Applying Integer Operations

8.5

***MathLinks 8*, pages 312–317**

Suggested Timing
50–60 minutes

Materials
- calculator (optional)

Blackline Masters
BLM 8–3 Chapter 8 Warm-Up
BLM 8–11 Section 8.5 Extra Practice

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☐ Visualization (V)

Specific Outcomes
N7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, 6, 8, 12, 15
Typical	1–4, 6, 8–21
Extension/Enrichment	1–3, 22–24

Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce material learned in previous sections.

Inform students that in section 8.5 they will need to decide when to use multiplication and when to use division of integers. Because the section includes the order of operations with integers, students will also draw on their knowledge of addition and subtraction.

As a class, read and discuss the information about the *Ben Franklin* submersible. Students may be interested in the origin of its name, as described in the Did You Know? feature. Ask students to consider the two questions at the end of the description. Students will likely agree on the use of the integers −600 and −200 to represent the two depths. Ask students which operation is required to answer the second question. The word *times* in the question may suggest to some that multiplication is required, but division is usually used to determine how many times as great one quantity is as another. Once the decision is made to use division, have students complete the calculation, with negative integers representing the depths: $(-600) \div (-200) = +3$. Thus, a depth of 600 m is three times as great as a depth of 200 m.

8.5 Applying Integer Operations

Focus on...
After this lesson, you will be able to...
- apply the order of operations to solve problems involving integers

A famous submersible, the *Ben Franklin*, is at the Vancouver Maritime Museum. This submersible was built to study the currents and sea life along the east coast of North America in the Gulf Stream.

The *Ben Franklin* explored the Gulf Stream at depths from 200 m to 600 m. How would you represent these depths with integers? What operation would you use to find how many times as great a depth of 600 m is as a depth of 200 m?

Did You Know?
The *Ben Franklin* is named after the American scientist and diplomat Benjamin Franklin, 1706–1790. While a passenger on transatlantic voyages, he spent his time measuring ocean temperatures. Franklin mapped a stream of warm water about 70 km wide flowing across the cold Atlantic. He named this stream the Gulf Stream.

Explore the Math

Why is it important to know the order of operations when solving problems involving integers?

Laura, Abeni, and Rob were discussing the following problem.

A submersible dives from the surface at 15 m/min for 6 min and then at 25 m/min for 20 min. What is the depth of the submersible after the dive?

They worked together to write the following expression to solve the problem.
$6 \times (-15) + 20 \times (-25)$

Then they evaluated the expression independently.
Laura evaluated the expression as +1750.
Abeni evaluated the expression as −590.
Rob evaluated the expression as −750.

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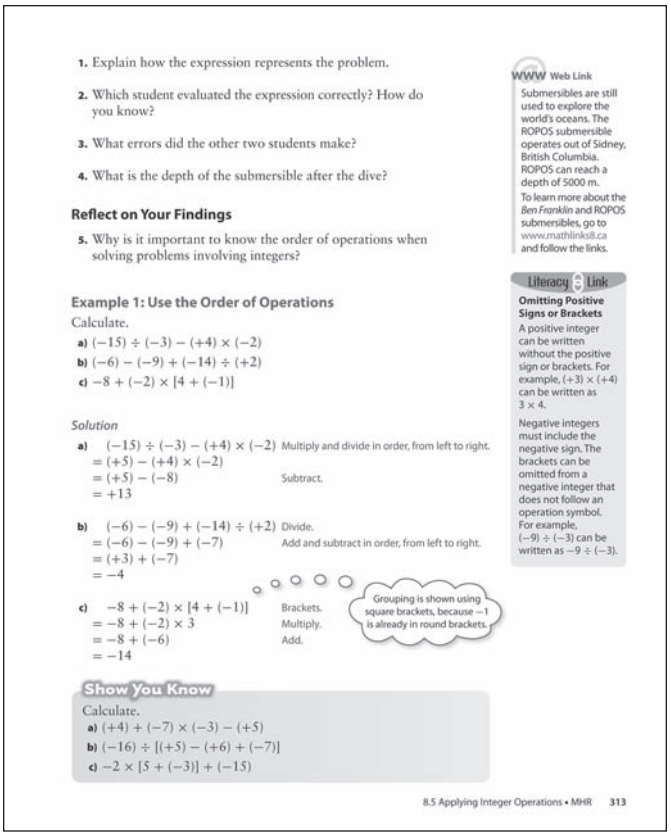
Explore the Math

In this exploration, students explore the importance of knowing the order of operations when solving problems involving integers.



1. Explain how the expression represents the problem.
2. Which student evaluated the expression correctly? How do you know?
3. What errors did the other two students make?
4. What is the depth of the submersible after the dive?

Reflect on Your Findings

5. Why is it important to know the order of operations when solving problems involving integers?

WWW Web Link
Submersibles are still used to explore the world's oceans. The ROPOS submersible operates out of Sidney, British Columbia. ROPOS can reach a depth of 5000 m.
To learn more about the *Ben Franklin* and ROPOS submersibles, go to www.mathlinks8.ca and follow the links.

Literacy Link
Omitting Positive Signs or Brackets
A positive integer can be written without the positive sign or brackets. For example, $(+3) \times (+4)$ can be written as 3×4.
Negative integers must include the negative sign. The brackets can be omitted from a negative integer that does not follow an operation symbol. For example, $(-9) \div (-3)$ can be written as $-9 \div (-3)$.

Example 1: Use the Order of Operations
Calculate.
a) $(-15) \div (-3) - (+4) \times (-2)$
b) $(-6) - (-9) + (-14) \div (+2)$
c) $-8 + (-2) \times [4 + (-1)]$

Solution
a) $(-15) \div (-3) - (+4) \times (-2)$ Multiply and divide in order, from left to right.
$= (+5) - (+4) \times (-2)$
$= (+5) - (-8)$ Subtract.
$= +13$

b) $(-6) - (-9) + (-14) \div (+2)$ Divide.
$= (-6) - (-9) + (-7)$ Add and subtract in order, from left to right.
$= (+3) + (-7)$
$= -4$

c) $-8 + (-2) \times [4 + (-1)]$ Brackets.
$= -8 + (-2) \times 3$ Multiply.
$= -8 + (-6)$ Add.
$= -14$

Grouping is shown using square brackets, because -1 is already in round brackets.

Show You Know
Calculate.
a) $(+4) + (-7) \times (-3) - (+5)$
b) $(-16) \div [(+5) - (+6) + (-7)]$
c) $-2 \times [5 + (-3)] + (-15)$

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Method 1 Have students complete the exploration in groups of three, assuming the roles of Laura, Abeni, and Rob. The student assigned to each role can determine how that person evaluated the expression and describe the method to the group. The three students can collectively decide which value of the expression is correct. Some students may reason from the wording of the problem that the descent can be considered in two stages, first a descent to a depth of 90 m and then a descent of a further 500 m to a total depth of 590 m. Other students may invoke the order of operations and state that both multiplication operations in the expression must be performed before the addition.

Method 2 Have students complete the exploration individually. Then, poll the class to determine which value of the expression students believe is correct. If there are differences of opinion, have a class or group discussion to decide on the correct value.

Example 1

This example shows students how to apply the order of operations to integer calculations.

Literacy Link Note that in part c) positive signs and unnecessary brackets are omitted from the expressions, as described in the Literacy Link beside the exploration. Stress the situations in which signs and brackets can and cannot be omitted. Part c) also includes the use of square brackets as a grouping symbol to avoid the use of one set of round brackets around another set.

After discussing Example 1, have students complete the Show You Know question to make sure that they are ready to move on.

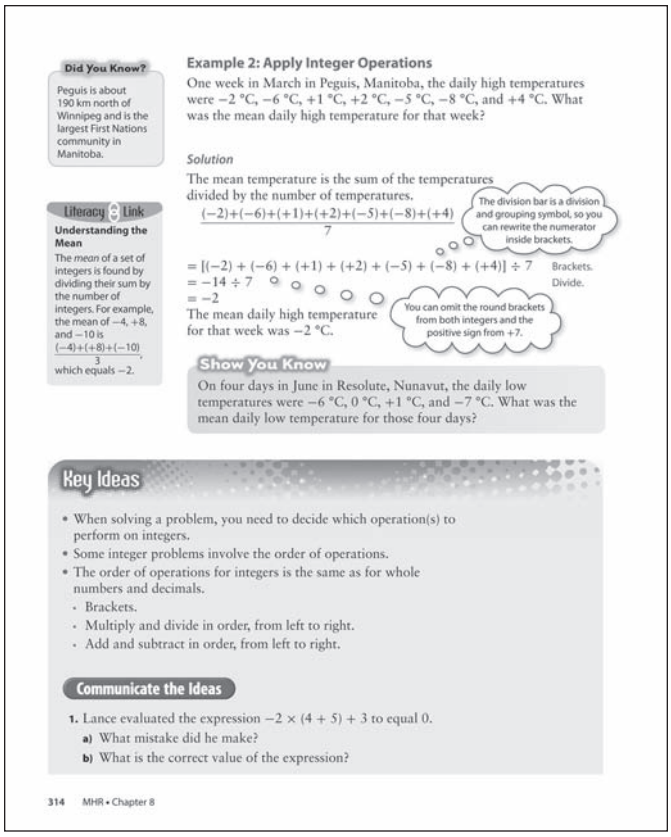
Example 2

Example 2 shows an application of integer calculations and involves the order of operations in calculating a mean value.

Literacy Link Draw students' attention to the Literacy Link beside this example, reminding students of the definition of *mean* and indicating the mean of three integers.

Stress the role of the division bar as a grouping symbol. Also, point out the selective use of signs and brackets in the example. Check that students clearly understand the situations in which signs and brackets can be omitted. As in earlier examples of applications of integer calculations in this chapter, stress the importance of the summary statement to explain the meaning of the integer result.

After discussing Example 2, have students complete the Show You Know question to make sure that they are ready to move on.



Did You Know?

Peguis is about 190 km north of Winnipeg and is the largest First Nations community in Manitoba.

Literacy Link

Understanding the Mean

The *mean* of a set of integers is found by dividing their sum by the number of integers. For example, the mean of −4, +8, and −10 is $\frac{(-4)+(+8)+(-10)}{3}$, which equals −2.

Example 2: Apply Integer Operations

One week in March in Peguis, Manitoba, the daily high temperatures were −2 °C, −6 °C, +1 °C, +2 °C, −5 °C, −8 °C, and +4 °C. What was the mean daily high temperature for that week?

Solution

The mean temperature is the sum of the temperatures divided by the number of temperatures.

$$\frac{(-2)+(-6)+(+1)+(+2)+(-5)+(-8)+(+4)}{7}$$

The division bar is a division and grouping symbol, so you can rewrite the numerator inside brackets.

$= [(-2) + (-6) + (+1) + (+2) + (-5) + (-8) + (+4)] \div 7$ Brackets.

$= -14 \div 7$ Divide.

$= -2$

You can omit the round brackets from both integers and the positive sign from +7.

The mean daily high temperature for that week was −2 °C.

Show You Know

On four days in June in Resolute, Nunavut, the daily low temperatures were −6 °C, 0 °C, +1 °C, and −7 °C. What was the mean daily low temperature for those four days?

Key Ideas

- When solving a problem, you need to decide which operation(s) to perform on integers.
- Some integer problems involve the order of operations.
- The order of operations for integers is the same as for whole numbers and decimals.
  - Brackets.
  - Multiply and divide in order, from left to right.
  - Add and subtract in order, from left to right.

Communicate the Ideas

1. Lance evaluated the expression $-2 \times (4 + 5) + 3$ to equal 0.
   a) What mistake did he make?
   b) What is the correct value of the expression?

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Meeting Student Needs

- Show the students a picture of *Ben Franklin* and locate the Gulf Stream on a map.
- Describe the way we use sea level as the zero point for measuring how high land is. Discuss how high above sea level the students' community is. How would we write numbers showing how deep the ocean is?
- For Example 1, you may wish to quickly reactivate students' skills related to the order of operations.
- For Example 1, complete at least one additional problem of each type of division shown in Example 1, working first as a whole class and then with partners, before doing the Show You Know activity.
- You may wish to have student work in small groups. Model examples showing the steps used to complete the equations. Allow time for discussion among the members of each group and between groups.

Common Errors

- Some students may have difficulty in deciding on the correct value of the expression in the Explore the Math.

R_x Encourage students to think of the descent of the submersible in two stages, represented by $6 \times (-15)$ and $20 \times (-25)$, and to add the two products to determine the total descent.

- Some students may have difficulty in deciding how each value of the expression was determined in the Explore the Math.

R_x Assist students in recognizing the possible calculation methods. The most obvious ones are to perform the operations in order from left to right (which results in an incorrect value of +1750); to multiply before adding (which results in the correct value of −590); and to add before multiplying (which results in an incorrect value of −750).

Answers

Explore the Math

1. Explanations may vary. Example: $6 \times (-15)$ represents the dive for the first 6 min when the rate was 15 m/min. Similarly, $20 \times (-25)$ represents the dive for the next 20 min when the rate was 25 m/min.
2. Abeni evaluated the expression correctly. Explanations may vary. Example: The multiplication is performed before the addition.
3. Explanations may vary. Example: Laura evaluated the expression by performing the operations from left to right. Rob added −15 to +20 before carrying out the multiplication.
4. The depth of the submersible after the dive is 590 m.
5. Answers may vary. Example: It is important to know the order of operations when solving problems involving integers because you will get different answers depending on the order in which you perform the different operations.

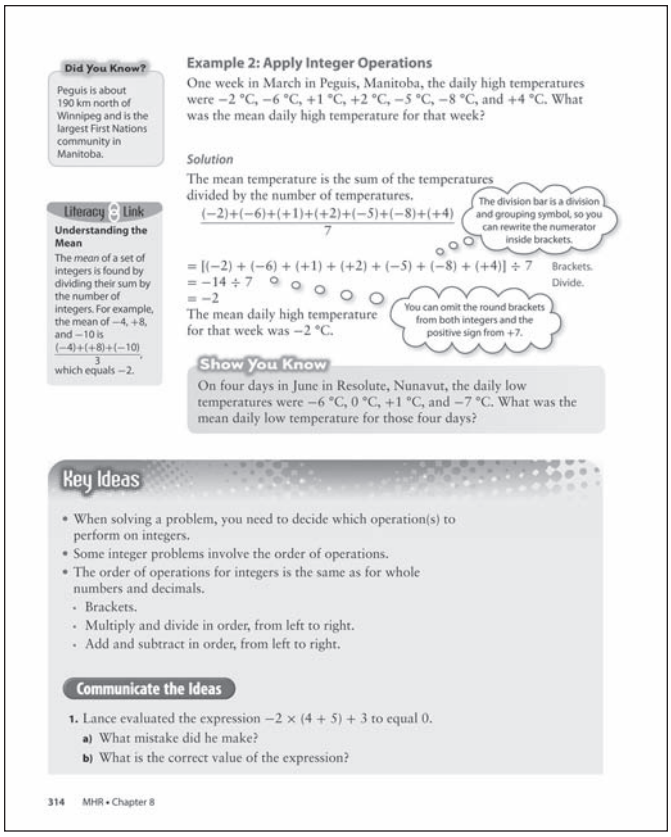
Show You Know: Example 1

a) 20 **b)** 2 **c)** −19

Show You Know: Example 2

−3 °C

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss the answer to #5. Clarify any misunderstandings.	• Students may benefit from additional questions to help them understand why the order of operations is important in solving problems. Allow them to verbalize how they would solve them and why they knew to carry out the work in the order they did. • Clarify any misunderstandings.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Have students work with a partner and talk through their thinking. • Have students refer back to the previous examples that model a question pattern similar to those in Example 1. • Some students may benefit from being coached through the steps represented by the order of operations for each of parts a) to c) in Example 1. • Students may benefit from working through additional problems. Have them verbalize their thinking for solving the problems. • Have students record the steps involved in the order of operations in their chapter Foldable.
Example 2 Have students do the Show You Know related to Example 2.	• Some students may benefit from working with a partner and talking through their thinking. • You may need to help some students recall the meaning of *mean* and the processes involved in integer addition. • Calculators may be beneficial for some students.



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One week in March in Peguis, Manitoba, the daily high temperatures were −2 °C, −6 °C, +1 °C, +2 °C, −5 °C, −8 °C, and +4 °C. What was the mean daily high temperature for that week?

Solution

The mean temperature is the sum of the temperatures divided by the number of temperatures.

$$\frac{(-2)+(-6)+(+1)+(+2)+(-5)+(-8)+(+4)}{7}$$

The division bar is a division and grouping symbol, so you can rewrite the numerator inside brackets.

$= [(-2) + (-6) + (+1) + (+2) + (-5) + (-8) + (+4)] \div 7$ Brackets.

$= -14 \div 7$ Divide.

$= -2$

You can omit the round brackets from both integers and the positive sign from +7.

The mean daily high temperature for that week was −2 °C.

Show You Know

On four days in June in Resolute, Nunavut, the daily low temperatures were −6 °C, 0 °C, +1 °C, and −7 °C. What was the mean daily low temperature for those four days?

Key Ideas

- When solving a problem, you need to decide which operation(s) to perform on integers.
- Some integer problems involve the order of operations.
- The order of operations for integers is the same as for whole numbers and decimals.
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  - Multiply and divide in order, from left to right.
  - Add and subtract in order, from left to right.

Communicate the Ideas

1. Lance evaluated the expression $-2 \times (4 + 5) + 3$ to equal 0.
   a) What mistake did he make?
   b) What is the correct value of the expression?

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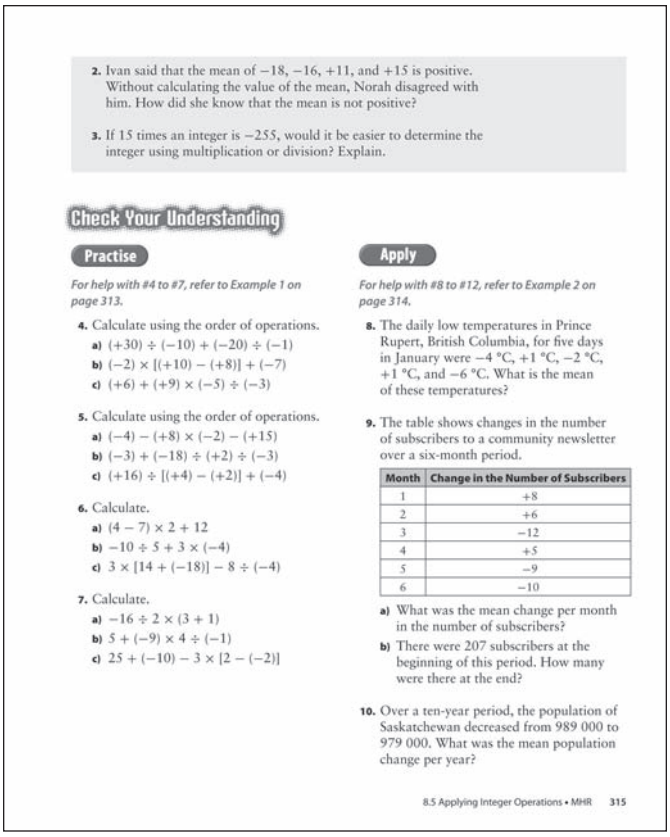

2. Ivan said that the mean of −18, −16, +11, and +15 is positive. Without calculating the value of the mean, Norah disagreed with him. How did she know that the mean is not positive?

3. If 15 times an integer is −255, would it be easier to determine the integer using multiplication or division? Explain.

Check Your Understanding

Practise

For help with #4 to #7, refer to Example 1 on page 313.

4. Calculate using the order of operations.
   a) $(+30) \div (-10) + (-20) \div (-1)$
   b) $(-2) \times [(+10) - (+8)] + (-7)$
   c) $(+6) + (+9) \times (-5) \div (-3)$

5. Calculate using the order of operations.
   a) $(-4) - (+8) \times (-2) - (+15)$
   b) $(-3) + (-18) \div (+2) \div (-3)$
   c) $(+16) \div [(+4) - (+2)] + (-4)$

6. Calculate.
   a) $(4 - 7) \times 2 + 12$
   b) $-10 \div 5 + 3 \times (-4)$
   c) $3 \times [14 + (-18)] - 8 \div (-4)$

7. Calculate.
   a) $-16 \div 2 \times (3 + 1)$
   b) $5 + (-9) \times 4 \div (-1)$
   c) $25 + (-10) - 3 \times [2 - (-2)]$

Apply

For help with #8 to #12, refer to Example 2 on page 314.

8. The daily low temperatures in Prince Rupert, British Columbia, for five days in January were −4 °C, +1 °C, −2 °C, +1 °C, and −6 °C. What is the mean of these temperatures?

9. The table shows changes in the number of subscribers to a community newsletter over a six-month period.

Month	Change in the Number of Subscribers
1	+8
2	+6
3	−12
4	+5
5	−9
6	−10

   a) What was the mean change per month in the number of subscribers?
   b) There were 207 subscribers at the beginning of this period. How many were there at the end?

10. Over a ten-year period, the population of Saskatchewan decreased from 989 000 to 979 000. What was the mean population change per year?

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Key Ideas

Have students read and review the Key Ideas. Stress that the order of operations for integers is the same as the order that students previously used for whole numbers and decimals.

Communicate the Ideas

You may wish to have students complete the questions in groups and discuss their answers. In #1, students need to correct an error in applying the order of operations. In #2, students use mental math to decide whether the mean of a set of integers is positive or negative. In #3, multiplication would involve systematic trial and be more time consuming.

Answers

Communicate the Ideas

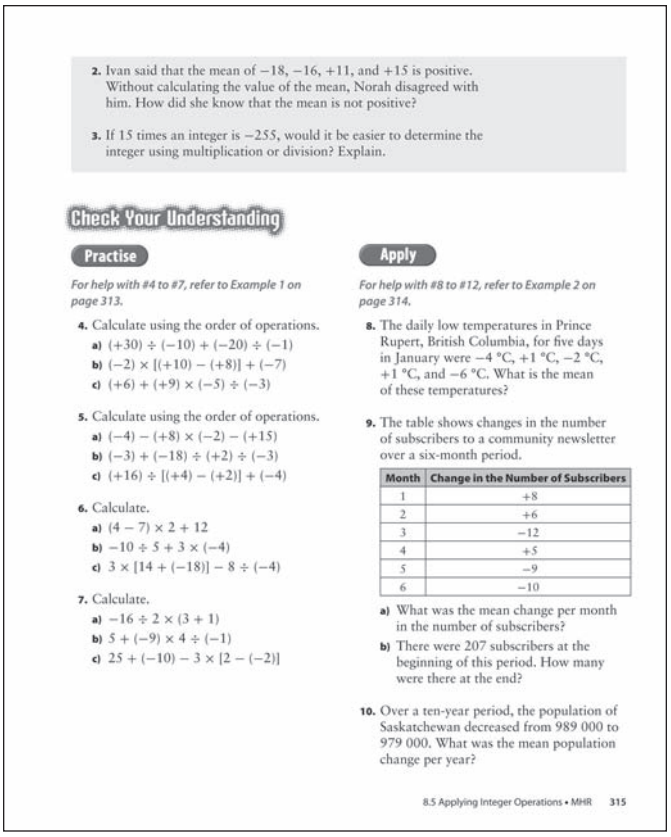
1. a) Explanations may vary. Example: He multiplied –2 by 4 first, and then added 5 and 3 to get the answer 0.

b) –15

2. Explanations may vary. Example: Since –18 and –16 are further from zero than +11 and +15, the sum of the four numbers must be negative. Therefore, their mean is also negative.

3. Answers may vary. Example: It would be easier to use division. With –255 as the dividend and 15 as the divisor, the quotient is the unknown integer.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to #3.	• Partner students so that they can practise verbalizing their thinking. • In #1, students need to apply their understanding of the order of operations. Students may benefit from referring back to previous examples. • Students who need assistance with #2 may benefit from writing the meaning of the word *mean* and the processes associated with it in their chapter Foldable. • For #3, have students refer back to the previous examples and encourage them to write mathematical symbols for the appropriate word in the question.



2. Ivan said that the mean of −18, −16, +11, and +15 is positive. Without calculating the value of the mean, Norah disagreed with him. How did she know that the mean is not positive?

3. If 15 times an integer is −255, would it be easier to determine the integer using multiplication or division? Explain.

Check Your Understanding

Practise

For help with #4 to #7, refer to Example 1 on page 313.

4. Calculate using the order of operations.
   a) (+30) ÷ (−10) + (−20) ÷ (−1)
   b) (−2) × [(+10) − (+8)] + (−7)
   c) (+6) + (+9) × (−5) ÷ (−3)

5. Calculate using the order of operations.
   a) (−4) − (+8) × (−2) − (+15)
   b) (−3) + (−18) ÷ (+2) ÷ (−3)
   c) (+16) ÷ [(+4) − (+2)] + (−4)

6. Calculate.
   a) (4 − 7) × 2 + 12
   b) −10 ÷ 5 + 3 × (−4)
   c) 3 × [14 + (−18)] − 8 ÷ (−4)

7. Calculate.
   a) −16 ÷ 2 × (3 + 1)
   b) 5 + (−9) × 4 ÷ (−1)
   c) 25 + (−10) − 3 × [2 − (−2)]

Apply

For help with #8 to #12, refer to Example 2 on page 314.

8. The daily low temperatures in Prince Rupert, British Columbia, for five days in January were −4 °C, +1 °C, −2 °C, +1 °C, and −6 °C. What is the mean of these temperatures?

9. The table shows changes in the number of subscribers to a community newsletter over a six-month period.

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   a) What was the mean change per month in the number of subscribers?
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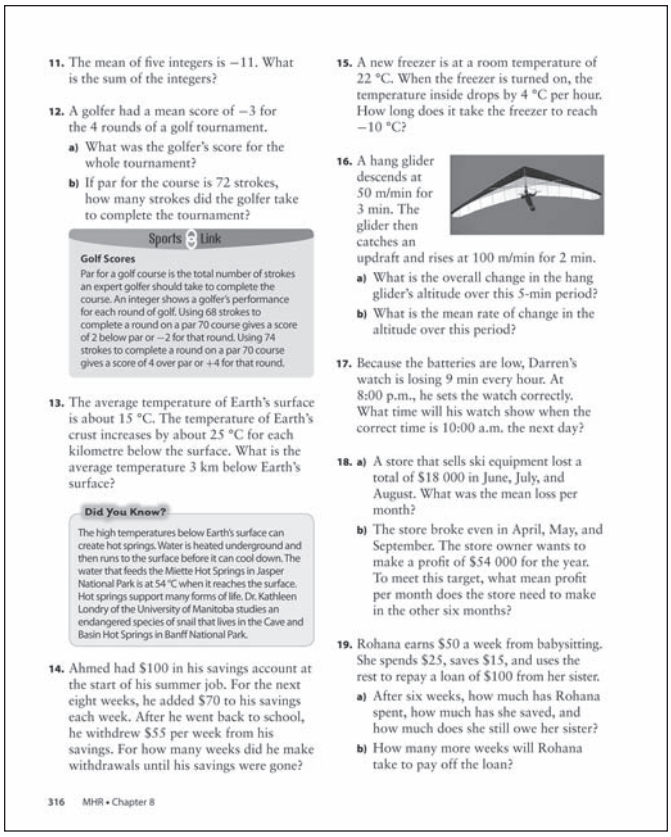
10. Over a ten-year period, the population of Saskatchewan decreased from 989 000 to 979 000. What was the mean population change per year?



11. The mean of five integers is −11. What is the sum of the integers?

12. A golfer had a mean score of −3 for the 4 rounds of a golf tournament.
   a) What was the golfer's score for the whole tournament?
   b) If par for the course is 72 strokes, how many strokes did the golfer take to complete the tournament?

Sports Link

Golf Scores

Par for a golf course is the total number of strokes an expert golfer should take to complete the course. An integer shows a golfer's performance for each round of golf. Using 68 strokes to complete a round on a par 70 course gives a score of 2 below par or −2 for that round. Using 74 strokes to complete a round on a par 70 course gives a score of 4 over par or +4 for that round.

13. The average temperature of Earth's surface is about 15 °C. The temperature of Earth's crust increases by about 25 °C for each kilometre below the surface. What is the average temperature 3 km below Earth's surface?

Did You Know?

The high temperatures below Earth's surface can create hot springs. Water is heated underground and then runs to the surface before it can cool down. The water that feeds the Miette Hot Springs in Jasper National Park is at 54 °C when it reaches the surface. Hot springs support many forms of life. Dr. Kathleen Londry of the University of Manitoba studies an endangered species of snail that lives in the Cave and Basin Hot Springs in Banff National Park.

14. Ahmed had $100 in his savings account at the start of his summer job. For the next eight weeks, he added $70 to his savings each week. After he went back to school, he withdrew $55 per week from his savings. For how many weeks did he make withdrawals until his savings were gone?

15. A new freezer is at a room temperature of 22 °C. When the freezer is turned on, the temperature inside drops by 4 °C per hour. How long does it take the freezer to reach −10 °C?

16. A hang glider descends at 50 m/min for 3 min. The glider then catches an updraft and rises at 100 m/min for 2 min.
   a) What is the overall change in the hang glider's altitude over this 5-min period?
   b) What is the mean rate of change in the altitude over this period?

17. Because the batteries are low, Darren's watch is losing 9 min every hour. At 8:00 p.m., he sets the watch correctly. What time will his watch show when the correct time is 10:00 a.m. the next day?

18. a) A store that sells ski equipment lost a total of $18 000 in June, July, and August. What was the mean loss per month?
   b) The store broke even in April, May, and September. The store owner wants to make a profit of $54 000 for the year. To meet this target, what mean profit per month does the store need to make in the other six months?

19. Rohana earns $50 a week from babysitting. She spends $25, saves $15, and uses the rest to repay a loan of $100 from her sister.
   a) After six weeks, how much has Rohana spent, how much has she saved, and how much does she still owe her sister?
   b) How many more weeks will Rohana take to pay off the loan?

Check Your Understanding

Practise

All of these questions involve using the order of operations.

Apply

For #8 to #20, students must model the descriptions of real-world applications mathematically. For these questions, stress the need for a summary statement to explain the meaning of the integer result.

For #12, refer students to the Sports Link, which is intended to assist those who are not familiar with the scoring system used in golf.

The Did You Know? connects the context in #13 to the occurrence of hot springs and research taking place in Banff National Park.

For #20, draw students' attention to the need for careful use of language to describe the expressions unambiguously. You may wish to give an example of an ambiguous statement, e.g., if the question is written as 20 minus 3 times −8, students will not know whether the intended mathematical expression is $(20 - 3) \times (-8)$, which equals −136, or $20 - 3 \times (-8)$, which equals 44. Challenge students to make up more expressions like those in #20, making sure that they are unambiguous, and then have a classmate evaluate them.

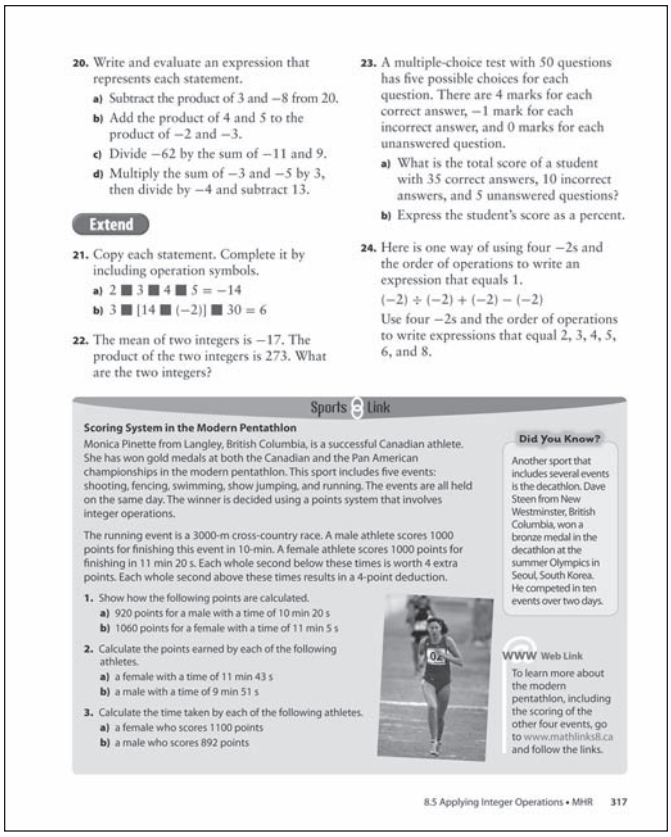
Extend

In #21, students can use systematic trial to determine the operation symbol that belongs in each location.

Students can solve #22 by systematic trial. If students have difficulty, you might prompt them to consider the signs of the two integers. (Their positive product shows that they must have the same sign. Their negative mean then establishes that both integers must be negative.)

For #23, you might wish to discuss how a different test could be graded to give an expected score of 0 from random guesses. For example, one possibility would be a test with 40 questions, with 4 possible choices for each question. There could be 3 marks for each correct answer, −1 mark for each incorrect answer, and 0 for each unanswered question.

In #24, challenge students to make up more questions of this type. Students could use a different integer (e.g., four −3s) and/or a different number of that integer (e.g., five −3s) to make up similar problems and have a classmate solve them.



20. Write and evaluate an expression that represents each statement.
a) Subtract the product of 3 and −8 from 20.
b) Add the product of 4 and 5 to the product of −2 and −3.
c) Divide −62 by the sum of −11 and 9.
d) Multiply the sum of −3 and −5 by 3, then divide by −4 and subtract 13.

Extend

21. Copy each statement. Complete it by including operation symbols.
a) 2 ■ 3 ■ 4 ■ 5 = −14
b) 3 ■ [14 ■ (−2)] ■ 30 = 6

22. The mean of two integers is −17. The product of the two integers is 273. What are the two integers?

23. A multiple-choice test with 50 questions has five possible choices for each question. There are 4 marks for each correct answer, −1 mark for each incorrect answer, and 0 marks for each unanswered question.
a) What is the total score of a student with 35 correct answers, 10 incorrect answers, and 5 unanswered questions?
b) Express the student's score as a percent.

24. Here is one way of using four −2s and the order of operations to write an expression that equals 1.
(−2) ÷ (−2) + (−2) − (−2)
Use four −2s and the order of operations to write expressions that equal 2, 3, 4, 5, 6, and 8.

Sports Link

Scoring System in the Modern Pentathlon

Monica Pinette from Langley, British Columbia, is a successful Canadian athlete. She has won gold medals at both the Canadian and the Pan American championships in the modern pentathlon. This sport includes five events: shooting, fencing, swimming, show jumping, and running. The events are all held on the same day. The winner is decided using a points system that involves integer operations.

The running event is a 3000-m cross-country race. A male athlete scores 1000 points for finishing this event in 10-min. A female athlete scores 1000 points for finishing in 11 min 20 s. Each whole second below these times is worth 4 extra points. Each whole second above these times results in a 4-point deduction.

1. Show how the following points are calculated.
a) 920 points for a male with a time of 10 min 20 s
b) 1060 points for a female with a time of 11 min 5 s

2. Calculate the points earned by each of the following athletes.
a) a female with a time of 11 min 43 s
b) a male with a time of 9 min 51 s

3. Calculate the time taken by each of the following athletes.
a) a female who scores 1100 points
b) a male who scores 892 points

Did You Know?

Another sport that includes several events is the decathlon. Dave Steen from New Westminster, British Columbia, won a bronze medal in the decathlon at the summer Olympics in Seoul, South Korea. He competed in ten events over two days.

WWW Web Link

To learn more about the modern pentathlon, including the scoring of the other four events, go to www.mathlinks8.ca and follow the links.

8.5 Applying Integer Operations • MHR 317

The Sports Link gives students the opportunity to apply integer operations to the scoring system in the modern pentathlon. In each question in the Sports Link, students may calculate answers in stages or write a single expression and evaluate it using the order of operations. For example in #1a), students may evaluate $20 \times (-4)$ and then evaluate $1000 + (-80)$, or they may write and evaluate the expression $1000 + 20 \times (-4)$.

Draw students' attention to the Web Link, which provides an opportunity to learn more about the modern pentathlon, and to the Did You Know?, which identifies another successful Canadian athlete.

Meeting Student Needs

- The temperature questions in #13 are useful for practising the multiplication of integers, but they do not always reflect reality. It is true that the mean temperature change could be 2 °C/h, for example, but temperatures would rarely change at such an even rate. Discuss with students the assumption that underlies these kinds of questions.
- Have students make a number line from (–10) to (+10) and use three dice of one colour to represent positive integers and three dice of another colour to represent negative integers. Students can cover up the results on their number line. The first student to cover up all of their number line wins. Ask students to keep track of which numbers were easy to cover up and which ones were more difficult.
- Provide **BLM 8–11 Section 8.5 Extra Practice** to students who would benefit from more practice.

Gifted and Enrichment

- Have students prepare a report of sports that use integers in their reporting of player performance. Explain the significance of the statistics used (e.g., golf scores, hockey plus/minus scores).
- Have students play integer golf. Students roll a six-sided die for each hole and represent their score on the scorecards as an integer. Have students calculate their score above and below par for 9 and 18 holes.

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- For #12, ensure that students understand that a negative score is good as it means that the golfer was under par.
- For #14, ensure that students understand that *withdrew* implies a negative integer.
- For #18, clarify the meaning of a store losing money. Some students may apply the definition for *lost* as cannot find. The store can find the money; it just did not make a profit. Discuss the term *broke even*.

Common Errors

- Some students may have difficulty in deciding when to multiply and when to divide in solving problems.

R$_x$ Encourage students to try many problems and to look for the most efficient way to solve them. Also, encourage students to check that the answer to a problem makes sense in relation to the given data.

- Some students may ignore the order of operations and solve all expressions from left to right.

R$_x$ Remind students that the order of operations is the same for integers as for whole numbers and decimals. Provide examples in which failing to follow the order of operations results in incorrect answers. Assign additional practice to assist familiarity.

- Some students may forget the correct order of operations.

R$_x$ Encourage the use of a mnemonic, such as BEDMAS, as a memory aid.

Answers

Sports Link

1. a) $1000 - 20 \times 4$ **b)** $1000 + 15 \times 4$

2. a) 908 **b)** 1036

3. a) 10 min, 55 s **b)** 10 min, 27 s

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #4, #6, and #8. Students who have no problems with these questions can go on to the remaining Apply questions.	• Coach students who need assistance with #4 or #6 through the application of the order of operations. All of these question parts have positive signs and brackets included with positive integers. Have students verbalize their understanding and solution to #5a) before assigning parts b) and c), and #7a) before assigning parts b) and c). • Some students may need assistance with #8, either in terms of writing out mathematical equivalents and/or underlining key words in the question. Help students recall the meaning of the term *mean*. • Some students may need a calculator when working with large numbers in parts of #9. Have students verbalize their process for #9a) before completing it.
Assessment *as* Learning	
Math Learning Log Have students complete the following statements: • When I complete a problem that requires the order of operations, the order in which I approach the calculations is … • What I find confusing about integer problems is …	• Depending on students' learning styles, have them provide oral or written answers. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulty with.