 m.

a) Does he have enough small prisms?

b) What is the volume of the new prism to the nearest hundredth of a metre?

For help with #4 to #6, refer to Example 2 on pages 270–271.

4. Two cylinders have the same volume. The first cylinder has a diameter of 10 cm and a height of 30 cm. The second cylinder has a diameter of 8 cm. What is the height of the second cylinder, to the nearest tenth of a centimetre?

5. A concrete culvert that is 10 m long has an outside diameter of 1 m and an inside diameter of 0.8 m. Determine the volume of concrete required to make the culvert, to the nearest tenth of a cubic centimetre.

6. A pipe has an outside diameter of 10 cm, an inside diameter of 8 cm, and a height of 40 cm. What is the capacity of the pipe, to the nearest tenth of a cubic centimetre?

For help with #7, refer to Example 3 on pages 271–272.

7. A clay planter has the shape of a right triangular prism as shown. Inside the planter is a cylindrical hole. Calculate the volume of clay needed to make the planter, to the nearest tenth of a cubic centimetre.

Apply

8. Manuel's company uses shipping crates with dimensions 3 m × 3 m × 7 m. He has to ship 25 000 boxes with dimensions 10 cm × 10 cm × 20 cm. Calculate whether one crate will be enough.

9. Laura, an office manager, has purchased a carton that is 300 cm × 400 cm × 600 cm to store 9000 boxes of files. Each box has dimensions 30 cm × 26 cm × 10 cm. Calculate whether all of the files will fit in the carton.

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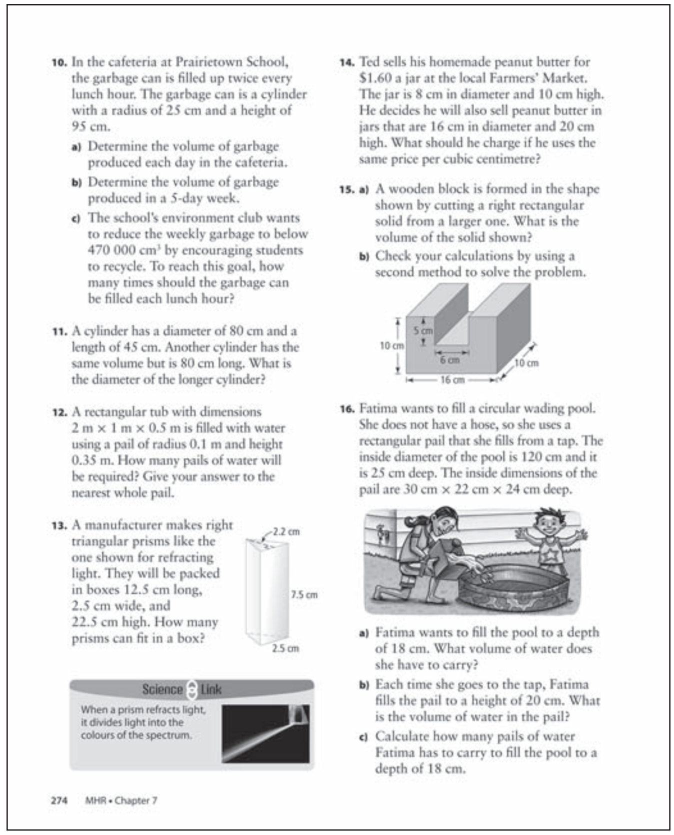

10. In the cafeteria at Prairietown School, the garbage can is filled up twice every lunch hour. The garbage can is a cylinder with a radius of 25 cm and a height of 95 cm.

a) Determine the volume of garbage produced each day in the cafeteria.

b) Determine the volume of garbage produced in a 5-day week.

c) The school's environment club wants to reduce the weekly garbage to below 470 000 cm^3 by encouraging students to recycle. To reach this goal, how many times should the garbage can be filled each lunch hour?

11. A cylinder has a diameter of 80 cm and a length of 45 cm. Another cylinder has the same volume but is 80 cm long. What is the diameter of the longer cylinder?

12. A rectangular tub with dimensions 2 m × 1 m × 0.5 m is filled with water using a pail of radius 0.1 m and height 0.35 m. How many pails of water will be required? Give your answer to the nearest whole pail.

13. A manufacturer makes right triangular prisms like the one shown for refracting light. They will be packed in boxes 12.5 cm long, 2.5 cm wide, and 22.5 cm high. How many prisms can fit in a box?

Science Link

When a prism refracts light, it divides light into the colours of the spectrum.

14. Ted sells his homemade peanut butter for $1.60 a jar at the local Farmers' Market. The jar is 8 cm in diameter and 10 cm high. He decides he will also sell peanut butter in jars that are 16 cm in diameter and 20 cm high. What should he charge if he uses the same price per cubic centimetre?

15. a) A wooden block is formed in the shape shown by cutting a right rectangular solid from a larger one. What is the volume of the solid shown?

b) Check your calculations by using a second method to solve the problem.

16. Fatima wants to fill a circular wading pool. She does not have a hose, so she uses a rectangular pail that she fills from a tap. The inside diameter of the pool is 120 cm and it is 25 cm deep. The inside dimensions of the pail are 30 cm × 22 cm × 24 cm deep.

a) Fatima wants to fill the pool to a depth of 18 cm. What volume of water does she have to carry?

b) Each time she goes to the tap, Fatima fills the pail to a height of 20 cm. What is the volume of water in the pail?

c) Calculate how many pails of water Fatima has to carry to fill the pool to a depth of 18 cm.

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Check Your Understanding

Practise

These questions offer students the opportunity to revisit problems similar to the examples. For #3, have students consider the best way to orient the prisms in such a structure. It might be time-consuming but useful to have them sketch the entire structure.

Encourage students to sketch and label diagrams for the scenarios in #4, #8, and #9.

Apply

The Apply questions provide a range of contexts for students to apply their problem solving skills. Consider giving students some choice in the questions they do. For #13, direct students to the Science Link that helps explain prisms that refract light.

Extend

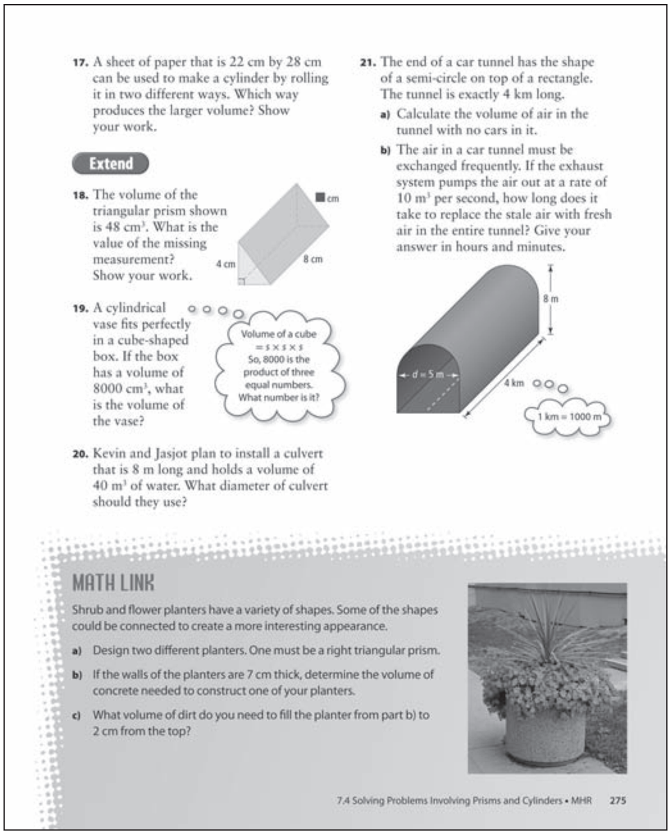
Most of these questions extend thinking by providing the volume and asking for a missing dimension. For #21, students combine what they have learned about solving problems involving volume with earlier work on rates.

Math Link

The Math Link provides another opportunity for students to solve multi-step problems involving the volume of prisms and cylinders. Students who have difficulty visualizing the size of a planter may benefit from using a tape measure to measure actual planters or objects that are about the same size as planters.

Meeting Student Needs

- Some students may struggle with the text-dense questions in the Practice, Apply, and Extend sections. Help students extract the information they need to answer each question.
- Consider allowing students to work in pairs. They might work on one question together and then work individually on the next one. Ensure that students complete a number of questions individually.
- Provide **BLM 7–11 Section 7.4 Extra Practice** to students who would benefit from more practice.


17. A sheet of paper that is 22 cm by 28 cm can be used to make a cylinder by rolling it in two different ways. Which way produces the larger volume? Show your work.

Extend

18. The volume of the triangular prism shown is 48 cm^3. What is the value of the missing measurement? Show your work.

19. A cylindrical vase fits perfectly in a cube-shaped box. If the box has a volume of 8000 cm^3, what is the volume of the vase?

20. Kevin and Jasjot plan to install a culvert that is 8 m long and holds a volume of 40 m^3 of water. What diameter of culvert should they use?

21. The end of a car tunnel has the shape of a semi-circle on top of a rectangle. The tunnel is exactly 4 km long.
   a) Calculate the volume of air in the tunnel with no cars in it.
   b) The air in a car tunnel must be exchanged frequently. If the exhaust system pumps the air out at a rate of 10 m^3 per second, how long does it take to replace the stale air with fresh air in the entire tunnel? Give your answer in hours and minutes.

MATH LINK

Shrub and flower planters have a variety of shapes. Some of the shapes could be connected to create a more interesting appearance.

a) Design two different planters. One must be a right triangular prism.
b) If the walls of the planters are 7 cm thick, determine the volume of concrete needed to construct one of your planters.
c) What volume of dirt do you need to fill the planter from part b) to 2 cm from the top?

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ELL

- Assign fewer questions to English language learners so they can focus on understanding the math.
- English language learners may not be familiar with the following terms: *clay planter*, *carton*, *refracting light*, *Farmer's Market*, *wading pool*, *hose*, *car tunnel*, *exchanged*, *exhaust system*, and *stale air*. Use the visuals in the student resource and visuals from other sources to help describe each of these terms.

Gifted and Enrichment

- Have students do the following question: Martin wants to build a square pool 2 m deep. He wants it to hold 140 m^3 of water when it is 70% full. The walls and floor will be 1 m thick. The walls will be placed on the floor of the pool. What is the volume of concrete he will need? (Answer: 232 m^3)

Common Errors

- Some students may calculate the inner dimensions of a planter as the outer dimension minus the thickness of one wall.

R_x Have them draw a picture of the planter and identify the number of dimensions affected by the width of the container walls.

Answers

Math Link

a) Designs and dimensions will vary. Ensure that they are reasonable. Ask students who have difficulty with one of the shapes to do the calculations for that shape of the planter.

b) Volumes will vary depending on dimensions. Ensure that students correctly list the dimensions. Remind them that all the walls, including the bottom, are 7 cm thick.

c) Volumes will vary depending on dimensions. Ensure that students subtract 2 cm from the inside depth of the planter before calculating.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise Have students do #3, #4, and #7. Students who have no problems with these questions can go on to the Apply questions.	• Encourage students who need support in solving contextual problems to use the following strategies: – Draw and label a diagram. – Draw a second diagram in cases when a 3-D object sits inside another 3-D object. – Write out all the steps to help track errors, since many problems require more than one calculation. • It may benefit students if you have them verbalize what they are trying to determine, what 3-D objects are involved, and what they know about each 3-D object. • Provide additional coaching with Example 1 to students who need help with #3. Have students explain their thinking; clarify any misunderstandings. • Provide additional coaching with Example 2 to students who need help with #4. Have students explain their thinking; clarify any misunderstandings. Then, have students complete #5 on their own. Check back with them several times to make sure that they understand how to solve similar problems. • Provide additional coaching with Example 3 to students who need help with #7. Have students explain their thinking; clarify any misunderstandings. As they go on to the Apply questions, check back with them several times to make sure that they understand how to solve similar problems.
Math Link The Math Link on page 275 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 279.	• Coach students who need help to draw and label the diagrams of planters. Have them review their diagrams with you and orally explain the process for solving the volume problems. • Consider assigning only one planter and allow students to choose the 3-D shape. Generally, a rectangular prism is the easiest shape for students to use. • Encourage artistic students to create models of their planter designs. • To help them get started, some students may benefit from using **BLM 7–12 Section 7.4 Math Link**, which provides scaffolding for this activity.
Assessment *as* Learning	
Math Learning Log Have students answer the following question: • Think about how you could apply what you have learned about volume of prisms and cylinders in different situations in your daily life. Develop and solve a problem involving one of these situations.	• Allow students who cannot think of a situation from their daily life to create an imaginary situation. You might provide some suggestions such as the following: building and filling a sand box; building a retaining wall; or packing a cooler for a picnic. • Encourage students to draw and label a diagram before solving the problem. • Depending on students' learning style, have them provide any combination of oral or written answers. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.