Analysing Graphs of Linear Relations



Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 3, 4, 6, 9, Math Link
Typical	1, 3, 4, 6, 8–13, Math Link
Extension/Enrichment	1, 3, 13–18

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce material learned in previous sections.

As students examine and discuss the hieroglyphics photo, remind them that graphs and hieroglyphics are ways to tell a story. Ask students questions such as

- How do graphs tell a story?
- How are graphs and hieroglyphics similar? different?

Literacy Link Read the Literacy Link on page 332 with students. Invite students to share what they remember about relationships from their work in grade 7.

Explore the Math

In this exploration, students examine the cost of pizzas according to the number of toppings. In #4a), students are asked to identify three ways of representing data. Some students may be able to identify four methods:

- in words
- with a graph
- using a concrete model
- in a table of values

Note that the cardboard circles should have an approximate radius of 5 cm.

Method 1 Divide the class into groups of three and have them work through the exploration and questions together. Once they have completed the steps, have each group compare their results with those of another group.



Method 2 Divide the class into four groups and have each group represent the data about Tony's Pizzeria using one of the four methods (see bulleted list on previous page). Have each group share their results, explaining what they did and how their method shows the data. As a class, discuss which methods are more effective and less effective for representing this particular data and why.

Literacy Link Direct students' attention to the Literacy Link on page 333. Also, discuss the thought bubble on that page, which gives more information about tables of values.

Example 1

Example 1 leads students to look at the information on a graph in several ways and identify different patterns on the graph.

The activity includes making a table of values from the graph. Students must understand that tables of values can be arranged vertically or horizontally. They should also realize that the data from each axis of the graph must be placed in specific positions in a table of values. Make sure that all students understand this convention. As practice, you might want to give a graph and ask students to create a horizontal or vertical table of values for that graph.

Part c) requires students to continue the pattern shown on the graph. Students could find the answer by extending the graph or by determining an expression for cost in terms of the number of baseballs.



Example 2

This example is similar to Example 1 and reinforces the skills introduced there. Part d) is included to get students thinking about whether it is possible, from a common sense perspective, to have points between those shown on the graph. This is not intended as a formal study. Students are not expected to understand discrete and continuous variables but rather simply to identify an argument for or against additional points. This may help them to understand the information that is available from the graph.

Literacy Link With students, read the Literacy Link on page 335. Students studied linear equations in grade 7, so they may recall that linear relations result in a graph that lies along a straight line. Have them share examples of linear equations that they remember from prior learning.

Meeting Student Needs

- Have students work through Example 1 in pairs. After they feel confident, students should individually try the Show You Know, checking their responses against those of their partner.
- Teach this section over two periods, focusing on one example in each period. Give an additional set of questions with each example.
- If students have difficulty remembering what the term *linear relation* means, point out to them that *linear* contains the word *line*.
- Have students work with data that relates to something that they are familiar with. For example, show students a picture of a beaded article, such as a traditional Inuit parka, and have them make a table of values relating the number of red beads to the number of blue beads.

ELL

- Explain the following terms in context: *pictorial writings, tombs, relationships, sets of data, toppings, headings, triangles, part-time,* and *hourly rate of pay.*
- Have students list a variety of different pizza toppings so that English language learners understand what the graph is about.
- For the Show You Know activities, allow English language learners to write their ideas in their first language, and then discuss them in English orally using visuals. You may also wish to give them a word bank that includes words such as *increasing*, *horizontally*, and *vertically*.

Common Errors

- Some students may reverse the order of the *x*-coordinates and *y*-coordinates when making their table of values.
- R_x Remind students that the first coordinate in an ordered pair is the *x*-coordinate or horizontal coordinate. It should always be placed in the first column or row of the table. The second coordinate is the *y*-coordinate or vertical coordinate.
- Some students may incorrectly describe patterns on a graph.
- **R**_x Remind students that when you describe a pattern, you tell what it relates to, where it starts, and how it changes.

Answers

Explore the Math

- 1. a) Answers may vary.
 - **b)** Answers may vary. Example: The pizza costs \$9 and each of the three toppings that are represented cost \$1, which makes a total of \$12. The cost shown by the model is \$12.
- **2.** The cost of a medium pizza with three toppings is \$12. Answers may vary. Example: The cost is the same as the answer in #1b).

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3. a)	Number of Toppings	Cost (\$)
	0	9
	1	10
	2	11
	3	12
	4	13
	5	14
	6	15

- **b)** Answers may vary. Example: I used the headings Number of Toppings and Cost because they were used on the graph.
- **4.** a) Answers may vary. Check that students identify three of four possible ways to represent data: on a graph, with a concrete model, in a table of values, with words.
 - **b)** Answers may vary. Example: I prefer a graph because you can see the patterns on the graph.

Show You Know: Example 1

- a) Answers may vary. Example: The graph provides data on the number of triangles in a pattern. Figure 1 has three triangles, Figure 2 has six triangles, Figure 3 has nine triangles, The points appear to lie in a straight line. The graph shows that to move from one point to the next, you go one unit horizontally and three units vertically.
- b) Answers may vary. Example:



c)	Figure Number	Number of Triangles
	1	3
	2	6
	3	9
	4	12

d) 297

Show You Know: Example 2

- a) Answers may vary. Example: The graph provides data on the cost of notebooks. The cost of one notebook is \$2, the cost of two notebooks is \$4, the cost of three notebooks is \$6, ... The points appear to lie in a straight line. The graph shows a linear relation. To move from one point to the next on the graph, you go one unit horizontally and two units vertically.
- **b)** The cost per notebook is \$2. Answers may vary. Example: The cost for any number of notebooks is \$2 times the number of notebooks

:)	Number of Notebooks	Cost (\$)
	1	2
	2	4
	3	6
	4	8

d) Answers may vary. Example: No, it is not possible to have points between the ones on the graph. You can only buy a whole number of notebooks.

Assessment	Supporting Learning
Assessment <i>as</i> Learning	
Reflect on Your Findings Listen as students discuss what they discovered during Explore the Math. Try to have students explain what is most important in the activity and generalize the conclusions about their findings.	 Show students that the steps they completed in the Explore the Math will help them to answer #4a). How is the information presented in the question? (In words) What way is used to represent the data in question #1a)? (Cardboard model) In #2? (Graph) In #3a)? (Table of values) Students should feel comfortable with all three formats: graphs, tables, and models. Some students may benefit from verbalizing what is different about each format. In #4b), encourage students to take a position and support it in some way. Remind students that their position can change depending on the situation. In their groups, have students share and discuss the responses to #4b) with the class. It may be beneficial to discuss the advantages and disadvantages of each. This discussion may assist students who are still uncertain about one of the formats.
Assessment for Learning	
Example 1 Have students do the Show You Know related to Example 1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Encourage alternative wordings for describing the patterns formed on a graph. Ensure that all students use the correct labels for rows and columns in their tables of values and that the labelling is based on the axes of the graph. Refer students to Example 1, which provides a similar question pattern. Some learners may find it beneficial to verbalize the pattern in the graph. Ask them questions such as "How do you get from one point to the next?" Having some students draw or model the triangle pattern may help them to describe the pattern and lead them to determine the answer to part d). Students may find it helpful to use the class responses as springboards to prepare their own answers.
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Allow students to use manipulatives, such as square tiles or coloured counters, to model the situation. Suggest that students use a different layout for their table of values from what they used in Example 1. If students have difficulty with part b), remind them that <i>cost per notebook</i> is found by determining the value of the second or vertical coordinate when the first or horizontal coordinate is 1. It may be beneficial to discuss the response to part d) with the entire class. Have students generate further examples of when it would and would not be appropriate to use points between those on the graph. Some learners may benefit from making a list of the suggestions in their Foldable.





Key Ideas

The Key Ideas provide a detailed summary of what is most important in section 9.1. Have students prepare their own summary of the Key Ideas and record them in their chapter Foldable. Encourage them to use different examples from the ones in the student resource.

Communicate the Ideas

Have students work individually or in small groups to answer the questions. In #1, students indicate whether they think it is reasonable to have points between those provided on two different graphs. This question requires an interpretation of what each graph is about.

For #2, students make a graph of a linear relation and then must come up with a description that matches their graph.

In #3, students are asked to show one way that a graph and a table of values are similar and one way that they are different.

Meeting Student Needs

• Students should be discouraged from copying the Key Ideas and should limit their summary to what is important to them and/or what they are less familiar or comfortable with.

ELL

• For #1 of Communicate the Ideas, make sure that English language learners understand that *carton* is a word for the container/box that ice cream comes in.

Answers

Communicate the Ideas

- 1. Answers may vary. Example: It is reasonable to have points between the ones on Graph A since it is possible to travel distances that include a part of a kilometre. It is not reasonable to have points between the ones on Graph B since it is not possible to buy part of a carton of ice cream.
- **2.** Answers may vary. Example: The graph shows the cost of tickets to the school dance.



3. Answers may vary. Example: A graph and a table of values are different because the graph shows a picture of the data, which can help to determine a pattern, whereas the table of values does not. A graph and a table of values are similar because the values in the table are represented by the points on the graph. The graph from #2 shows the same data as the following table of values:

Number of Tickets	Cost (\$)
1	5
2	10
3	15
4	20
5	25
6	30

Assessment	Supporting Learning
Assessment <i>as</i> Learning	
Communicate the Ideas Have all students complete #1 and #3. Check student responses for conceptual understanding.	 Consider having students work in pairs or groups. Check each student's answer to #1. If students find this question challenging, encourage them to look at Graph B first, remind them that values between those given would be fractional, and ask them to think about whether you can buy part of a carton of ice cream. Have students share their answers to #3 in a class discussion.



Check Your Understanding

Practise

Students may be given a choice between #4 or #5 and #6 or #7 since these pairs of questions are similar. Ask students which questions they selected and why.

Students may work individually or in pairs on the questions in the Practise.

Apply

The Apply questions provide a range of contexts for the graphs of linear relations. Select questions based on whether the context is of interest or familiar to students. Also, let students have some choice in the questions they will complete. For example, assign #9, #10, and #14, and then have students select two additional questions from those remaining. **Literacy Link** The Literacy Link on page 339 will help students recall how to determine the perimeter of a square, which will assist them in understanding Apply #12. You might have students draw on grid paper a few of the other squares in the table that accompanies #12. This way, they can model that a square with a side length of 2 cm has a perimeter of 8 cm, a square with a side length of 3 cm has a perimeter of 12 cm, etc.

Extend

Both Extend questions provide a graph showing data for two individuals. In #17, students are asked to determine where the lines of points will meet. In #18, students are asked questions about when each individual's money reaches a specified level. This requires students to extend the graphs or use other strategies to determine the answers.



Math Link

This Math Link involves a polar bear adventure tour. It parallels the mathematical content covered in section 9.1 and may assist students to come up with their own ideas for the Wrap It Up! at the end of the chapter.



Encourage students' interest in the Math Link by having them visit sites that offer polar bear adventure tours. Go to www.mathlinks8.ca and follow the links.

Meeting Student Needs

- Providing choices in what questions students will complete helps them take responsibility for their learning and may improve their attitude toward mathematics.
- Encourage all students to use materials to model the situation depicted on a graph. This helps develop a connection between a real-life situation and the graph or table of values.

- It may be better for your class to have students work in pairs to complete the Practise questions for Example 1 immediately after working on that example. Follow the same procedure for Example 2. Then, have students complete the Apply activities independently after the Example 1 and 2 work is completed.
- For Practise #5b), some students may need assistance in understanding that the values for *t* are found on the horizontal axis.
- For Apply #8d), some students may need assistance in recalling their work with ordered pairs in grade 7.
- Provide **BLM 9–5 Section 9.1 Extra Practice** to students who would benefit from more practice.

ELL

• For Apply #10, you may need to help some students to understand what sugar flowers are and what they are used for. Point them out in the photo in the student resource.



Common Errors

- Some students may describe a pattern incorrectly.
- $\mathbf{R}_{\mathbf{x}}$ Emphasize a three-step approach:
 - What is the pattern about?
 - Where does it begin?
 - How does it change from one item to the next?
- Some students may extend patterns incorrectly.
- $\mathbf{R}_{\mathbf{x}}$ Reinforce that each element in a pattern is generated from the previous one but that it may be necessary to look at three or more consecutive items in the pattern.

Answers

Math Link

a) Answers may vary. Example: The cost of the polar bear adventure tour increases \$400 for every extra day of the adventure. The points appear to lie in a straight line. To move from one point to the next on the graph, you go 1 unit horizontally and 400 units vertically.

b),	c)
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Number of Days	Cost (\$)
0	0
1	400
2	800
3	1200
4	1600
5	2000
6	2400
7	2800
8	3200

d)	Number of Days	Cost (\$)
	0	300
	1	700
	2	1100
	3	1500
	4	1900
	5	2300
	6	2700
	7	3100
	8	3500

e) Answers may vary. Example: The data in the two tables of values are similar because the rate of increase for both tables of values is \$400 per day. The data in the two tables of values are different because the starting value for the first table is (0, 0) and the starting value for the second table is (0, 300).

Assessment	Supporting Learning
Assessment for Learning	
Practise and Apply Have students do #4, #6, and #9. Students who have no problems with these questions can go on to the remaining Apply questions.	 Students who find #4 and #6 challenging will need additional coaching with Examples 1 and 2. Coach students through #4 and #6 and then have them complete #5 and #7. Check back with students to make sure that they are more confident in their understanding. Encourage students to use concrete materials, such as square tiles or coloured counters, to model what is shown on the graphs. This will help students to understand what the graph is telling them. Some students may benefit from orally describing the graphs. Copying the graphs and drawing in the vertical and horizontal moves may also assist some learners to identify the pattern. As they draw the moves, have them verbalize their thinking. Students who find #9 challenging should employ similar strategies as listed above. Have students verbalize when it is appropriate to use values between points before they attempt to answer part c).
Math Link The Math Link on page 341 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 363.	 It is not necessary for students to do the Math Link. However, it may be beneficial for some students to complete it as it relates to the Wrap It Up! at the end of the chapter. Reinforce that the Math Link is mathematically identical to both Examples 1 and 2 but simply involves a different situation. Students who need help getting started could use BLM 9–6 Section 9.1 Math Link, which provides scaffolding for this activity.
Assessment as Learning	
 Math Learning Log Have students respond to the following prompts: How are graphs and tables of values similar and different? I can get the following information from a graph 	 Encourage students to select an example of a table of values and accompanying graph from section 9.1 and use it to help identify similarities and differences. Encourage concrete and kinesthetic learners to use manipulatives to model graphs. Depending on students' learning styles, have them provide oral or written answers. Encourage students to look back over their work for ideas to help them answer the questions. Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.