8.4 Dividing Integers

***MathLinks 8*, pages 306–311**

Suggested Timing
50–60 minutes

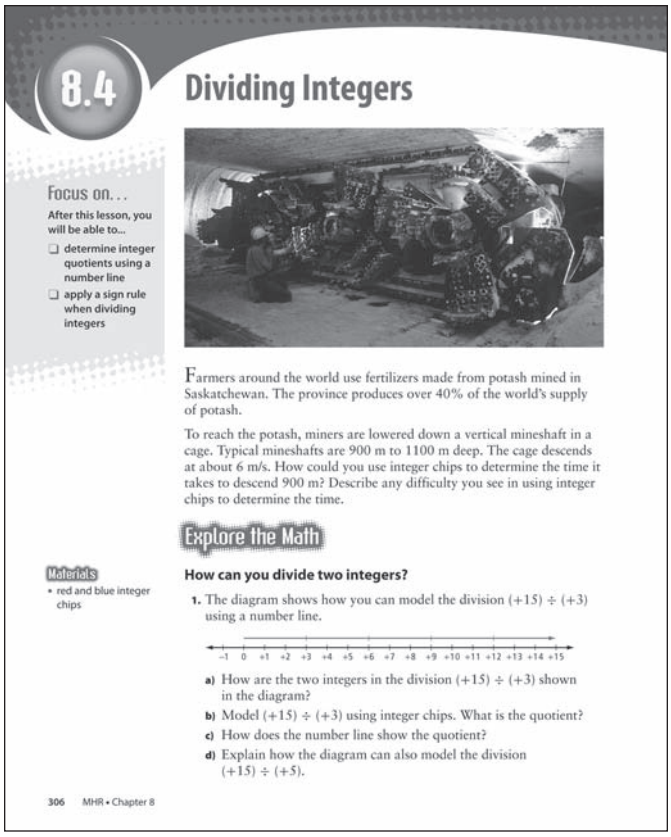
Materials
- red and blue integer chips
- coloured pencils
- red and blue construction paper (optional)
- scissors (optional)
- transparent plastic strips (optional)
- red and blue markers (optional)
- calculator (optional)

Blackline Masters
Master 4 Vertical and Horizontal Number Lines
Master 19 Multiplication Chart
BLM 8–3 Chapter 8 Warm-Up
BLM 8–9 Section 8.4 Extra Practice
BLM 8–10 Section 8.4 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
N7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, 9, 11, 19, Math Link
Typical	1–3, 5, 7, 9, 11–20, Math Link
Extension/Enrichment	1–4, 21, 22



8.4 Dividing Integers

Focus on...
After this lesson, you will be able to...
- determine integer quotients using a number line
- apply a sign rule when dividing integers

Farmers around the world use fertilizers made from potash mined in Saskatchewan. The province produces over 40% of the world's supply of potash.

To reach the potash, miners are lowered down a vertical mineshaft in a cage. Typical mineshafts are 900 m to 1100 m deep. The cage descends at about 6 m/s. How could you use integer chips to determine the time it takes to descend 900 m? Describe any difficulty you see in using integer chips to determine the time.

Explore the Math

Materials
- red and blue integer chips

How can you divide two integers?

1. The diagram shows how you can model the division (+15) ÷ (+3) using a number line.

−1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9 +10 +11 +12 +13 +14 +15

a) How are the two integers in the division (+15) ÷ (+3) shown in the diagram?
b) Model (+15) ÷ (+3) using integer chips. What is the quotient?
c) How does the number line show the quotient?
d) Explain how the diagram can also model the division (+15) ÷ (+5).

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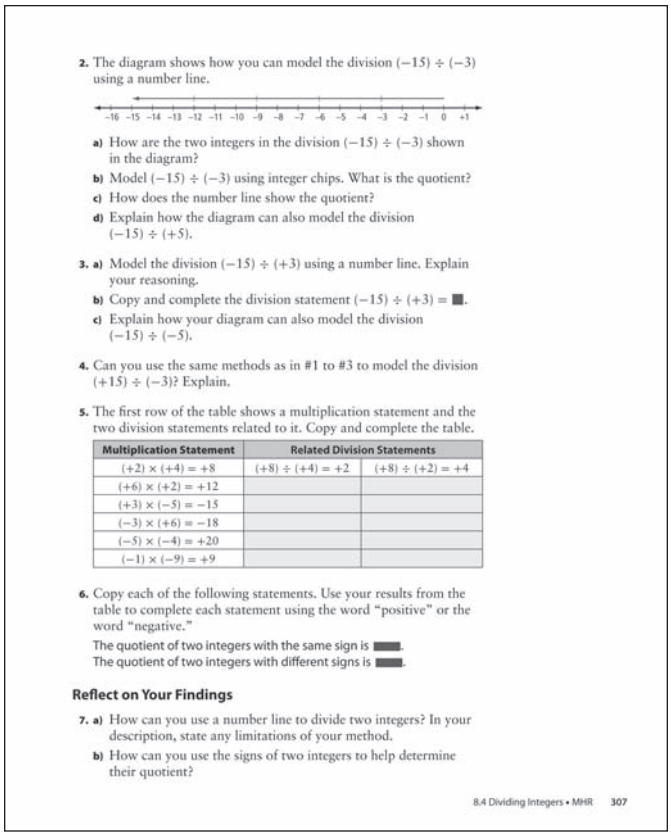
Planning Notes

As a class, read and discuss the information about potash mining in Saskatchewan provided in the student resource. Have students discuss the answers to the questions. There will likely be agreement on the idea of using the integer −900 to represent a descent of 900 m and the integer −6 to represent the 6 m/s rate of descent. Therefore, the time, in seconds, it takes to descend is given by (−900) ÷ (−6). Using integer chips to determine the time is problematic because of the need to separate 900 blue chips into groups of six chips to count the number of groups.

You may wish to stop the discussion at this point. However, some students may use whole numbers to reason that the time taken is 150 s. Other students may realize from their observations in section 8.3 that the quotient must be positive and that its numerical value must be 150. You may wish to return to the chapter opener and determine the time taken after students have more systematically established and applied the sign rules for integer division.



2. The diagram shows how you can model the division $(-15) \div (-3)$ using a number line.

a) How are the two integers in the division $(-15) \div (-3)$ shown in the diagram?
b) Model $(-15) \div (-3)$ using integer chips. What is the quotient?
c) How does the number line show the quotient?
d) Explain how the diagram can also model the division $(-15) \div (+5)$.

3. a) Model the division $(-15) \div (+3)$ using a number line. Explain your reasoning.
b) Copy and complete the division statement $(-15) \div (+3) = \blacksquare$.
c) Explain how your diagram can also model the division $(-15) \div (-5)$.

4. Can you use the same methods as in #1 to #3 to model the division $(+15) \div (-3)$? Explain.

5. The first row of the table shows a multiplication statement and the two division statements related to it. Copy and complete the table.

Multiplication Statement	Related Division Statements	
$(+2) \times (+4) = +8$	$(+8) \div (+4) = +2$	$(+8) \div (+2) = +4$
$(+6) \times (+2) = +12$		
$(+3) \times (-5) = -15$		
$(-3) \times (+6) = -18$		
$(-5) \times (-4) = +20$		
$(-1) \times (-9) = +9$		

6. Copy each of the following statements. Use your results from the table to complete each statement using the word "positive" or the word "negative."
The quotient of two integers with the same sign is ▬.
The quotient of two integers with different signs is ▬.

Reflect on Your Findings

7. a) How can you use a number line to divide two integers? In your description, state any limitations of your method.
b) How can you use the signs of two integers to help determine their quotient?

Explore the Math

In this exploration, students model integer division on a number line and develop the sign rules for dividing integers.

Method 1 Have students work on the exploration in pairs or small groups and discuss their answers. Give each pair of students a supply of red and blue integer chips (at least 20 of each colour), or other suitable manipulatives, so that students can compare this concrete method with the semi-concrete representations shown on a number line.

Provide students with copies of **Master 4 Vertical and Horizontal Number Lines**. Have students cut out paper strips of appropriate lengths to represent integers on the number lines, with red strips representing positive integers and blue strips representing negative integers. Have students draw an arrow along the length of each strip and label each strip with the appropriate integer, so that the positive values are right side up when the arrow points to the right and the negative values are right side up when the arrow points to the left. Have students complete #1 to #4 to become familiar with using the strips. Then, have students use the strips to model the division in the table in #5.

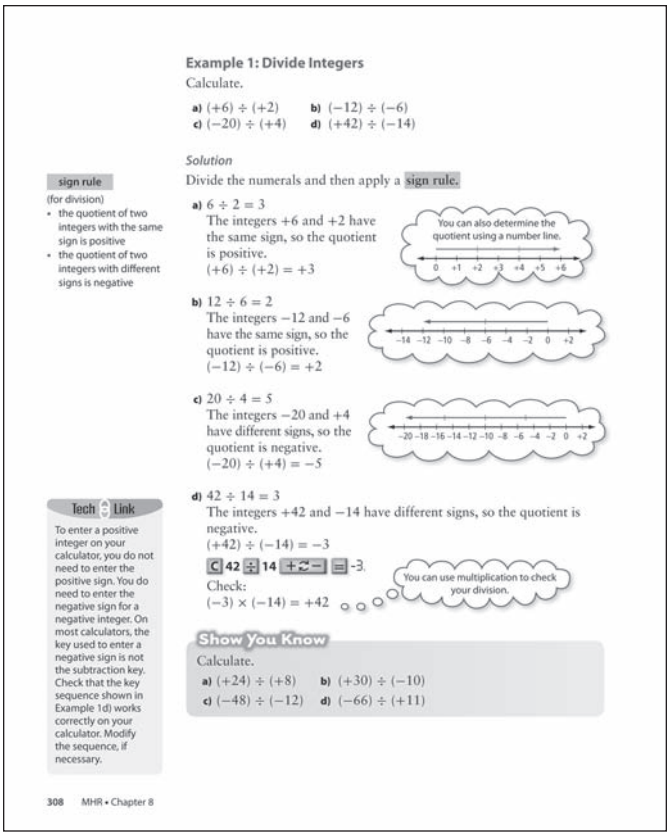
Where necessary, provide guidance to assist students in answering the questions. The diagrams provided in #1 and #2 model integer division by two methods. The first method is to cut the arrow that represents the dividend into equal sections, each representing the divisor, and count the number of sections, as in #1c), #2c), and #3c). The second method is to cut the arrow that represents the dividend into a number of equal sections, each representing the divisor, and determine the integer represented by each section, as in #1d), #2d), and #3a). Neither of these methods works in #4. (An arrow that represents $+15$ cannot be cut into -3 sections or into sections that each represents -3.) You may wish to point out the similarities between these methods and the integer-chip methods used in section 8.3.

It is important for students to make sure that the table in #5 is accurate before they make generalizations in #6. When students have done so, you might ask how the rules for quotients compare with the previously established rules for products.

Method 2 As in Method 1, have students work in pairs or small groups, and supply them with integer chips. Alternatively, have students draw coloured diagrams of integer chips. Students can draw coloured diagrams and use number lines to model integer division. You may wish to provide students with **Master 4 Vertical and Horizontal Number Lines** to complete this activity.

Example 1

Reinforce using the concrete or semi-concrete models in parts a) to c). Note that part d) of the example includes a calculator key sequence for the two-digit by two-digit integer division. The Tech Link beside this part of the example gives students some pointers on calculator use. Clarify that on most calculators the integer sign key and the subtraction key are not the same. Encourage students to write



Example 1: Divide Integers

Calculate.

a) $(+6) \div (+2)$ b) $(-12) \div (-6)$
c) $(-20) \div (+4)$ d) $(+42) \div (-14)$

Solution

Divide the numerals and then apply a sign rule.

sign rule
(for division)
- the quotient of two integers with the same sign is positive
- the quotient of two integers with different signs is negative

a) $6 \div 2 = 3$
The integers $+6$ and $+2$ have the same sign, so the quotient is positive.
$(+6) \div (+2) = +3$

You can also determine the quotient using a number line.

b) $12 \div 6 = 2$
The integers -12 and -6 have the same sign, so the quotient is positive.
$(-12) \div (-6) = +2$

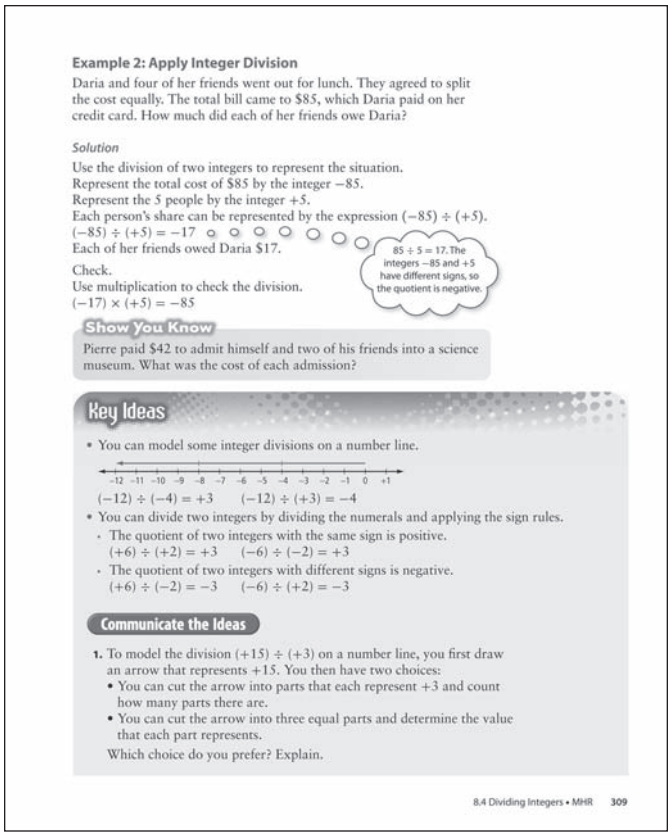
c) $20 \div 4 = 5$
The integers -20 and $+4$ have different signs, so the quotient is negative.
$(-20) \div (+4) = -5$

Tech Link
To enter a positive integer on your calculator, you do not need to enter the positive sign. You do need to enter the negative sign for a negative integer. On most calculators, the key used to enter a negative sign is not the subtraction key. Check that the key sequence shown in Example 1d) works correctly on your calculator. Modify the sequence, if necessary.

d) $42 \div 14 = 3$
The integers $+42$ and -14 have different signs, so the quotient is negative.
$(+42) \div (-14) = -3$
C 42 ÷ 14 +/− = -3.
Check:
$(-3) \times (-14) = +42$

You can use multiplication to check your division.

Show You Know
Calculate.
a) $(+24) \div (+8)$ b) $(+30) \div (-10)$
c) $(-48) \div (-12)$ d) $(-66) \div (+11)$



Example 2: Apply Integer Division

Daria and four of her friends went out for lunch. They agreed to split the cost equally. The total bill came to $85, which Daria paid on her credit card. How much did each of her friends owe Daria?

Solution

Use the division of two integers to represent the situation.
Represent the total cost of $85 by the integer -85.
Represent the 5 people by the integer $+5$.
Each person's share can be represented by the expression $(-85) \div (+5)$.
$(-85) \div (+5) = -17$
Each of her friends owed Daria $17.

$85 \div 5 = 17$. The integers -85 and $+5$ have different signs, so the quotient is negative.

Check.
Use multiplication to check the division.
$(-17) \times (+5) = -85$

Show You Know
Pierre paid $42 to admit himself and two of his friends into a science museum. What was the cost of each admission?

Key Ideas
- You can model some integer divisions on a number line.

$(-12) \div (-4) = +3$ $(-12) \div (+3) = -4$

- You can divide two integers by dividing the numerals and applying the sign rules.
  - The quotient of two integers with the same sign is positive.
    $(+6) \div (+2) = +3$ $(-6) \div (-2) = +3$
  - The quotient of two integers with different signs is negative.
    $(+6) \div (-2) = -3$ $(-6) \div (+2) = -3$

Communicate the Ideas
1. To model the division $(+15) \div (+3)$ on a number line, you first draw an arrow that represents $+15$. You then have two choices:
   - You can cut the arrow into parts that each represent $+3$ and count how many parts there are.
   - You can cut the arrow into three equal parts and determine the value that each part represents.

   Which choice do you prefer? Explain.

out the correct key sequence for their own calculator and to compare with other students. When students can successfully complete the calculation in part d) on their calculators, ask them to verify the answers to the other three parts of this example. To reinforce the connection between division and multiplication, draw attention to the use of multiplication to check the division in part d).

After discussing Example 1, have students complete the Show You Know question to make sure that they are ready to move on.

Example 2

Make sure that students understand the use of the integer -85 to represent the cost of lunch and the use of the integer $+5$ to represent the number of people. The check at the end of the solution reinforces the connection between division and multiplication. Emphasize the importance of including the summary statement in the solution to explain the meaning of the integer quotient.

After discussing Example 2, have students complete the Show You Know question to make sure that they are ready to move on.

Meeting Student Needs

- Add the sign rules for dividing integers to the poster showing the sign rules for multiplying integers. Point out that they are the same.
- Many cultures use patterns, such as bead work or quilting in their fine or decorative arts and crafts. Invite a community member or Elder to bring in some beadwork or display some art or handiwork. Have student discuss in small groups some of the patterns that they can see, such as multiples of repeated design elements, and in what ways they could use division to represent the patterns.
- When using manipulatives, consider using objects that are familiar from students' cultures, such as shells (e.g., dentalia), beads (e.g., pony beads), seeds, and buttons.

ELL

- Some students may not be familiar with terms such as *vertical mine shaft* and *cage* as they relate to mining. Use a picture to help students understand what a vertical mine shaft looks like and explain that a cage is similar to an elevator.
- For the Example 2 Show You Know, ensure that students understand that *admission* refers to how much it costs to get into an attraction.

Common Errors

- Some students may have difficulty in modelling integer division on a number line.

R_x Encourage the continued use of integer chips or diagrams of chips for as long as necessary. Assist students in comparing the integer-chip and number-line models in simple cases. Point out the resemblance between the models (i.e., the separation of integer chips into equal groups and the cutting of integer arrows into equal sections).

- Some students may not understand how the same number-line model can represent two different division statements.

R_x Stress the two general methods for modelling integer division using a number line:
- Cut the arrow that represents the dividend into equal sections, so that each section represents the divisor. Count the number of sections to determine the quotient.
- Cut the arrow that represents the dividend into equal sections, so that the number of sections represents the divisor. Determine the integer represented by each section to determine the quotient.

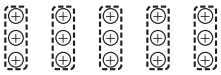
- Some students may have difficulty recognizing that two division statements are related to each multiplication statement.

R_x Use whole-number examples to remind students of this concept. For example, $3 \times 2 = 6$ has the related division statements $6 \div 2 = 3$ and $6 \div 3 = 2$.

Answers

Explore the Math

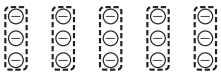
1. a) Answers may vary. Example: The dividend +15 is shown as an arrow 15 units long extending to the right from 0 on the number line. The divisor +3 is shown by sections three units long marked on the dividend arrow.

b)  The quotient is +5.

c) Answers may vary. Example: The quotient is shown by the number of three-unit sections in the dividend arrow.

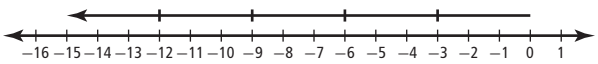
d) Answers may vary. Example: Since the dividend arrow is divided into five sections, the number of units in each section represents the quotient +3.

2. a) Answers may vary. Example: The dividend –15 is shown as an arrow 15 units long extending to the left from zero on the number line. The divisor –3 is shown by sections three units long marked on the dividend arrow.

b)  The quotient is +5.

c) Answers may vary. Example: The quotient is shown by the number of three-unit sections in the dividend arrow.

d) Answers may vary. Example: Since the dividend arrow is divided into five sections, the total value of the units in each section represents the quotient –3.

3. a) 

Explanations may vary. Example: The dividend –15 is shown as an arrow 15 units long drawn to the left of zero. The divisor +3 is shown by sections three units long marked on the dividend arrow.

b) $(-15) \div (+3) = -5$

c) Explanations may vary. Example: Since the dividend arrow is divided into five sections, the number of units in each section represents the quotient +3.

4. Explanations may vary. Example: No, a value of +15 cannot be divided using a number line into sections of –3, nor can it be divided into –3 sections.

5.

Multiplication Statement	Related Division Statements	
$(+2) \times (+4) = +8$	$(+8) \div (+4) = +2$	$(+8) \div (+2) = +4$
$(+6) \times (+2) = +12$	$(+12) \div (+2) = +6$	$(+12) \div (+6) = +2$
$(+3) \times (-5) = -15$	$(-15) \div (-5) = +3$	$(-15) \div (+3) = -5$
$(-3) \times (+6) = -18$	$(-18) \div (+6) = -3$	$(-18) \div (-3) = +6$
$(-5) \times (-4) = +20$	$(+20) \div (-4) = -5$	$(+20) \div (-5) = -4$
$(-1) \times (-9) = +9$	$(+9) \div (-9) = -1$	$(+9) \div (-1) = -9$

6. The quotient of two integers with the same sign is positive. The quotient of two integers with different signs is negative.

7. Explanations may vary. Example:

a) Show the dividend as an arrow drawn to the right from 0 if the dividend is positive, or drawn to the left from 0 if the dividend is negative. Then, either cut the dividend arrow into parts that each represent the divisor and count the number of parts, or cut the dividend arrow into the number of equal parts indicated by the divisor and determine the value that each part represents. This method does not work when the dividend is positive and the divisor is negative.

b) The quotient of two integers with the same sign is positive. The quotient of two integers with different signs is negative.

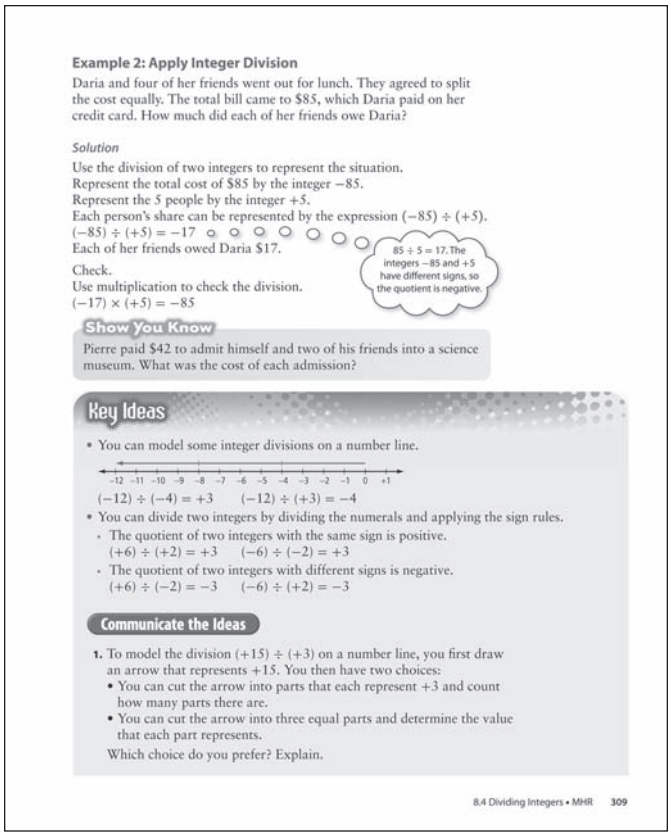
Show You Know: Example 1

a) +3 **b)** –3 **c)** +4 **d)** –6

Show You Know: Example 2

$14

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss the answers to #7a) and b). Try to have students generalize conclusions about their findings. Clarify any misunderstandings.	• Some students may need assistance generalizing how to use a number line to divide integers and how to use signs. Have these students model their understanding with integer chips or a number line and then verbalize the similarities and differences between the two methods. They could then apply their results to the sign rules. • Refer students who are unsure how to answer #6 back to #5. • Some students will benefit from referring back to the two methods and verbalizing their thinking in #7a). • Help students recall the sign rules as required. Encourage students to set up a reference chart in their chapter Foldable.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Have students work with a partner and talk through their thinking. • Have students refer back to the previous examples that model a question pattern similar to those in Example 1. • Some students may benefit from being coached through the two methods. The first method is to cut the arrow that represents the dividend into equal sections, each representing the divisor, and count the number of sections. The second method is to cut the arrow that represents the dividend into a number of equal sections, so that each section represents the divisor, and determine the integer represented by each section. • Remind students to link their results to the sign rules. • It may be beneficial to show students how to check their work using multiplication. • Some students may benefit from working with a multiplication table chart such as **Master 19 Multiplication Chart**.
Example 2 Have students do the Show You Know related to Example 2.	• Help students remember the terminology used to assign signs to integer values. It may be beneficial to discuss words that cannot be translated into a mathematical expression with a negative value. For example, *people* cannot take on a negative value. • Some students may benefit from writing out the problem and underlining the words that reflect integer values, or writing the word and the mathematical equivalent beside it. • Some students may find using a calculator beneficial. It may be necessary to help students recall the correct keystrokes.



Example 2: Apply Integer Division

Daria and four of her friends went out for lunch. They agreed to split the cost equally. The total bill came to $85, which Daria paid on her credit card. How much did each of her friends owe Daria?

Solution

Use the division of two integers to represent the situation.
Represent the total cost of $85 by the integer −85.
Represent the 5 people by the integer +5.
Each person's share can be represented by the expression (−85) ÷ (+5).
(−85) ÷ (+5) = −17
Each of her friends owed Daria $17.

85 ÷ 5 = 17. The integers −85 and +5 have different signs, so the quotient is negative.

Check.
Use multiplication to check the division.
(−17) × (+5) = −85

Show You Know

Pierre paid $42 to admit himself and two of his friends into a science museum. What was the cost of each admission?

Key Ideas

- You can model some integer divisions on a number line.

(−12) ÷ (−4) = +3 (−12) ÷ (+3) = −4

- You can divide two integers by dividing the numerals and applying the sign rules.
  - The quotient of two integers with the same sign is positive.
    (+6) ÷ (+2) = +3 (−6) ÷ (−2) = +3
  - The quotient of two integers with different signs is negative.
    (+6) ÷ (−2) = −3 (−6) ÷ (+2) = −3

Communicate the Ideas

1. To model the division (+15) ÷ (+3) on a number line, you first draw an arrow that represents +15. You then have two choices:
   - You can cut the arrow into parts that each represent +3 and count how many parts there are.
   - You can cut the arrow into three equal parts and determine the value that each part represents.

   Which choice do you prefer? Explain.

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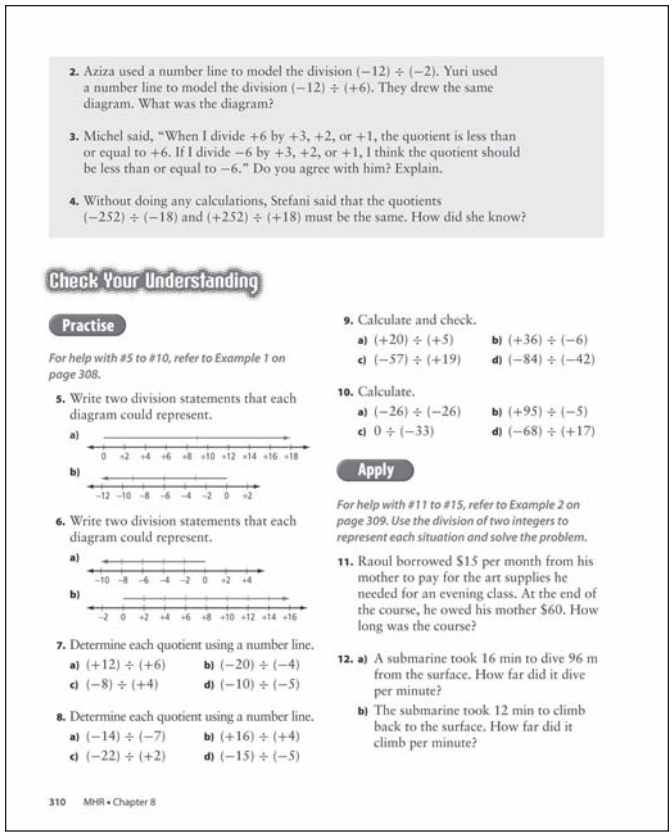

2. Aziza used a number line to model the division (−12) ÷ (−2). Yuri used a number line to model the division (−12) ÷ (+6). They drew the same diagram. What was the diagram?

3. Michel said, "When I divide +6 by +3, +2, or +1, the quotient is less than or equal to +6. If I divide −6 by +3, +2, or +1, I think the quotient should be less than or equal to −6." Do you agree with him? Explain.

4. Without doing any calculations, Stefani said that the quotients (−252) ÷ (−18) and (+252) ÷ (+18) must be the same. How did she know?

Check Your Understanding

Practise

For help with #5 to #10, refer to Example 1 on page 308.

5. Write two division statements that each diagram could represent.

a)

b)

6. Write two division statements that each diagram could represent.

a)

b)

7. Determine each quotient using a number line.
   a) (+12) ÷ (+6) b) (−20) ÷ (−4)
   c) (−8) ÷ (+4) d) (−10) ÷ (−5)

8. Determine each quotient using a number line.
   a) (−14) ÷ (−7) b) (+16) ÷ (+4)
   c) (−22) ÷ (+2) d) (−15) ÷ (−5)

9. Calculate and check.
   a) (+20) ÷ (+5) b) (+36) ÷ (−6)
   c) (−57) ÷ (+19) d) (−84) ÷ (−42)

10. Calculate.
    a) (−26) ÷ (−26) b) (+95) ÷ (−5)
    c) 0 ÷ (−33) d) (−68) ÷ (+17)

Apply

For help with #11 to #15, refer to Example 2 on page 309. Use the division of two integers to represent each situation and solve the problem.

11. Raoul borrowed $15 per month from his mother to pay for the art supplies he needed for an evening class. At the end of the course, he owed his mother $60. How long was the course?

12. a) A submarine took 16 min to dive 96 m from the surface. How far did it dive per minute?
    b) The submarine took 12 min to climb back to the surface. How far did it climb per minute?

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Key Ideas

Have students read and review the Key Ideas. Remind students that they know how to model the division of two integers with the same sign and the division of a negative integer by a positive integer on a number line, but not the division of a positive integer by a negative integer. Have students explain how the same number-line model represents two different division statements. Provide additional examples of number-line models. For the second Key Idea, you may wish to reinforce the similarity of the sign rules for integer division with those for integer multiplication.

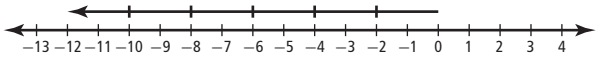
Communicate the Ideas

You may wish to have students complete the questions in groups and discuss their answers. For #1, students compare the two methods for modelling an integer division on a number line. In #2, the idea that the same number-line diagram can be used to model two different divisions is reinforced. Use #3 and #4 to prompt discussion of the sign rules.

Answers

Communicate the Ideas

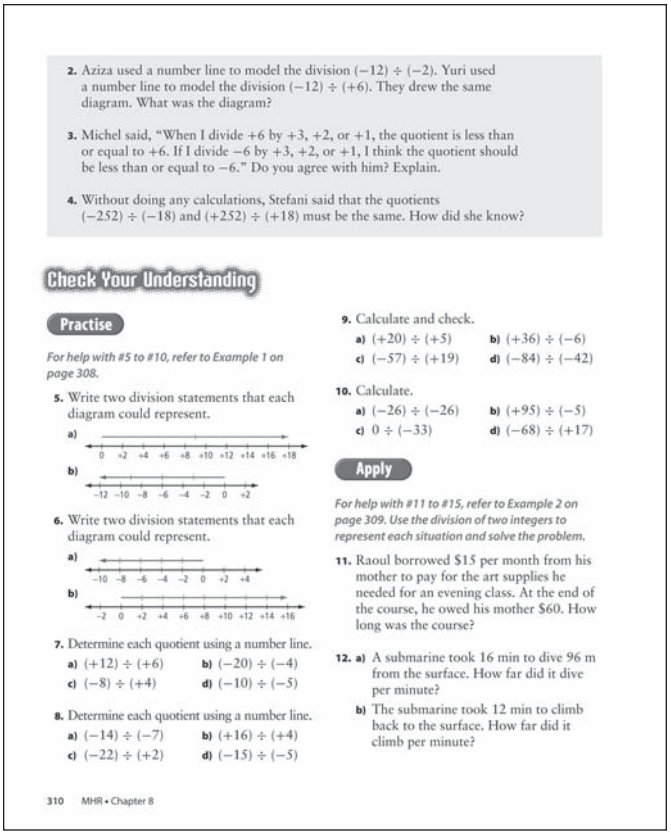
1. Answers may vary. Example: I prefer the first method because it is easier to count off the units for each part than to calculate how to divide the arrow into equal parts.

2. 

3. Explanations may vary. Example: No. The quotient is –2, –3, or –6, each of which is a negative number greater than or equal to the dividend.

4. Explanations may vary. Example: In each expression, the pairs of integers have the same sign. Therefore, both quotients are positive.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to #3 to reinforce the two general methods, as well as when to apply the integer-chip model.	• Refer students who need assistance with #1 to the Explore the Math. • Encourage students who need assistance with #2 and #3 to use number lines and integer chips to model the questions. Provide students with a similar problem to solve, such as $(-10) \div (-5)$ and $(-10) \div (+2)$ to check for understanding. • Have students write the sign rules into chart form to use for quick reference. • Students may benefit from using a multiplication table or chart such as **Master 19 Multiplication Chart**.



2. Aziza used a number line to model the division $(-12) \div (-2)$. Yuri used a number line to model the division $(-12) \div (+6)$. They drew the same diagram. What was the diagram?

3. Michel said, "When I divide +6 by +3, +2, or +1, the quotient is less than or equal to +6. If I divide −6 by +3, +2, or +1, I think the quotient should be less than or equal to −6." Do you agree with him? Explain.

4. Without doing any calculations, Stefani said that the quotients $(-252) \div (-18)$ and $(+252) \div (+18)$ must be the same. How did she know?

Check Your Understanding

Practise

For help with #5 to #10, refer to Example 1 on page 308.

5. Write two division statements that each diagram could represent.
a)

b)

6. Write two division statements that each diagram could represent.
a)

b)

7. Determine each quotient using a number line.
a) $(+12) \div (+6)$ b) $(-20) \div (-4)$
c) $(-8) \div (+4)$ d) $(-10) \div (-5)$

8. Determine each quotient using a number line.
a) $(-14) \div (-7)$ b) $(+16) \div (+4)$
c) $(-22) \div (+2)$ d) $(-15) \div (-5)$

9. Calculate and check.
a) $(+20) \div (+5)$ b) $(+36) \div (-6)$
c) $(-57) \div (+19)$ d) $(-84) \div (-42)$

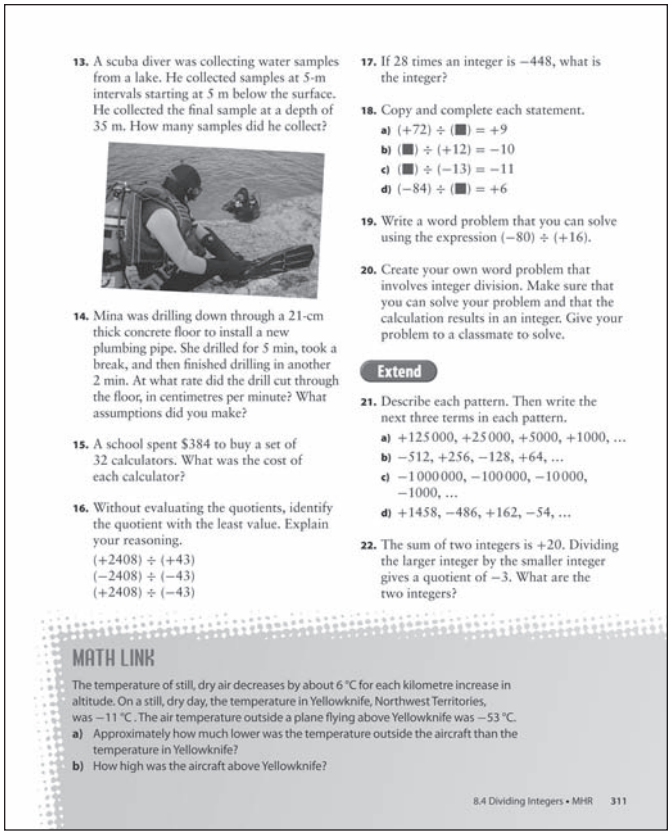
10. Calculate.
a) $(-26) \div (-26)$ b) $(+95) \div (-5)$
c) $0 \div (-33)$ d) $(-68) \div (+17)$

Apply

For help with #11 to #15, refer to Example 2 on page 309. Use the division of two integers to represent each situation and solve the problem.

11. Raoul borrowed $15 per month from his mother to pay for the art supplies he needed for an evening class. At the end of the course, he owed his mother $60. How long was the course?

12. a) A submarine took 16 min to dive 96 m from the surface. How far did it dive per minute?
b) The submarine took 12 min to climb back to the surface. How far did it climb per minute?



13. A scuba diver was collecting water samples from a lake. He collected samples at 5-m intervals starting at 5 m below the surface. He collected the final sample at a depth of 35 m. How many samples did he collect?

14. Mina was drilling down through a 21-cm thick concrete floor to install a new plumbing pipe. She drilled for 5 min, took a break, and then finished drilling in another 2 min. At what rate did the drill cut through the floor, in centimetres per minute? What assumptions did you make?

15. A school spent $384 to buy a set of 32 calculators. What was the cost of each calculator?

16. Without evaluating the quotients, identify the quotient with the least value. Explain your reasoning.
$(+2408) \div (+43)$
$(-2408) \div (-43)$
$(+2408) \div (-43)$

17. If 28 times an integer is −448, what is the integer?

18. Copy and complete each statement.
a) $(+72) \div (\blacksquare) = +9$
b) $(\blacksquare) \div (+12) = -10$
c) $(\blacksquare) \div (-13) = -11$
d) $(-84) \div (\blacksquare) = +6$

19. Write a word problem that you can solve using the expression $(-80) \div (+16)$.

20. Create your own word problem that involves integer division. Make sure that you can solve your problem and that the calculation results in an integer. Give your problem to a classmate to solve.

Extend

21. Describe each pattern. Then write the next three terms in each pattern.
a) +125 000, +25 000, +5000, +1000, ...
b) −512, +256, −128, +64, ...
c) −1 000 000, −100 000, −10 000, −1000, ...
d) +1458, −486, +162, −54, ...

22. The sum of two integers is +20. Dividing the larger integer by the smaller integer gives a quotient of −3. What are the two integers?

MATH LINK

The temperature of still, dry air decreases by about 6 °C for each kilometre increase in altitude. On a still, dry day, the temperature in Yellowknife, Northwest Territories, was −11 °C. The air temperature outside a plane flying above Yellowknife was −53 °C.
a) Approximately how much lower was the temperature outside the aircraft than the temperature in Yellowknife?
b) How high was the aircraft above Yellowknife?

Check Your Understanding

Practise

Students will differ in the extent to which they continue to rely on concrete or semi-concrete models to divide integers. Some may continue to need to use integer chips and number lines, while others may be comfortable with symbolic representations and the use of the sign rules. Assign Practise questions accordingly, and support as necessary. Some students may benefit from working with a partner or in a group. Note that students may require a calculator for the division of a two-digit integer by a two-digit integer in parts of #9 and #10. Have them estimate before calculating the answer.

Apply

For #11 to #15, students must model the descriptions of real-world applications mathematically. Stress the need for a summary statement that explains the meaning of the integer quotient.

For #17 and #18, the pre-algebraic determination of missing integers in multiplication and division statements is included. Some students may complete these questions by systematic trial, whereas others may already be able to think more abstractly. Encourage students to use their knowledge of multiplication and division to write related statements.

Students will differ in their abilities to create word problems in #19 and #20. Suggest that students model their problems on previous examples or questions. As students gain more confidence, encourage creativity. You may wish to discuss students' problems with the class.

Extend

In #21, the patterns are all based on the use of integer divisors. Students will likely answer #22 by systematic trial. In this question, the sign of the quotient is an important clue because it indicates that the integers have different signs.

Math Link

In this Math Link, students continue to use integers to solve temperature problems. Students use their understanding of integer division to determine the temperature outside of the aircraft and the altitude of the aircraft.

Meeting Student Needs

- Provide **BLM 8–9 Section 8.4 Extra Practice** to students who would benefit from more practice.

Common Errors

- Some students may ignore the sign rules for dividing integers.

R_x Emphasize the need to consider the sign of the product in all integer division problems. If necessary, use concrete or semi-concrete models to emphasize that some integer quotients are positive and others are negative.

- Some students may forget the sign rules for dividing integers.

R_x Stress the similarity to the rules for multiplication. You might also post the sign rules for integer multiplication and division in the classroom until students are thoroughly familiar with them.

- Some students may provide incomplete solutions to application problems by failing to explain the meaning of the integer quotient.

R_x Stress the need to include a summary statement at the end of the solution to an application problem.

- Some students may make keying errors when dividing integers on a calculator.

R_x Make sure that students understand how to enter negative integers on their calculators. Encourage the use of multiplication to check numerical answers from integer divisions.

Answers

Math Link

a) 42 °C **b)** 7 km

Assessment	Supporting Learning
Assessment *for* Learning	
Practise Have students do #5, #7, #9, and #11. Students who have no problems with these questions can go on to the remaining Apply questions.	• Students who need assistance with #5 could be coached through #5a). Have them verbalize their understanding. Students may benefit from working in pairs and explaining their thinking to each other. • Encourage students to use integer chips to model the mathematical processes, then translate the process to one that they can model using arrows on a number line. Have students try the remaining parts of #5 or #6 before going on. • Encourage students who need assistance with #7 to verbalize their thinking. They may benefit from having a multiplication table such as **Master 19 Multiplication Chart.** Coach students through the sign rules relating to integers, and then have them complete #8 on their own. • Students who need assistance with #9 may require coaching in the rules that apply to estimation. A calculator may be of assistance for the estimation and actual calculation. Ensure students are clear on keystrokes if using a calculator. Make sure that they check each calculation by multiplying. Have students try #10 on their own. • When doing #11, some students may need help to recall the meanings of mathematical words and how they translate into mathematical expressions and operations. Have students list such words, along with their mathematical meanings, in their chapter Foldable. • Have students try #12 before moving on.
Math Link The Math Link on page 311 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 321.	• You may wish to have students complete the Math Link to apply their understanding of integer division. Listen to any discussion about how students solve the problem or for any terminology that may cause them difficulty. • Students may wish to refer back to the Explore the Math section or previous questions in the Practise sections for assistance. • Encourage students to write an expression representing the problem. • Remind students of the importance of a final statement. • Some students may benefit from using **BLM 8–10 Section 8.4 Math Link**, which provides scaffolding for this activity.
Assessment *as* Learning	
Math Learning Log Have students answer the following questions: • What methods can you use to divide two integers? • What method do you like best? Why?	• Have students discuss each answer with a partner before recording the answer on their own. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with.