

Multiplying Integers

MathLinks 8, pages 293-299

Suggested Timing

50-60 minutes

Materials

- red and blue integer chips
- coloured pencils
- red and blue construction paper (optional)
- scissors (optional)
- transparent plastic strips (optional)
- red and blue markers (optional)
- calculator (optional)

Blackline Masters

Master 3 Integer Number Lines Master 4 Vertical and Horizontal Number Lines Master 19 Multiplication Chart BLM 8–3 Chapter 8 Warm-Up BLM 8–6 Section 8.2 Extra Practice BLM 8–7 Section 8.2 Math Link

Mathematical Processes

- Communication (C)
- ✓ Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- ✓ Reasoning (R)
- Technology (T)
- ✓ Visualization (V)

Specific Outcomes

N7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 10, 12, 23, Math Link
Typical	1-4, 6, 8, 10, 12-24, Math Link
Extension/Enrichment	1-3, 25-29



Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce material learned in previous sections.

As a class, read and discuss the information about the migration of sooty shearwaters. To provide some insight into the astonishing distances that these birds cover, mention that the circumference of the Earth is about 40 000 km. The sooty shearwater migrates a distance that is almost twice the circumference of the Earth each year.



Have students describe and discuss how they could use integers to determine the depths of the deepest dives of sooty shearwaters. Students' typical description will include representing the depth of the average dive by the integer -14. The deepest dives are about five times as deep. Students' descriptions of the rest of the process may vary. Some students may suggest modelling with integer chips. If so, ask if there are any concerns with this approach. (It requires quite a large number of integer chips.) Other students may suggest the use of repeated addition. You may also find that some students have already moved toward a generalization of what happens in integer multiplication (multiply 5 and 14 and predict the sign of the product as being negative).

Later in the lesson, after students have determined and applied the sign rules, you might again ask the class to consider how integers could be used to determine the depth of the deepest dives of sooty shearwaters. Have students compare their methods with the ones they originally chose.

Explore the Math

In this exploration, students use a number line to multiply two integers.

Method 1 Have students work in pairs or small groups and discuss their answers. Give each pair of students a supply of red and blue integer chips (at least 20 of each colour) or other suitable manipulatives, so that students can use a concrete method wherever necessary and can compare this method with the semi-concrete representations shown on a number line.

Provide students with copies of **Master 3 Integer Number Lines**. Have students cut out paper strips of appropriate lengths to represent integers on the number lines, with red strips representing positive integers and blue strips representing negative integers. Have students draw an arrow along the length of each strip and label each strip with the appropriate integer, so that the positive values are right side up when the arrow points to the right and negative values are right side up when the arrow points to the left. Have students model the additions in #1 and #2 to become familiar with using the strips. Then, have students use the strips to model the multiplications in #4, where possible.

It is important to make sure that the table is accurate before students make generalizations in #4b) and #5a). Students need to realize that the two statements on each line of the table give the same product, and therefore two integers can be multiplied in either order to determine the product.

Method 2 As in Method 1, have students work in pairs or small groups, and supply them with integer chips. Alternatively, have students draw coloured diagrams of integer chips. To use number lines to model integer multiplication, students can draw coloured diagrams. You may wish to provide students with **Master 4 Vertical and Horizontal Number Lines**.



Example 1

Example 1 shows students how to use the sign rules to multiply two integers. Reinforce the concrete or semi-concrete models of the same multiplications, and the idea that the result is the same if the integers are multiplied in either order.

Some students may find it beneficial to continue to use a number line to complete the Show You Know questions.

Example 2

Example 2 shows an application of integer multiplication. Make sure that students understand the use of the integer -35 to represent the monthly \$35 deduction and the use of the integer +12 to represent the number of months in a year.

Point out that this example includes estimating the product. Students can apply their estimation techniques from the multiplication of whole numbers, except for the additional step of determining the sign of an integer product. Stress the importance of the summary statement at the end of the solution to Example 2. Some students may expect to write the final line of the solution as -\$420, but a complete solution requires an explanation of the negative sign in the context of the problem.

You could also include the use of technology to show the completion of the calculation in Example 2 and to prepare students for calculator use in the division of two-digit by two-digit integers in section 8.4. The following key sequence is for the TI-30Xa calculator:

C 12 × 35 +/- =

Students using other calculators may need to explore the method for entering negative integers on their calculators and may need assistance with this. Make it clear that, on most calculators, the integer sign key and the subtraction key are not the same.

After discussing Example 2, have students complete the Show You Know question to make sure that they are ready to move on.

Meeting Student Needs

- Have students compare the migration of local animals, such as whales, eagles, hawks, or any other local bird. The students can compare the depth of dives of local birds with that of the sooty shearwater.
- Make a poster for display in the classroom showing the sign rules for multiplying integers.
- You may need to remind some students of the meaning of the ≈ symbol, which is used in the estimation of the product in Example 2.

ELL

- Ensure that students understand the following words: *migration*, *deep/deepest*, *annual*, and *monthly*.
- Example 2 refers to Tina who supports her favourite charity with an *automatic deduction*. Explain that this means that \$35 comes out of her bank account each month; in other words, she is giving money.

Common Errors

- Some students may have difficulty modelling integer multiplication on a number line.
- $\mathbf{R}_{\mathbf{x}}$ Encourage the continued use of integer chips or diagrams of chips for as long as necessary. Assist students in comparing the integer-chip and number-line models in simple cases, e.g., (+3) × (+2). Point out the resemblance between the models, i.e., the representation of multiplication as repeated addition.
- Some students may not recognize when the numberline model shown in the exploration is inappropriate to use for integer multiplication.
- $\mathbf{R}_{\mathbf{x}}$ Remind students that this model represents repeated addition. Therefore, they can use the number-line method whenever they can rewrite the multiplication as a repeated addition (i.e., whenever there is at least one positive integer in the multiplication).

Answers

Explore the Math

- **1.** a) Answers may vary. Example: The integers are shown as three arrows that are each two units long and pointed in the positive direction on the number line.
 - **b)** The model would consist of three groups of two red chips each. The product is six.
 - c) Answers may vary. Example: The product is represented by the point reached by moving two units in the positive direction three times.
- **2.** a) Answers may vary. Example: Move three units to the left four times. Since each move represents -2, the point reached after all four moves shows the product.

- **b)** The product is -12. Explanations may vary. Example: The four moves to the left finish at -12 on the number line.
- **3.** Answers may vary. Example: The method used in #1 and #2 will not work because it is not possible to have a negative number of moves.

4. a) $(+6) \times (+2) = +12$	$(+2) \times (+6) = +12$
$(+4) \times (-5) = -20$	$(-5) \times (+4) = -20$
$(-4) \times (-3) = +12$	$(-3) \times (-4) = +12$

b) Answers may vary. Example: The order of multiplying two integers does not affect the product. For example, $(+6) \times (-5) = (-5) \times (+6)$ and $(-2) \times (-11) = (-11) \times (-2)$.

- c) Answers may vary. Example: Change the order to $(+2) \times (-3)$, then use the same method as in #1 and #2.
- **5.** a) The product of two integers with the same sign is positive. The product of two integers with different signs is negative.
 - **b)** Answers may vary. Example: $(-3) \times (-7) = 21$, $(-8) \times (-9) = 72$, $(-5) \times (+3) = -15$, and $(+7) \times (-7) = -49$.
- **6.** Answers may vary. Example:
 - a) Move to the right or left of zero in steps that correspond to the number of units in the second factor. The first factor indicates the number of steps. This method can be used only when the first factor is positive.
 - **b)** If both factors have the same sign, the product is positive. If the factors have different signs, the product is negative.

Show You Know: Example 1

a) 28 **b)** -30 **c)** 16 **d)** -36

Show You Know: Example 2

The deductions in 18 months total \$1170.

Assessment	Supporting Learning	
Assessment as Learning		
Reflect on Your Findings Listen as students discuss the answers to #6a) and b). Clarify any misunderstandings.	 Some students may need assistance generalizing the number-line method in #6a). Have them model their understanding with integer chips and compare their work to a number line. Have them verbalize the similarities and differences between the two methods. Students who need assistance with #6b) should be referred back to #5. 	
Assessment <i>for</i> Learning		
Example 1 Have students do the Show You Know related to Example 1.	 Have students work with a partner and talk through their thinking. Have students use the number line and integer chips to assist them in their thinking. Coach students through the sign rules developed previously. Reinforce the idea that the result is the same if integers are multiplied in either order. 	
Example 2 Have students do the Show You Know related to Example 2.	 Have students work with a partner and talk through their thinking. Help students recall the terminology used to assign signs to integer values. Students may benefit from writing out the problem and underlining the words that reflect integer values or writing the words and the mathematical equivalent beside it. Some students may find using a calculator beneficial. It may be necessary to help students recall the correct keystrokes. 	



Key Ideas

Have students read and review the Key Ideas. Emphasize the importance of knowing how to model the multiplication of two integers on a number line if at least one of the integers is positive, but not if both integers are negative.

Communicate the Ideas

You may wish to have students complete these questions in groups and discuss their answers. In #1, students need to consider both the use of a number line as a model and the order in which two integers are multiplied. This question provides an opportunity to reinforce the fact that multiplying two integers in either order gives the same product. Use #2 and #3 to prompt discussion of the sign rules.

Answers

Communicate the Ideas

- 1. Answers may vary. Example: Ishnan rewrote the product as $(+3) \times (+7)$. This product can be represented by three arrows that are each seven units long.
- **2.** No, the product of two negative integers is a positive integer. A positive integer is greater than –5.
- **3.** Answers may vary. Example: She knew that the product of two numbers with different signs results in a negative number. So, the product is the same negative integer no matter which of the two factors has the negative sign.

Assessment	Supporting Learning
Assessment as Learning	
Communicate the Ideas Have all students complete #1 to #3.	 Encourage students who need assistance with #1 to refer to the Explore the Math #4a) and review their thinking. Encourage students who need assistance with #2 to use number lines and integer chips to model an example such as (+5) × (-2). Encourage students who need assistance with #3 to model the question using smaller values before making any corrections. Have them complete examples, such as (-3) × (+2) and (+4) × (-1). Reinforce the sign rules. Students could write the sign rules in chart form and use the chart as a quick reference. Students who need coaching in multiplying integers may benefit from using Master 19 Multiplication Chart.



Check Your Understanding

Practise

Students will differ in the extent to which they continue to rely on concrete or semi-concrete models to multiply integers. Some students may already be very comfortable with symbolic representations and the use of the sign rules. Assign Practise questions accordingly, and support students as necessary. Some students may benefit from working with a partner or in a group.

You may wish to have students discuss their estimates in #10 and #11 and their strategies for arriving at them. If students do not mention them, help them recall the use of front-end and relative-size estimation.

Apply

For #12 to #15, students need to model the real-world applications mathematically. For these questions, stress that the final line of the solution should be a summary statement that explains the meaning of the integer product.

In #18, a different approach to multiplication involving negative integers is introduced: the use of patterns. Patterning will also be used with integer division in the next section, and #18 may provide useful preparation.

For #19, some students may identify +2 as an integer that can be multiplied by itself to give the product +4, but they may overlook -2 as the other possibility. To extend #19, discuss the answers to parts a) and b) in the context of square roots.

In #20, the pre-algebraic determination of missing integers in multiplication statements is included. Some students may complete this question by systematic trial, whereas others may be able to think more abstractly. At this stage, you may wish to point out connections to integer division.

Students will differ in their ability to create word problems in #23 and #24. Encourage students who are having difficulty to model their problems on previous examples or questions. As students gain more confidence, encourage creativity.



Extend

Students are required to determine the product of three integers for #26. Some students may do this by multiplying two of the integers and then multiplying the product and the third integer. Other students may think in more general terms about the sign rules for multiplying three integers.

Literacy Link For #26, draw students' attention to the Literacy Link, which explains the meaning of *opposite integers*.

Control of Control

Math Link

In this Math Link, students explore how to use integers to solve problems that involve temperature changes. Clarify the meaning of the term *altitude* (i.e., height above sea level). Give examples with pictures on the chalkboard. Encourage students to use integer chips or a number line to model the problem.

Meeting Student Needs

• Provide **BLM 8–6 Section 8.2 Extra Practice** to students who would benefit from more practice.

ELL

- For #12, ensure students understand the terms *annual* and *discount*.
- For #13, ensure students understand that *descending* means going down (the numbers are decreasing, getting smaller, becoming less).
- For #27, provide students with oral and written examples of consecutive numbers.

Common Errors

- Some students may ignore the sign rules for multiplying integers.
- R_x Emphasize the need to consider the sign of the product in all integer multiplication problems. If necessary, use concrete or semi-concrete models to demonstrate that some products of integer multiplications are positive and others are negative.
- Some students may forget the sign rules for multiplying integers.
- R_x Encourage students to visualize models of examples of integer multiplications that illustrate the sign rules. You might also post the sign rules for integer multiplication in the classroom until students are thoroughly familiar with them.
- Some students may provide incomplete solutions to application problems by failing to explain the meaning of the integer product.
- $\mathbf{R}_{\mathbf{x}}$ Stress the need to include a summary statement at the end of the solution to an application problem.

Web Link

Have students explore another way to model multiplication of integers. Note that the website model uses red to represent negative integers and blue to represent positive integers. Go to www.mathlinks8.ca and follow the links.

Answers

Math Link

a) -62 °C b) The temperature increased by 36 °C.

Assessment	Supporting Learning	
Assessment for Learning		
Practise and Apply Have students do #4, #6, #8, #10 and #12. Students who have no problems with these questions can go on to the rest of the Apply questions.	 Coach students who need assistance with #4a). Have them verbalize their understanding. Encourage students to use integer chips to explain the mathematical processes and then translate the process to the meaning of arrows on a number line. Have students try #5 before going on. Encourage students who need assistance with #6 to verbalize their thinking. They may find it beneficial to refer to a multiplication table such as the one on Master 19 Multiplication Chart. Coach students through the sign rules relating to integers. Have students complete #7a) before beginning #8. Students who need assistance with #10 may benefit from being coached through the rules that apply to estimation. A calculator may be of assistance for the estimation and actual calculation. You may need to help students recall the correct keystrokes. Students who need assistance with #12 may benefit from recalling what they know about the meaning of mathematical words and how they translate into mathematical expressions and operations. Make a list of words along with their mathematical meanings, and have students record them in their chapter Foldable. Have students try #13 before moving on. 	
Math Link The Math Link on page 299 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 321.	 You may wish to have students complete the Math Link in order to apply their understanding of integer multiplication. Listen to any discussion about how the students solve the problem or for any terminology that may cause confusion. Some students may benefit from using BLM 8–7 Section 8.2 Math Link, which provides scaffolding for this activity. 	
Assessment <i>as</i> Learning		
 Math Learning Log Have students reflect on how well they succeeded in multiplying integers by completing the following questions: What methods can you use to multiply two integers? What method do you like best? Why? 	 Depending on students' learning style, have them provide oral or written answers. Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with. 	