Exploring Integer Multiplication



their answers and their methods. Because students have not multiplied integers before, they may determine the answer by multiplying 12 m by 4, without thinking in terms of integers. Remind students to consider the type of integer values usually assigned to backward motion.

Ask if there is another way to determine the total distance that the glacier recedes in four years. The word *total* should trigger the idea of addition. Remind students that multiplication is repeated addition and that they already know how to add integers. Students can think of the total distance that the glacier recedes in terms of the sum (-12) + (-12) + (-12) + (-12).

Later in the lesson, after students have multiplied some integers, you might ask the class to consider the receding glacier again. Students will then be able to represent the total distance by the expression $(+4) \times (-12)$ to give -48, indicating that the glacier recedes 48 m in four years.

MathLinks 8, pages 286–292

- Suggested Timing 50-60 minutes

Materials

- red and blue integer chips
- coloured pencils (optional)
- scissors (optional)
- transparent chips (optional)

Blackline Masters

Master 2 Two Stars and One Wish Master 20 Integer Chips BLM 8-3 Chapter 8 Warm-Up BLM 8–5 Section 8.1 Extra Practice

Mathematical Processes

- Communication (C)
- ✓ Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- Reasoning (R)
- Technology (T)
- Visualization (V)

Specific Outcomes

N7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

| Category | Question Numbers |
|---|-------------------------|
| Essential (minimum questions to cover the outcomes) | 1-4, 5, 7, 9, 13, 14 |
| Typical | 1, 2, 4, 5, 7, 9, 14–18 |
| Extension/Enrichment | 1, 2, 4, 19, 20 |

Planning Notes

Have students complete the warm-up questions on BLM 8–3 Chapter 8 Warm-Up to reinforce material learned in previous sections.

As a class, read and discuss the information about the Columbia Icefield and the Athabasca Glacier in the student resource. Have students determine individually the distance that the Athabasca Glacier recedes in four years. Then, have students compare



Explore the Math

In this exploration, students use integer chips to multiply two integers.

Method 1 Have students work on the exploration in pairs and discuss their answers. Give each pair of students a supply of red and blue integer chips (at least 15 of each colour), so that they can learn to multiply integers concretely. If you do not have a supply of integer chips, students might use other manipulatives, such as:

- algebra tiles (unit tiles can be used to represent +1 and -1)
- pennies, with heads representing +1 and tails representing -1
- square cutouts of equal sizes made from sheets of red and blue paper
- circles cut from Master 20 Integer Chips and coloured red and blue

Method 2 Have students use a semi-concrete method by drawing and colouring circles to represent the integers. Students can cross out or cover circles that need to be removed from the diagram.

Literacy Link Draw students' attention to the Literacy Links on pages 286 and 287. The first Literacy Link is a reminder that red chips represent positive integers and blue chips represent negative integers. Discuss how hands may turn blue when they get very cold, and suggest that this may be one way to remember that blue represents negative integers. Temperatures in the negative digits can turn our digits blue.

The second Literacy Link reinforces the meanings of *product* and *multiplication statement*, and reminds students that multiplication can be performed by repeated addition. You may wish to remind students that positive integers are often written without the positive sign. Point out that the multiplication statement $4 \times 2 = 8$ can also be interpreted as two groups of four make eight, with the corresponding addition 4 + 4.

The third Literacy Link is a reminder of the meaning of *zero pair* and how students can model a zero pair with integer chips.

The negative sign in -2 shows that you need to remove 2 groups. Each group will contain 4 blue chips. So, you need to remove a total of 8 blue chips. Start with 8 zero pa 00000 The product is +8. The multiplication statement is $(-2) \times (-4) = +8$ Show You Know Determine each product using integer chips. Use diagrams to show your thinking. a) $(+4) \times (+2)$ b) $(+5) \times (-2)$ c) $(-4) \times (+2)$ d) $(-6) \times (-1)$ Example 2: Apply Integer Multiplication For 5 h, the temperature in Flin Flon fell by 3 °C each hour. What was the total change in temperature? Did You Know? Solution Use the multiplication of two integers to represent the situation. Represent the time of 5 h by the integer +5. Represent the 3°C decrease in each hour by the integer -3. The total change in temperature can be represented by the expression $(+5) \times (-3)$. $\bigcirc \bigcirc \bigcirc$ Multiply $(+5) \times (-3)$ using integer chips. Flin Flon lies along the Manitobafictional character alled Professor losiah Flintabbatey lonatin. He was th tero of a science e Sunless City. In s novel, Josiah plored a ttomless lake in The product is -15. ubmarine and covered a tunne ed with gold. Flir in was named The total change in temperature was a decrease of 15 °C. Show You Know For 4 h, the temperature in Victoria fell by 2 °C each hour. What was the total change in temperature? 8.1 Exploring Integer Multiplication • MHR 289

Example 1

Example 1 shows students how to multiply integers using integer chips. Remind students that zero pairs represent 0. For parts c) and d), stress that the presence of a minus sign in the first integer in a multiplication indicates that chips are to be removed. Reinforce the idea that this example uses a familiar problem solving strategy (Model It), and ask students if they can think of another strategy they could use (e.g., Draw a Diagram).

Students will need to use integer chips and diagrams to complete the Show You Know questions.

Example 2

Example 2 shows an application of integer multiplication. Make sure that students understand the use of positive and negative integers to represent the quantities that are multiplied. Emphasize the importance of including the summary statement to explain the meaning of the integer product. Negative integers can be used to represent a negative temperature value, such as -15 °C, or a temperature decrease. The summary statement in the example explains that the integer product, -15, represents a decrease of 15 °C in this case. Some students may need integer chips or diagrams to complete the Show You Know question.

Meeting Student Needs

- For #6a) in Explore the Math, some students may need assistance to determine how many zero pairs to use and what to remove.
- Have students work through the examples in pairs, then complete the Show You Know independently.
- In Example 2, ensure students understand that the negative sign means the temperature went down.

ELL

- Students may need assistance identifying inferences in the Explore the Math section.
- Ensure students understand the following words: *icefields*, *border*, *melting*, and *receding*.
- When working through the examples, ensure that students know the required math language, such as *positive*, *negative*, *product*, *multiplication statement*, *minus*, *decrease*, and *increase*.
- Before students work on word problems, have the class brainstorm different words that refer to different operations. Post these words in the classroom (e.g., addition: *more*, *increase*, *sum*, *taller*, *longer*, and, *together*).

Common Errors

- Some students may not understand when to model with zero pairs.
- **R**_x Point out that modelling the multiplication (+4) × (+2) involves the insertion of four groups with two red chips in each group. In contrast, modelling the multiplication (-4) × (+2), in which a negative integer appears first, involves the removal of four groups with two red chips in each group. In order for these chips to be removed, zero pairs are used to include sufficient chips.
- Some students may be unsure of how many zero pairs to use.
- $\mathbf{R}_{\mathbf{x}}$ Explain that the number of chips to be removed determines the number of zero pairs. For example, modelling $(-4) \times (+2)$ involves the removal of four groups, each containing two red chips (i.e., the removal of eight red chips altogether). Beginning with eight zero pairs provides sufficient red chips for removal.

Answers

Explore the Math

1. a) $\oplus \oplus \oplus \oplus \oplus \oplus \oplus$ b) 6

- 2. a) 3 + 3 + 3 + 3 b) ⊕⊕⊕ ⊕⊕⊕ ⊕⊕⊕
 c) 12
- **3.** a) (-5) + (-5) + (-5) b) ⊖⊖⊖⊖⊖ ⊖⊖⊖⊖⊖ ⊖⊖⊖⊖⊖ c) -15
- - **b)** $\oplus \oplus \oplus \oplus \oplus \oplus \oplus (+2) \times (+3) = 6$
 - c) $\bigcirc \bigcirc (+4) \times (-3) = -12$
 - **d)** $\bigcirc \bigcirc (+3) \times (-4) = -12$
- **5.** Answers may vary. Example: No, you cannot have a negative number of groups.
- **6.** a) Answers may vary. Example: The diagram representing (-2) × (+3) shows adding six zero pairs. Then, removing two groups of three red chips leaves six blue chips. The diagram representing (-3) × (-4) shows the addition of 12 zero pairs. Removing three groups of four blue chips leaves 12 red chips.
 - **b)** -6; 12



8. Answers may vary. Example: To model the multiplication of an integer by a positive integer, insert integer chips of the appropriate colour. To model the multiplication of an integer by a negative integer, remove integer chips of the appropriate colour from zero pairs. To find the number of zero pairs needed, first remove any negative signs from the two numbers you are multiplying so that both are positive. Then, multiply the two positive numbers.

Show You Know: Example 1



Show You Know: Example 2

The temperature fell by 8 °C.

| Assessment | Supporting Learning |
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| Assessment as Learning | |
| Reflect on Your Findings Listen to students' descriptions and explanations to both questions. A discussion may be necessary to clarify student thinking to ensure understanding of the concept of integer multiplication using integer chips. | • Ask students who need assistance with #8 to use the class responses as a springboard to respond to this question. |
| Assessment for Learning | |
| Example 1 Have students do the Show You Know related to Example 1. | Have students work with a partner and talk through their thinking. Have students refer back to the previous examples that model a question pattern similar to those asked in Example 1. Some students may benefit from guiding rules. For example: If the leading factor is positive, the question can be completed as repeated addition. If the leading factor has a negative sign, you need zero pairs. The negative sign tells you that you will be removing that quantity of the second factor, e.g., (-2) × (-3) means remove two groups of (-3) or blue chips. |
| Example 2 Have students do the Show You Know related to Example 2. | Have students work with a partner and discuss their thinking.Have students represent the temperature and time as integers and use integer chips and diagrams. |

| | the appropriate co | r by a positive integer, you can blour. |
|---|---|--|
| | | |
| $(+2) \times (-3) = -6$ | | |
| To model the multipli can remove integer ch | ication of an intege tips of the appropr | r by a negative integer, you iate colour from zero pairs. |
| 000000 0 | | + ***** |
| $(-2) \times (-3) = +6$ | , , | |
| Communicate the lo | deas | |
| David said that he using 3 red chips and | could model the m nd 7 blue chips. | ultiplication $(+3) \times (-7)$ |
| a) Do you agree wi | ith David? Explain | • |
| b) What chips wou | ld you use to mode | $(+3) \times (-7)$? Explain. |
| 2. To model $(-3) \times (-3)$ | -5), Raini places | 0000000000000000 |
| 15 zero pairs on he | r desk. | |
| a) Why did she use b) What should she | e do next? | |
| of what should she | e do nexti | |
| 3. a) Paolo models (- determines the c | $(-2) \times (+3)$ as show correct product, -6 | n in the diagram. He Explain why his method works. |
| | 00 0 0000 00 0 0000 | 000000 0 0000 |
| 00000000 00000000 | | as started with 4 yero pairs? Evplain |
| b) Could Paolo mo | del the product if h | ie started with 4 zero pairs. Explain. |

Key Ideas

Have students read and review the Key Ideas. Stress that modelling multiplication in which the first integer is negative involves the use of zero pairs because of the need to remove integer chips.

Communicate the Ideas

You may wish to have students complete the questions in groups and then discuss their answers. When students have considered #3, reinforce the idea that the minimum possible number of zero pairs is usually chosen to model a multiplication in which a negative integer appears first.

Meeting Student Needs

- Connect multiplying using integer chips to other areas of students' lives. For example, you might use pony beads, feathers, or willow sticks to help teach this concept.
- Invite a community member or an Elder to talk about counting systems used in other parts of the world or by First Nations or Inuit communities prior to European contact.

Answers

Communicate the Ideas

- **1.** Answers may vary. Example:
 - a) No, he is showing the sum of 3 and -7.
 - **b)** Three groups of seven blue chips each are needed to represent $(+3) \times (-7)$. Each blue chip represents a negative unit.
- **2.** Answers may vary. Example:
 - a) She needs to remove 15 chips.
 - b) She should remove three groups of five blue chips.
- **3.** Answers may vary. Example:
 - a) After he removes the four zero pairs, he has six blue chips left, which represent the product, -6.
 - **b)** No, he needs to remove six red chips; four zero models have only four red chips.
- **4.** Answers may vary. Example: Zero groups of two chips contain zero chips.

| Assessment | Supporting Learning |
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| Assessment as Learning | |
| Communicate the Ideas Have all students complete #1, #2, and #4. Essential level students should also do #3 as it combines the conceptual understanding of #1 and #2. You may wish to discuss #4 as a class. Have students explain their thinking and how integer chips may or may not be used. | Consider having students work in groups or pairs. Check answers to questions #1 and #2. These are key questions; make sure students understand the concepts before proceeding. Have students continue to use integer chips and diagrams as needed. |



Check Your Understanding

Practise

For #13, encourage students to work with a partner and check the answers using integer chips. You may wish to have students work in small groups for other Practise questions at this early stage of integer multiplication. Some students may be able to visualize the chips without handling them physically or drawing them by the end of the Practise questions. Some students may begin to make their own generalizations about the sign of the product of an integer multiplication.

Apply

Students must model the descriptions mathematically for #14 to #17, which involve real-world applications. For these questions, students may disagree on the order in which the integers should be written in a multiplication. If so, have students compare their answers to see that the order of multiplication does not affect the product. Stress that the final line of the solution should be a summary statement that explains the meaning of the integer product. For #18, encourage students to think about the signs of integers and to compare and order integer values. By considering examples, many students will conclude that the answer depends on the sign of the integer, but they may fail to consider zero, which is also an integer.

Extend

The Extend questions involve the determination of the product of three integers. Students who are modelling with integer chips or diagrams will need to multiply two of the integers and then multiply their product and the third integer. Students who are able to think more abstractly may already be arriving at generalizations that will allow them to decide both the sign and magnitude of the product. You may wish to have students compare their methods to see that the order in which the three integers are multiplied does not affect the product.

Meeting Student Needs

• Provide **BLM 8–5 Section 8.1 Extra Practice** to students who would benefit from more practice.

Common Errors

- Some students may not allow for the signs of the integers they are multiplying.
- R_x Reinforce the use of the two colours of manipulatives to account for positive and negative integers and to determine the sign of the product.
- Some students may confuse the multiplication and addition of integers.
- **R**_x Remind students that multiplication is *repeated* addition, and provide an example with whole numbers to distinguish addition and multiplication. For example, 3 + 2 = 5, but $3 \times 2 = 6$, because $3 \times 2 = 2 + 2 + 2$.
- Some students may be unsure of how to represent real-world quantities with integer values.
- R_x Provide more practice in assigning integer values in familiar contexts, such as temperature increases and decreases, distances above and below sea level, and money earned and spent.

- Some students may be unsure of where to use brackets in writing multiplication statements.
- R_x Reinforce the idea that brackets are used to distinguish integer signs from operation symbols. Therefore, brackets go around a number and its positive or negative sign.
- Some students may omit the summary statement from solutions to problems involving real-world data.
- R_x Stress that integers representing measurements must be explained in order to complete the solution. Giving the final answer to a problem involving temperatures as -20 does not indicate whether the temperature is -20 °C or whether the temperature has decreased by 20 °C. Giving the final answer to a problem involving distance as -12 does not indicate the units in which the distance is measured or whether the distance is below a certain level (e.g., sea level) or is a distance an object has moved downward or backward.

| Assessment | Supporting Learning |
|--|---|
| Assessment for Learning | |
| Practise and Apply Have students do #5, #7, #9, #13, and #14. Students who have no problems with these questions can go on to complete the rest of the Apply questions. | Students who need assistance with #5 and #7 may benefit from additional coaching. Have students explain their thinking on these questions using integer chips if necessary, and clarify any misunderstandings. Coach students through #6a) and #8a), and then have them complete #6b) and #8b) on their own. Have students check back to previous examples in the student resource for additional prompting. Check back with students to make sure they understand the concepts. Coach students who need assistance with #9. Have them explain their thinking, and clarify any misunderstandings. Coach them through #10a), and then have them complete part b) and #11 and #12 on their own. Students who need assistance with #13 may benefit from additional coaching with Example 1. Have students explain their thinking using integer chips if necessary, and clarify any misunderstandings. Coach students through #13a), and then have them correct any errors in the remaining parts of the question. Check back with students to make sure they understand the concepts. Provide additional coaching with Example 2 to students who need assistance with #14. Coach students through #14a), and then have them correct any errors in part b). Have them do #16 on their own. Check back with students to make sure they understand the concepts. |
| Assessment as Learning | |
| Math Learning Log Have students answer the following questions: What do you understand about using integer chips to multiply two integers? What do you find difficult about multiplying integers using integer chips? | Encourage students to look back through their work for ideas to help answer the questions. Consider having students exchange their work with a classmate to check for errors and to suggest improvements. You may wish to provide students with Master 2 Two Stars and One Wish for recording their feedback. Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with. |