10 Solving Linear Equations

General Outcomes

- Represent algebraic expressions in multiple ways.

Specific Outcomes

PR2 Model and solve problems using linear equations of the form:

- $ax = b$
- $\frac{x}{a} = b, a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c, a \neq 0$
- $a(x + b) = c$

concretely, pictorially and symbolically, where a, b and c are integers.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
10.1	✓ model a problem with a one-step linear equation of the form $ax = b$, $\frac{x}{a} = b$
	✓ solve a one-step linear equation and record the process
	✓ verify the solution to a one-step linear equation
	✓ correct an error in a solution to a one-step linear equation
10.2	✓ model a problem with a two-step linear equation of the form $ax + b = c$
	✓ solve a two-step linear equation and record the process
	✓ verify the solution to a two-step linear equation
	✓ draw a diagram to solve a linear equation
10.3	✓ model a problem with a two-step linear equation of the form $\frac{x}{a} + b = c$
	✓ solve a two-step linear equation and record the process
	✓ verify the solution to a two-step linear equation
	✓ correct an error in a solution to a two-step linear equation
10.4	✓ model a problem with a two-step linear equation of the form $a(x + b)$
	✓ solve a two-step linear equation and record the process
	✓ verify the solution to a two-step linear equation
	✓ apply the distributive property to solve a linear equation
	✓ correct an error in a solution to a two-step linear equation

Assessment	Supporting Learning
Assessment *for* Learning	
Method 1: Use the Math Link introduction on page 369 in *MathLinks 8* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about integers, linear equations, and solving linear equations, including using order of operations to evaluate an expression and using reverse order of operations to solve for the unknown in a linear equation.	• **BLM 10–1 Chapter 10 Math Link Introduction** provides scaffolding for the Math Link introduction. • Have students use the What I Need to Work On section of their chapter Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete the Get Ready materials available on **BLM 10–2 Chapter 10 Get Ready**, in the *MathLinks 8 Practice and Homework Book*, and at the www.mathlinks8.ca book site.
Assessment *as* Learning	
Literacy Link (page 367) Before starting the chapter or after completing the Math Link introduction on page 369, have students use a KWL chart to identify what they know and want to learn about solving linear equations. As they complete each section, have them revisit their KWL chart and list what they have learned in the What I **Learned** column.	• Use student responses in the What I **Know** column to identify any misconceptions they may have about the topic. Deal with these when you come to an appropriate lesson during the chapter. • Before filling out the What I **Want** to Know column, have students scan the chapter by reading each section title, studying the picture, reading the opening text, and identifying the type of linear equation. Have them write down what they want to learn, sparked by this brief scan. • As students complete each section, you may wish to have them answer any relevant questions from their What I **Want** to Know column. • Before the practice test, have students fill out the What I **Learned** column.
Chapter 10 Foldable As students work on each section in Chapter 10, have them keep track of any problems they are having under the What I Need to Work On heading.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled.
Assessment *for* Learning	
BLM 10–3 Chapter 10 Warm-Up This BLM includes four warm-ups, one to be used at the beginning of each section. Each warm-up provides cumulative review questions for the entire student resource to that point, as well as mental math practice.	• As students complete questions from previous chapters, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing mental math calculations.

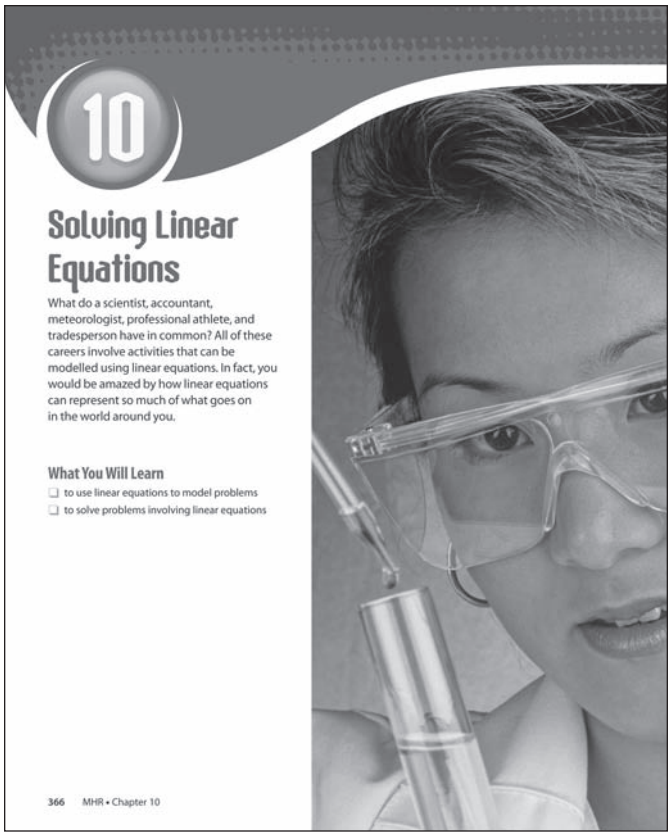
Problems of the Week

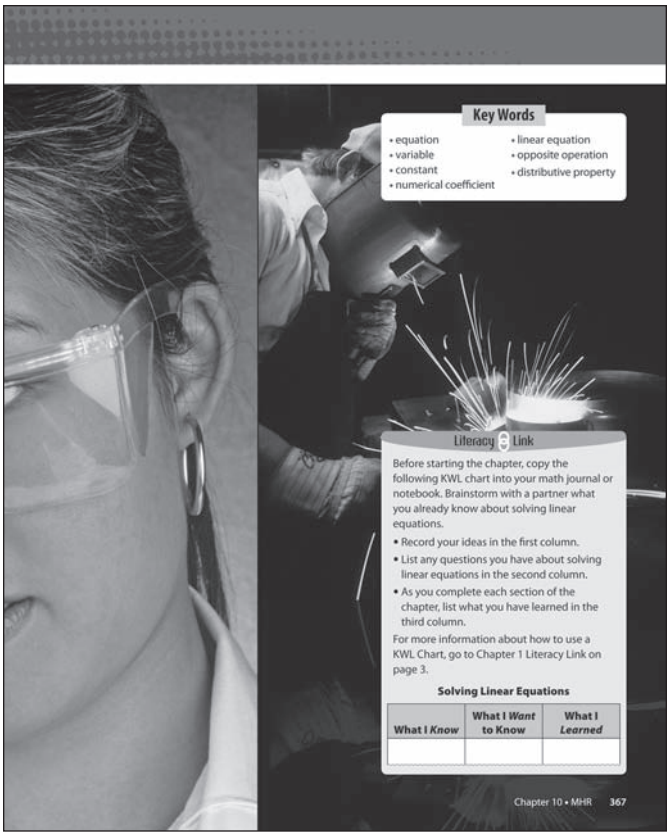
Have all students try at least one of the problems on **BLM 10–4 Chapter 10 Problems of the Week**. Many of these problems require students to think outside the box and experiment with a variety of approaches. Some have definitive answers; others can be answered in more than one way.

Students can take the problems home and consult with parents or guardians, work with other students when their work is completed, or try them on their own. The questions take a varying amount of time to solve, depending on the particular student and the problem itself. You may wish to give out these problems at the beginning of the chapter and discuss the solutions at appropriate times throughout your work on the chapter.

Chapter 10 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Extra Support	Assessment		
						Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 40–50 minutes (TR page 499)	Students should be familiar with • using the order of operations to evaluate algebraic expressions • using reverse order of operations to solve linear equations	• 11 × 17 sheet of paper • stapler • scissors • ruler	BLM 10–1 Chapter 10 Math Link Introduction BLM 10–2 Chapter 10 Get Ready BLM 10–4 Chapter 10 Problems of the Week		Online Learning Centre	TR page 498 Chapter 10 Foldable, TR page 498	TR page 498	
10.1 Modelling and Solving One-Step Equations: $ax = b$, $\frac{x}{a} = b$ • 80–100 minutes (TR page 502)	Students should be familiar with • order of operations and reverse order of operations • substituting into and solving linear equations using whole numbers	• spring scale and standard masses • grid paper • algebra tiles • integer chips • Silly Putty® • metre stick	Master 2 Two Stars and One Wish Master 8 Centimetre Grid Paper Master 15 Algebra Tiles Master 20 Integer Chips BLM 10–3 Chapter 10 Warm-Up BLM 10–5 Section 10.1 Extra Practice BLM 10–6 Section 10.1 Math Link	**Essential:** 1, 3, 4, 5a), b), 7b), d), 8a), c), 11a), d), 13, 15, 19, Math Link **Typical:** 1, 3, 4, 5a), b), 7b), d), 8a), c), 11a), d), 13, 15, 19–23, Math Link **Extension/Enrichment:** 1, 3, 4, 14, 18, 24, 25, 27, 28	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 506, 508 Math Learning Log, TR page 511 Chapter 10 Foldable, TR page 511 Master 2 Two Stars and One Wish	TR pages 506, 511	
10.2 Modelling and Solving Two-Step Equations: $ax + b = c$ • 80–100 minutes (TR page 512)	Students should be familiar with • order of operations and reverse order of operations • substituting into and solving linear equations using whole numbers	• blocks • coloured pencils • algebra tiles • integer chips	Master 15 Algebra Tiles Master 20 Integer Chips BLM 10–3 Chapter 10 Warm-Up BLM 10–7 Section 10.2 Extra Practice BLM 10–8 Section 10.2 Math Link	**Essential:** 1–3, 5, 7–9, 12, Math Link **Typical:** 1–3, 5, 7–9, 11–17, Math Link **Extension/Enrichment:** 1, 2, 11, 15, 17–21	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR page 517 Math Learning Log, TR page 519 Chapter 10 Foldable, TR page 519	TR pages 515, 519	
10.3 Modelling and Solving Two-Step Equations: $\frac{x}{a} + b = c$ • 80–100 minutes (TR page 520)	Students should be familiar with • order of operations and reverse order of operations	• algebra tiles • integer chips	BLM 10–3 Chapter 10 Warm-Up BLM 10–9 Section 10.3 Extra Practice BLM 10–10 Section 10.3 Math Link	**Essential:** 2–4, 6, 8–10, Math Link **Typical:** 2–4, 6, 8–9, 12–15, Math Link **Extension/Enrichment:** 2, 3, 12, 15–17	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 523, 525 Math Learning Log, TR page 526 Chapter 10 Foldable, TR page 526	TR pages 523, 526	
10.4 Modelling and Solving Two-Step Equations: $a(x + b) = c$ • 80–100 minutes (TR page 527)	Students should be familiar with • order of operations and reverse order of operations	• algebra tiles • integer chips	Master 15 Algebra Tiles Master 20 Integer Chips BLM 10–3 Chapter 10 Warm-Up BLM 10–11 Section 10.4 Extra Practice BLM 10–12 Section 10.4 Math Link	**Essential:** 1, 2, 4, 6, 8, 10a), b), 11, Math Link **Typical:** 1, 2, 4, 6, 8, 10–13, Math Link **Extension/Enrichment:** 1, 2, 12–16	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 530, 532 Math Learning Log, TR page 533 Chapter 10 Foldable, TR page 533	TR pages 530, 533	
Chapter 10 Review • 40–50 minutes (TR page 534)		• algebra tiles • cups and counters	Master 15 Algebra Tiles BLM 10–5 Section 10.1 Extra Practice BLM 10–7 Section 10.2 Extra Practice BLM 10–9 Section 10.3 Extra Practice BLM 10–11 Section 10.4 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	*MathLinks 8 Practice and Homework Book* *MathLinks 8 CAB*	Chapter 10 Foldable, TR page 535	TR page 535	
Chapter 10 Practice Test • 40–50 minutes (TR page 536)		• algebra tiles • cups and counters	Master 15 Algebra Tiles BLM 10–13 Chapter 10 Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1–5, 8, 10, 12, 14	*MathLinks 8 CAB*	TR page 538		TR page 538 BLM 10–13 Chapter 10 Test
Chapter 10 Wrap It Up! • 80–100 minutes (TR page 539)			Master 1 Project Rubric BLM 10–1 Chapter 10 Math Link Introduction BLM 10–6 Section 10.1 Math Link BLM 10–8 Section 10.2 Math Link BLM 10–10 Section 10.3 Math Link BLM 10–12 Section 10.4 Math Link BLM 10–14 Chapter 10 Wrap It Up!		Online Learning Centre			TR page 539 Master 1 Project Rubric
Chapter 10 Math Games • 30–40 minutes (TR page 541)		• algebra tiles • cups and counters	Master 15 Algebra Tiles				TR page 541	
Chapter 10 Challenge in Real Life • 80–100 minutes (TR page 542)		• grid paper	Master 1 Project Rubric Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper BLM 10–15 Chapter 10 BLM Answers		Online Learning Centre		TR page 543	TR page 543 Master 1 Project Rubric





***MathLinks 8*, pages 366–369**

Suggested Timing
40–50 minutes

Materials
- 11 × 17 sheet of paper
- stapler
- scissors
- ruler

Blackline Masters
BLM 10–1 Chapter10 Math Link Introduction
BLM 10–2 Chapter 10 Get Ready
BLM 10–4 Chapter 10 Problems of the Week

Key Words

equation	variable	constant
numerical coefficient	linear equation	opposite operation
distributive property		

What's the Math?

In this chapter, students use manipulatives, diagrams, and algebra to model and solve different forms of linear equations in a variety of real-world contexts. This chapter builds on students' exploration of solving linear equations in the previous grade and extends the numbers used to include integers.

Planning Notes

Begin the chapter by having a class discussion about the different situations students know about that can be modelled using linear equations. Ask pairs of students to describe scenarios they believe can be represented by growing patterns and linear equations. Show the class different types of linear equations they already know how to solve. Ask them to identify the steps they would use to solve these types of equations.

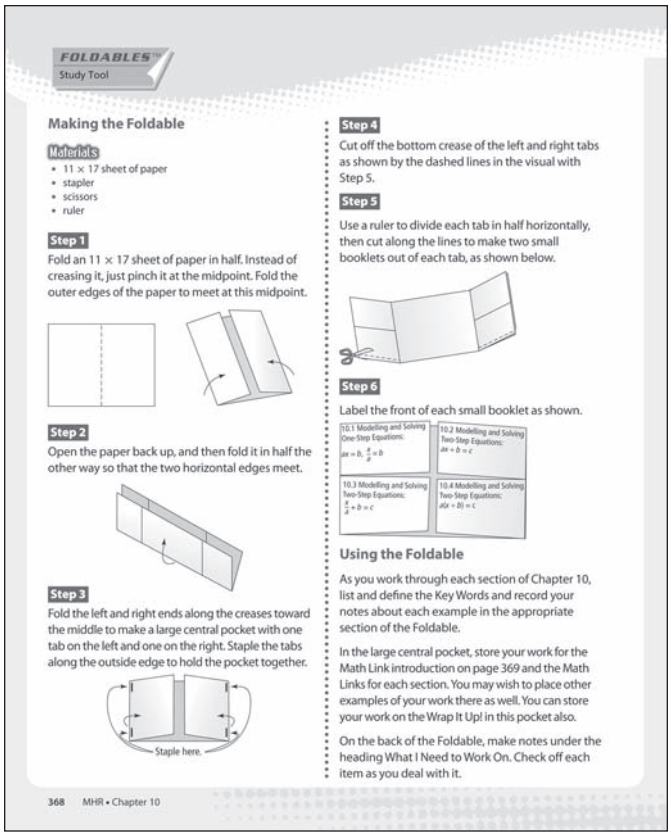
Literacy Link Have students fill out the first two columns of a KWL chart, either before starting work on the chapter or after they have completed the Math Link introduction on page 369. At the end of each section, have students list in the third column what they have learned.

Meeting Student Needs

- Discuss how aspects of a community or culture can be represented using linear equations. For example, for each person going on a hunting trip, k number of kilograms of food should be allocated per day. An equation that determines the total number of kilograms of food required for five days is $t = 5k$, where t is the total number of kilograms.
- Students who have difficulty understanding which operation is required to solve an equation could be encouraged to use manipulatives and diagrams to help them visualize what the equation is made up of. Demonstrate, with transparent pieces on an overhead, how algebra tiles can be used to help model and solve an equation.
- You may wish to pre-teach the Key Words. Some suggested activities might be to construct a memory game, write definitions, and have partners write quizzes for each other.
- Introduce each Key Word using a number of visuals and examples.
- Some students may benefit from having assistance in reactivating their skills and knowledge in the following areas:
  – sign rules for working with integers
  – order of operations
  – solving equations at the grade 7 level
- Consider having students complete the questions on **BLM 10–2 Chapter 10 Get Ready** to activate the prerequisite skills for this chapter.

ELL

- Some learners may not understand the names for the professions in the chapter opener and what they might involve. As a class, describe some of the things that people in these professions do.



FOLDABLES™
Study Tool

Making the Foldable

Materials
- 11 × 17 sheet of paper
- stapler
- scissors
- ruler

Step 1
Fold an 11 × 17 sheet of paper in half. Instead of creasing it, just pinch it at the midpoint. Fold the outer edges of the paper to meet at this midpoint.

Step 2
Open the paper back up, and then fold it in half the other way so that the two horizontal edges meet.

Step 3
Fold the left and right ends along the creases toward the middle to make a large central pocket with one tab on the left and one on the right. Staple the tabs along the outside edge to hold the pocket together.

Staple here.

Step 4
Cut off the bottom crease of the left and right tabs as shown by the dashed lines in the visual with Step 5.

Step 5
Use a ruler to divide each tab in half horizontally, then cut along the lines to make two small booklets out of each tab, as shown below.

Step 6
Label the front of each small booklet as shown.

10.1 Modelling and Solving One-Step Equations: $ax = b$, $\frac{x}{a} = b$

10.2 Modelling and Solving Two-Step Equations: $ax + b = c$

10.3 Modelling and Solving Two-Step Equations: $\frac{x}{a} + b = c$

10.4 Modelling and Solving Two-Step Equations: $a(x + b) = c$

Using the Foldable

As you work through each section of Chapter 10, list and define the Key Words and record your notes about each example in the appropriate section of the Foldable.

In the large central pocket, store your work for the Math Link introduction on page 369 and the Math Links for each section. You may wish to place other examples of your work there as well. You can store your work on the Wrap It Up! in this pocket also.

On the back of the Foldable, make notes under the heading What I Need to Work On. Check off each item as you deal with it.

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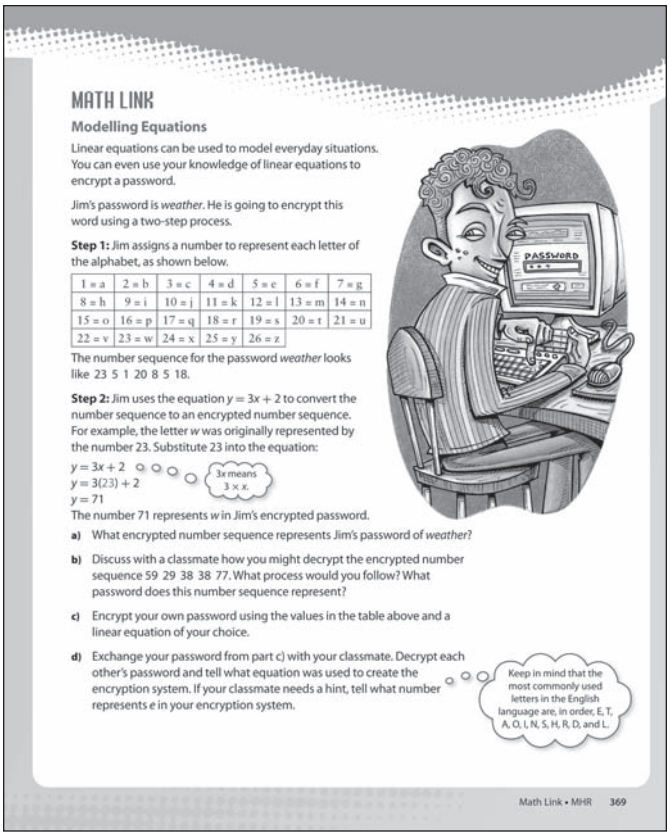

MATH LINK

Modelling Equations

Linear equations can be used to model everyday situations. You can even use your knowledge of linear equations to encrypt a password.

Jim's password is *weather*. He is going to encrypt this word using a two-step process.

Step 1: Jim assigns a number to represent each letter of the alphabet, as shown below.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g
8 = h	9 = i	10 = j	11 = k	12 = l	13 = m	14 = n
15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u
22 = v	23 = w	24 = x	25 = y	26 = z		

The number sequence for the password *weather* looks like 23 5 1 20 8 5 18.

Step 2: Jim uses the equation $y = 3x + 2$ to convert the number sequence to an encrypted number sequence. For example, the letter *w* was originally represented by the number 23. Substitute 23 into the equation:

$y = 3x + 2$
$y = 3(23) + 2$
$y = 71$

3x means 3 × x.

The number 71 represents *w* in Jim's encrypted password.

a) What encrypted number sequence represents Jim's password of *weather*?

b) Discuss with a classmate how you might decrypt the encrypted number sequence 59 29 38 38 77. What process would you follow? What password does this number sequence represent?

c) Encrypt your own password using the values in the table above and a linear equation of your choice.

d) Exchange your password from part c) with your classmate. Decrypt each other's password and tell what equation was used to create the encryption system. If your classmate needs a hint, tell what number represents *e* in your encryption system.

Keep in mind that the most commonly used letters in the English language are, in order, E, T, A, O, I, N, S, H, R, D, and L.

Math Link • MHR 369

Foldables Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter. They may wish to use the back page of the Foldable to keep track of what they need to work on as they progress through the chapter to assist them in identifying and solving any difficulties with concepts, skills, and processes.

Math Link

For part b) of the Math Link, you may wish to have the class explore strategies for decrypting before students attempt to decrypt on their own. For example:

Step 1: The first number in the encrypted number sequence is 59. It would be decrypted this way:

$$\begin{aligned} 59 &= 3x + 2 \\ 59 - 2 &= 3x + 2 - 2 \\ 57 &= 3x \\ \frac{57}{3} &= \frac{3x}{3} \\ 19 &= x \end{aligned}$$

Step 2: From the table in the student resource, the number 19 represents the letter s. The first letter of the password is s.

Have students read the Wrap It Up! on page 403 to give them a sense of what they will be expected to do at the end of the chapter. The Wrap It Up! problem is a summative assessment.

Meeting Student Needs

- Some students may benefit from using **BLM 10–1 Chapter 10 Math Link Introduction**, which provides scaffolding for this activity.
- Discuss with students the meaning of the words *encrypt* and *decrypt*.
- Brainstorm with students where you might find or use an encrypted message in other cultures.
- Have students work in small groups to create and encrypt a message.

ELL

- Assist students with the following terms: *password*, *weather*, *assigns*, *convert*, and *substitute*.

Answers

Math Link

a) 71 17 5 62 26 17 56

b) Answers may vary. Example: Subtract 2 from each encrypted number and then divide the result by 3. Locate each value in the table to find the letter. The password is *silly*.

c) Answers will vary. Example: Password: guitar. Linear equation: $y = 2x - 3$. Encrypted number sequence: 11 39 16 37 −1 33

d) Answers will vary.