

Exploring Integer Division

MathLinks 8, pages 300–305

Suggested Timing

50–60 minutes

Materials

- red and blue integer chips
- coloured pencils (optional)
- scissors (optional)
- transparent chips (optional)

Blackline Masters

Master 20 Integer Chips
BLM 8–3 Chapter 8 Warm-Up
BLM 8–8 Section 8.3 Extra Practice

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- Reasoning (R)
- Technology (T)
- Visualization (V)

Specific Outcomes

N7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 3, 5, 7, 10
Typical	1, 3, 5, 7, 10–14
Extension/Enrichment	1, 2, 15, 16

Planning Notes


Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce material learned in previous sections.

As a class, read and discuss the information about grizzly bears. Have students discuss the answers to the two questions. There will likely be agreement on the idea of using the integer -75 to represent a loss in mass of 75 kg. Students have previously worked with averages, so they should be aware that the average loss of mass in one month can be determined by dividing by the number of months.

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Exploring Integer Division

Grizzly bears lose much of their body mass during their winter sleep. A large male bear may enter his den at 300 kg in November. He may lose 75 kg by the time he emerges five months later. How would you represent a loss of 75 kg with an integer? What operation would you use to find the average loss of mass in one month?



FOCUS ON...
After this lesson, you will be able to...

- divide integers using integer chips


Explore the Math

Materials
• red and blue integer chips

Literacy Link
Understanding Division
In the division statement $6 \div 2 = 3$, the dividend is 6, the divisor is 2, and the quotient is 3. This division statement means that in 6 there are 3 groups of 2. It also means that when 6 is separated into 2 equal groups, there are 3 in each group.

How can you use integer chips to divide two integers?

- The diagram shows a way to model the division $(+8) \div (+2)$ using red integer chips.



 - Explain how the diagram shows the quotient $(+8) \div (+2)$.
 - Copy and complete the division statement $(+8) \div (+2) = \blacksquare$.
 - Explain how the same diagram can also model $(+8) \div (+4)$.
 - Copy and complete the division statement $(+8) \div (+4) = \blacksquare$.
- Use red integer chips to model the division $(+15) \div (+5)$.
 - Copy and complete the division statement $(+15) \div (+5) = \blacksquare$.
 - Write the other division statement that the model can represent.

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Because students have not divided integers before, you may wish to stop the discussion at this point. However, some students may use whole numbers to reason that the average loss of mass in one month is 15 kg. Later in the lesson, after students have modelled integer division, you may wish to ask the class to consider the average loss of mass again and to describe how the division $(-75) \div (+5)$ could be modelled. This example serves to show the limitation of using a large number of chips or a large diagram.

Explore the Math

In this exploration, students use integer chips to divide two integers.

Literacy Link The Literacy Link reminds students about the terminology used in division. It also suggests the idea of separating quantities into groups, which is the basis of modelling division with integer chips.

3. The diagram shows a way to model the division $(-8) \div (-2)$ using blue integer chips.



- Explain how the diagram shows the quotient $(-8) \div (-2)$.
 - Copy and complete the division statement $(-8) \div (-2) = \blacksquare$.
 - Explain how the same diagram can also model $(-8) \div (+4)$.
 - Copy and complete the division statement $(-8) \div (+4) = \blacksquare$.
4. a) Use blue integer chips to model the division $(-15) \div (-5)$.
- Copy and complete the division statement $(-15) \div (-5) = \blacksquare$.
 - Write the other division statement that the model can represent.
5. a) Model the division $(-8) \div (+2)$ using integer chips. Explain your method.
- Copy and complete the division statement $(-8) \div (+2) = \blacksquare$.

Reflect on Your Findings

- How can you use integer chips to divide two integers?

Example 1: Divide Using Integer Chips

Determine each quotient using integer chips. Copy and complete each division statement.

- $(+12) \div (+3)$
- $(-12) \div (-3)$
- $(-12) \div (+4)$

Solution

a)



There are 4 groups, so the quotient is +4. The division statement is $(+12) \div (+3) = +4$.

Separate the 12 red chips into groups of 3 red chips and count the number of groups.

Strategies
Model It

Another way to model this division is to separate the 12 red chips into 3 equal groups. There are 4 red chips in each group, so each group represents +4.

Strategies
What other strategy could you use?

Method 1 Have students work in pairs to complete the exploration and discuss their answers. Give each pair of students a supply of red and blue integer chips (at least 15 of each colour) so that they can learn to divide integers concretely. If you do not have a supply of integer chips, have students use other manipulatives, such as the following:

- algebra tiles (unit tiles can be used to represent +1 and -1)
- pennies, with a head representing +1 and a tail representing -1
- square cutouts of equal sizes made from sheets of red and blue paper
- circles cut from **Master 20 Integer Chips** and then coloured red and blue

Students should know from their work with whole numbers that the quotient in #1a) must be +4. Interpreting the integer-chip model involves counting the number of groups of two red chips that can be

formed from eight red chips. In #1c), students should know that the quotient must be +2. In this case, interpreting the integer-chip model involves determining the value represented by each of the four groups formed from eight red chips. Students apply these methods in #2 to #4. In #5, students should discover that only one of these methods works. (It is possible to separate eight blue chips into two equal groups, but it is not possible to separate eight blue chips into groups that each represent +2.)

No part of the exploration involves the division of a positive integer by a negative integer, e.g., $(+8) \div (-2)$. The reason is that neither of the methods used in #1 to #4 can be used in this situation. It is not possible to separate eight red chips into -2 groups or to separate eight red chips into groups that each represent -2. Integer chips can be used in conjunction with patterning to divide a positive integer by a negative integer, as indicated in #12 and #13 in this section.


Method 2 Have students use a semi-concrete method by drawing and colouring circles to represent the integers.

Example 1


Reinforce the idea that this example uses a familiar problem solving strategy (Model It) and ask students if they can think of another strategy they could use (e.g., Draw a Diagram, Look for a Pattern).

Point out that part a) of the example shows two methods for modelling the division of two positive integers, $(+12) \div (+3)$. In part b), only the first of these methods is possible, because 12 blue chips cannot be separated into -3 groups. In part c), only the second of these methods is possible because 12 blue chips can be separated into four equal groups but not into groups that each represent +4. Point out that the same integer-chip model represents two different division statements in parts b) and c).

After discussing Example 1, have students complete the Show You Know question to make sure that they are ready to move on.

b)  Separate the 12 blue chips into groups of 3 blue chips and count the number of groups.

There are 4 groups, so the quotient is $+4$. The division statement is $(-12) \div (-3) = +4$.

c)  Separate the 12 blue chips into 4 equal groups and count the number of blue chips in each group.


There are 3 blue chips in each group, so the quotient is -3 . The division statement is $(-12) \div (+4) = -3$.

Show You Know
Determine each quotient using integer chips. Use diagrams to show your thinking.

a) $(+14) \div (+7)$ **b)** $(-9) \div (-3)$ **c)** $(-16) \div (+2)$

Example 2: Apply Integer Division
One night, the temperature in Wetaskiwin, Alberta, was falling by 2°C each hour. How many hours did it take for the temperature to fall 10°C altogether? Show how you found your answer using integer chips.

Solution
Use the division of two integers to represent the situation. Represent the 2°C decrease each hour by the integer -2 . Represent the total decrease of 10°C by the integer -10 . The number of hours taken can be represented by the expression $(-10) \div (-2)$. Divide $(-10) \div (-2)$ using integer chips.

 Separate the 10 blue chips into groups of 2 blue chips. Count the number of groups.

There are 5 groups, so the quotient is $+5$. It took 5 h for the temperature to fall 10°C altogether.


Web Link
The name Wetaskiwin comes from the Cree term *wi-to-ski-oo-cha-ka-tin-ow*, which means "place of peace" or "hill of peace." To find out more about Aboriginal sources of Canadian place names, go to www.mathlinks.ca and follow the links.

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Show You Know
The temperature in Buffalo Narrows, Saskatchewan, was falling by 3°C each hour. How many hours did it take for the temperature to fall 12°C altogether? Show how you found your answer using integer chips.

Key Ideas

- You can use integer chips to model integer division.

 $(-6) \div (-2) = +3$
 $(-6) \div (+3) = -2$

Communicate the Ideas

- a)** Allison modelled the division $(+12) \div (+6)$ by separating 12 red chips into groups of 6. Tyler modelled the same division by separating 12 red chips into 6 equal groups. Explain how they each determined the correct quotient.

b) Explain how each of their methods also models the division $(+12) \div (+2)$.

c) Using blue chips, could you use Tyler's method to model $(-12) \div (+6)$? Explain.

d) Using blue chips, could you use Allison's method to model $(-12) \div (+6)$? Explain.
- a)** Wing modelled the division $0 \div (+4)$ by separating 8 zero pairs into 4 groups. There were 2 zero pairs in each group. Explain how his model shows the quotient.

b) Could you model the same division with a different number of zero pairs? Explain.

c) Would you use integer chips to divide 0 by a positive or negative integer? Explain.

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Example 2

Make sure that students understand the use of positive and negative integers to represent the quantities that are divided in the example. Emphasize the importance of including the summary statement as the last line of the example to explain the meaning of the integer quotient. In contrast with multiplication, students cannot generally divide two integers in either order (two equal integers are an exception).

Draw attention to the Web Link beside Example 2. It points out the origin of the place name in this example and gives students the opportunity to explore other place names of Aboriginal origin.

After discussing Example 2, have students complete the Show You Know question to make sure that they are ready to move on.

Meeting Student Needs

- Have students work through the examples in pairs, then complete the Show You Know questions independently.
- Encourage students to use manipulatives to explore dividing integers. Show a lot of visuals and draw steps to solve the equations.

ELL

- In the introduction, point out the picture of the grizzly bear and have a student explain the meaning of *body mass*, *winter sleep*, and *hibernation*. Some new Canadians may not be aware that some animals hibernate through the winter months.
- Example 2 refers to the temperature *falling*. Explain that in this sense, falling means that the temperature is dropping, or going down. Use a thermometer or picture on the chalkboard to show this.

Gifted and Enrichment

- Have students research grizzly bears. See the related Web Link below.

 **Web Link**
For information about grizzly bears, go to www.mathlinks8.ca and follow the links.

Common Errors

- Some students may not understand how the same integer-chip model can represent two different division statements.
- R_x** Emphasize the two general methods for modelling division using integer chips:
- Separate chips that represent the dividend into equal groups, so that each group represents the divisor. Count the number of groups to determine the quotient.
 - Separate chips that represent the dividend into equal groups, so that the number of groups represents the divisor. Count the number of chips in each group to determine the quotient.

Provide specific examples, such as a diagram of ten blue chips being separated into five groups, each containing two blue chips. Remind students that 10 blue chips represent -10 ; two blue chips represent -2 ; and five groups represent $+5$. Thus, the same model can be used to represent the following:

$$(-10) \div (-2) = +5 \text{ (See the first method above.)}$$

$$(-10) \div (+5) = -2 \text{ (See the second method above.)}$$

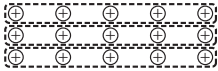
Answers

Explore the Math

- a) Answers may vary. Example: The eight positive chips are divided into four groups with two chips each.

b) $(+8) \div (+2) = +4$

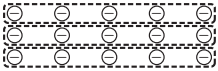
c) Answers may vary. Example: The eight positive chips could be separated into two groups with four chips each.


d) $(+8) \div (+4) = +2$
- a) 

b) $(+15) \div (+5) = +3$ c) $(+15) \div (+3) = +5$
- a) Answers may vary. Example: The eight negative chips are divided into four groups with two chips each.

b) $(-8) \div (-2) = +4$

c) Answers may vary. Example: The eight negative chips could be separated into two groups with four chips each.

d) $(-8) \div (+4) = -2$
- a) 

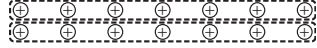
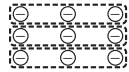

b) $(-15) \div (-5) = +3$ c) $(-15) \div (+3) = -5$
- a) 

Answers may vary. Example: Separate the eight negative chips into four groups with two negative chips each.

b) $(-8) \div (+2) = -4$

- Answers may vary. Example: Represent the dividend with positive or negative chips according to the sign of the dividend. If the divisor is positive, separate the chips into groups with each group containing a number of chips equal to the divisor. If the divisor is negative, separate the chips into groups with each group containing a number of chips equal to the divisor times -1 . If the dividend and the divisor have the same sign, the quotient is equal to the number of groups. If the dividend and the divisor have opposite signs, the quotient is equal to the number of groups times -1 .

Show You Know: Example 1

- 
The quotient is $+2$.
- 
The quotient is $+3$.
- 
The quotient is -8 .

Show You Know: Example 2

4 h. Explanations may vary. Example: Represent the fall of 12°C by 12 blue chips. Separate these 12 chips into groups containing three chips. Count the number of groups. This number corresponds to the number of hours it took for the temperature to fall 12°C .



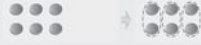
Assessment	Supporting Learning
Assessment <i>as</i> Learning	
<p>Reflect on Your Findings Listen as students discuss and describe how to solve the problems. Check students' responses to #6 for their ability to generalize what they learned in the Explore the Math.</p>	<ul style="list-style-type: none"> • Students who may not understand how the same integer-chip model can represent two different division statements should be coached through the two methods and then shown an additional example. • Have students verbalize their thinking, and assist them with any misunderstandings.
Assessment <i>for</i> Learning	
<p>Example 1 Have students do the Show You Know related to Example 1.</p>	<ul style="list-style-type: none"> • Have students work with a partner and talk through their thinking. • Encourage students to refer back to the previous examples that model a question pattern similar to those in Example 1. • Some students may benefit from being coached with some guiding rules, especially those having difficulty with knowing when the integer chip methods can be applied. Complete additional examples, such as $(-8) \div (+2)$ and $(-8) \div (-2)$. • Have students write out their thinking in their own words.
<p>Example 2 Have students do the Show You Know related to Example 2.</p>	<ul style="list-style-type: none"> • Have students work with a partner and discuss their thinking. • Have students represent the temperature and time as integers and use integer chips and diagrams to solve the problem.

Show You Know

The temperature in Buffalo Narrows, Saskatchewan, was falling by 3°C each hour. How many hours did it take for the temperature to fall 12°C altogether? Show how you found your answer using integer chips.

Key Ideas

- You can use integer chips to model integer division.



$$\begin{aligned}(-6) \div (-2) &= +3 \\ (-6) \div (+3) &= -2\end{aligned}$$

Communicate the Ideas

- Allison modelled the division $(+12) \div (+6)$ by separating 12 red chips into groups of 6. Tyler modelled the same division by separating 12 red chips into 6 equal groups. Explain how they each determined the correct quotient.
 - Explain how each of their methods also models the division $(+12) \div (+2)$.
 - Using blue chips, could you use Tyler's method to model $(-12) \div (+6)$? Explain.
 - Using blue chips, could you use Allison's method to model $(-12) \div (+6)$? Explain.
- Wing modelled the division $0 \div (+4)$ by separating 8 zero pairs into 4 groups. There were 2 zero pairs in each group. Explain how his model shows the quotient.
 - Could you model the same division with a different number of zero pairs? Explain.
 - Would you use integer chips to divide 0 by a positive or negative integer? Explain.

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Key Ideas

Have students read and review the Key Ideas. Have students explain how the same integer-chip model represents two different division statements. You may wish to provide additional examples of integer-chip models.

Communicate the Ideas

You may wish to have students complete the questions in groups and discuss their answers. In #1a), the two general methods for modelling division with integer chips are reinforced. Part b) reinforces the use of the same integer-chip model to represent two different divisions.

When answering #2, many students are likely to reason from their previous knowledge that dividing zero by an integer will result in a quotient of zero and that modelling with integer chips is unnecessary.

Answers

Communicate the Ideas

- Answers may vary. Example:

- Allison's method resulted in two groups of six chips per group. The number of groups corresponds to the quotient. Tyler's method resulted in six groups with two chips per group. With this method, the number of chips in each group corresponds to the quotient.
- With Allison's groupings, the number of chips per group represents the quotient for $(+12) \div (+2)$. With Tyler's groupings, the number of groups represents this quotient.
- Yes, the quotient equals the total of the chips in any one group: -2 .
- No. Since the blue chips are negative, they cannot be grouped to represent $+6$.

- Explanations may vary. Example:

- Each group with two zero pairs has a value of zero. Zero is the quotient for this division statement.
- Yes, you can model the division with any number of zero pairs that is a multiple of four.
- Integer chips can be used to represent the division of zero by any positive integer. However, this method does not work for division by negative integers since you cannot separate zero pairs into a negative number of groups.

Assessment

Supporting Learning

Assessment as Learning

Communicate the Ideas

Have all students complete #1 to reinforce the two general methods, as well as when to apply the integer-chip model.

- Consider having students work in groups to complete these answers.
- Encourage the use of integer chips to model each problem.
- You may need to coach students through #1c) and d) to point out how the two methods differ. In part c), separating 12 blue chips into six equal groups (Tyler's method) successfully models $(-12) \div (+6)$, because 12 blue chips represent -12 and the six groups represent $+6$. In part d), separating 12 blue chips into groups of six (Angel's method) does not model $(-12) \div (+6)$, because each of the resulting groups represents -6 , not $+6$. Have students complete similar problems before going on.

Check Your Understanding

Practise

For help with #3 to #8, refer to Example 1 on pages 301–302.

3. Copy each division statement. Use the diagrams to complete it.

a) $(+10) \div (+2) = \square$



b) $(-16) \div (-4) = \square$



c) $(-14) \div (+2) = \square$



4. Copy each division statement. Use the diagrams to complete it.

a) $(-4) \div (-2) = \square$



b) $(+9) \div (+3) = \square$



c) $(-12) \div (+6) = \square$



5. Copy both division statements. Use the diagrams to complete them.

a) $(+14) \div (+2) = \square$
 $(+14) \div (+7) = \square$



b) $(-10) \div (-2) = \square$
 $(-10) \div (+5) = \square$



6. Copy both division statements. Use the diagrams to complete them.

a) $(+15) \div (+5) = \square$
 $(+15) \div (+3) = \square$



b) $(-18) \div (-9) = \square$
 $(-18) \div (+2) = \square$



7. Determine each quotient using integer chips. Have a partner check your chips. Then copy and complete the division statement

a) $(+16) \div (+4) = \square$
 b) $(-7) \div (+7) = \square$
 c) $(-12) \div (-6) = \square$

8. Divide using integer chips. Then copy and complete the division statement.

a) $(-20) \div (-10) = \square$
 b) $(-10) \div (+2) = \square$
 c) $(+4) \div (+2) = \square$

Apply

For help with #9 to #11, refer to Example 2 on page 302. Use the division of two integers to represent each situation and solve the problem.

9. A submarine was diving at 3 m/min. How long did it take to dive 21 m?

10. From 11:00 p.m. to 5:00 a.m., the temperature in Saskatoon fell from -1°C to -19°C .

- a) What was the change in temperature?
 b) What was the change in temperature per hour? What assumption did you make?

11. Gary takes four bus trips on each day of the weekend. He spends \$16 each weekend on bus fares. How much does each trip cost?



12. Copy the pattern.

$$\begin{aligned} (-12) \div (-3) &= \square \\ (-9) \div (-3) &= \square \\ (-6) \div (-3) &= \square \\ (-3) \div (-3) &= \square \\ 0 \div (-3) &= \square \\ (+3) \div (-3) &= \square \\ (+6) \div (-3) &= \square \end{aligned}$$

- a) Use integer chips to complete the first four lines. Describe the pattern.
 b) Extend the pattern to determine the quotient $(+6) \div (-3)$.

13. Copy the pattern.

$$\begin{aligned} (-8) \div (-2) &= \square \\ (-6) \div (-2) &= \square \\ (-4) \div (-2) &= \square \\ (-2) \div (-2) &= \square \\ 0 \div (-2) &= \square \\ (+2) \div (-2) &= \square \\ (+4) \div (-2) &= \square \end{aligned}$$

- a) Use integer chips to complete the first four lines. Describe the pattern.
 b) Extend the pattern to determine the quotient $(+4) \div (-2)$.

14. The deepest recorded dive is 500 m for an emperor penguin and 2000 m for a sperm whale.



- a) Use the division of two integers to represent how many times as deep a sperm whale can dive as an emperor penguin.
 b) How can you model the division using only 20 integer chips?
 c) What is the quotient?

Extend

15. Divide each of the following using integer chips or diagrams of chips. Explain your reasoning.

a) $(+15) \div (+5) \div (+3)$
 b) $(-24) \div (-2) \div (+4)$
 c) $(-20) \div (+2) \div (-5)$
 d) $(-18) \div (+2) \div (+3)$

16. Since sunset 6 h ago, the temperature in Brandon, Manitoba, has decreased from $+1^\circ\text{C}$ to -11°C . Predict what the temperature will be 3 h from now. What assumptions did you make?

Check Your Understanding

Practise

For #3 and #4, encourage students to think about the two methods for using integer chips. For example, in #3a), the two red chips in each of the five groups represent the integer +2. In #3c), the number of groups (two) represents the integer +2.

Encourage students to interpret the same integer-chip model as representing two different division statements in #5 and #6.

For #7, encourage discussion of the math by suggesting that students have a partner check their integer chips.

Apply

Students must model the descriptions mathematically for the real-world applications in #9 to #11 and #14. Remind students to choose the integers and the order in which they are to be divided.

In #12 and #13, the use of patterning to model the division of a positive integer by a negative integer is introduced. The two methods used previously cannot be used in this situation because a group of red chips cannot be separated into groups of blue chips or into a negative number of groups.

Extend

Each part of #15 involves two division operations. Students who are modelling with integer chips or diagrams will need to divide the first two integers and then divide their quotient by the third integer. Students who are able to think more abstractly may be able to complete the divisions without using the models.

Meeting Student Needs

- Provide **BLM 8–8 Section 8.3 Extra Practice** to students who would benefit from more practice.

ELL

- For #9, explain what a *submarine* is and that when it goes under the water this is called diving. Ensure that students understand that diving refers to a negative integer.

Common Errors

- Some students may not allow for the signs of the integers they are dividing.

R_x Reinforce the use of the two colours of manipulatives to account for positive and negative integers and to determine the sign of the quotient.

- Some students may have difficulty modelling integer division.

R_x Demonstrate that there are two methods for modelling division with integer chips. They both involve separating chips into equal groups, but one method involves counting the number of groups and the other method involves counting the number of chips in each group. For the division of a positive integer by a positive integer, both methods work. The division of a negative integer by a negative integer involves the first method. The division of a negative integer by a positive integer involves the second method. Neither method works for the division of a positive integer by a negative integer.

- Some students may be unsure of where to use brackets in writing division statements.
- R_x** Stress that, as in multiplication statements, brackets are used to distinguish integer signs from operation symbols. Therefore, brackets go around a number and its positive or negative sign.
- Some students may not realize the importance of dividing integers in a specific order.
- R_x** Use whole-number examples to remind students that the order is important in division. For example, compare $6 \div 3 = 2$ with $3 \div 6 = \frac{3}{6}$ or $\frac{1}{2}$.

Assessment	Supporting Learning
Assessment for Learning	
<p>Practise and Apply Have students do #3, #5, #7, and #10. Students who have no problems with these questions can go on to the rest of the Apply questions.</p>	<ul style="list-style-type: none"> • Provide additional coaching with Example 1 to students who need assistance with #3, #5, and #7. Encourage them to use integer chips. Have students show you how to do #4a), #6a), and #8a) before having them complete the remaining parts of those questions. • Provide additional coaching with Example 2 to students who need assistance with #10. Coach them through corrections to #10 and then have them try #11 on their own. • Encourage students to refer back to the examples in the student resource.
Assessment as Learning	
<p>Math Learning Log Have students complete the following statements: <ul style="list-style-type: none"> • The two methods using integer chips that can be used to determine integer division are ... • What I find most confusing about the division of integers is ... </p>	<ul style="list-style-type: none"> • Concrete and kinesthetic learners may benefit from using integer chips to answer the questions. • Depending on students' learning styles, have them provide oral or written answers. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with.