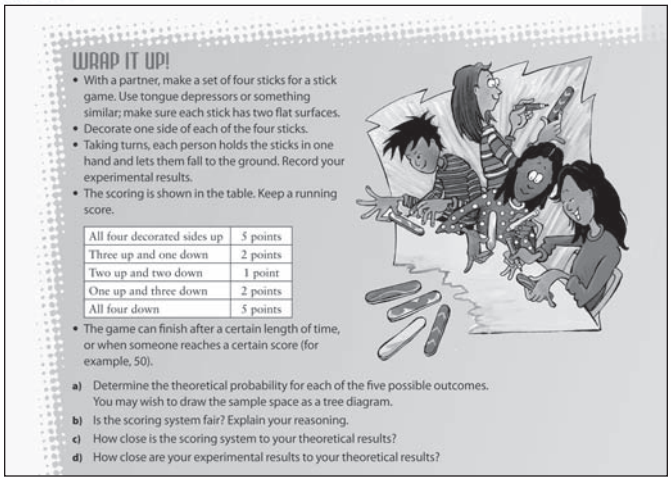
Wrap It Up!



WRAP IT UP!

- With a partner, make a set of four sticks for a stick game. Use tongue depressors or something similar; make sure each stick has two flat surfaces.
- Decorate one side of each of the four sticks.
- Taking turns, each person holds the sticks in one hand and lets them fall to the ground. Record your experimental results.
- The scoring is shown in the table. Keep a running score.

All four decorated sides up	5 points
Three up and one down	2 points
Two up and two down	1 point
One up and three down	2 points
All four down	5 points

- The game can finish after a certain length of time, or when someone reaches a certain score (for example, 50).

a) Determine the theoretical probability for each of the five possible outcomes. You may wish to draw the sample space as a tree diagram.
b) Is the scoring system fair? Explain your reasoning.
c) How close is the scoring system to your theoretical results?
d) How close are your experimental results to your theoretical results?

***MathLinks 8*, page 439**

Suggested Timing
60–90 minutes

Materials
- tongue depressors or craft sticks
- coloured pencils or markers

Blackline Masters
Master 1 Project Rubric
BLM 11–1 Chapter 11 Math Link Introduction
BLM 11–6 Section 11.1 Math Link
BLM 11–8 Section 11.2 Math Link
BLM 11–11 Section 11.3 Math Link
BLM 11–13 Chapter 11 Wrap It Up!

Specific Outcomes
SP2 Solve problems involving the probability of independent events.

Planning Notes

Introduce the problem and clarify the assessment criteria. Explain that students do not need to completely cover the one side of the stick. For part b), accept any opinion that is supported.

Some students may suggest making the scores correlate directly to their likelihood. Consider sharing the following possible solution after students have completed their solutions.

Theoretically, in 16 trials you would expect the following results: all four decorated sides up: 1; three up and one down: 4; two up and two down: 6; three down and one up: 4; all four down: 1.

Since all of these numbers are factors of 12, one could assign the following point system which is fair: all four decorated sides up: $1 \times 12 = 12$; three up and one down: $4 \times 3 = 12$; two up and two down: $6 \times 2 = 12$; three down and one up: $4 \times 3 = 12$; all four down: $1 \times 12 = 12$.

Notice that the numbers in the game are close to the ratios for the perfect scoring ratio. Consider discussing as a class scoring systems that are not exactly as probability would have them (e.g., casino scoring).

Common Errors

- Some students may not construct an appropriate tree diagram for part a).

R_x Coach students to create two branches for each of the four sticks. Have students revisit their answers to the Math Link in section 11.1.

Answers

Wrap It Up!

a) $P(\text{all four decorated sides up}) = \frac{1}{16}$; $P(\text{three up and one down}) = \frac{4}{16}$; $P(\text{two up and two down}) = \frac{6}{16}$; $P(\text{three down and one up}) = \frac{4}{16}$; $P(\text{all four down}) = \frac{1}{16}$

b) Answers will vary. Look for the idea that the scoring system is generally fair because the less probable combinations earn a player more points.

c), d) Answers will vary.

Assessment	Supporting Learning
Assessment *of* Learning	
Wrap It Up! This chapter problem wrap-up gives students an opportunity to apply and display their knowledge of probability. It is important for students to be able to determine theoretical probabilities using a tree diagram. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Wrap It Up! Page 584 in this TR provides notes on how to use this rubric for the Wrap It Up!	• You may wish to have students review the work they have completed in the Math Links in sections 11.1, 11.2, and 11.3 before they begin. • If students have not completed the Math Links earlier, you may wish to provide them with **BLM 11–1 Chapter 11 Math Link Introduction**, **BLM 11–6 Section 11.1 Math Link**, **BLM 11–8 Section 11.2 Math Link**, and **BLM 11–11 Section 11.3 Math Link**. • You may wish to have students use **BLM 11–13 Chapter 11 Wrap It Up!**, which provides scaffolding for the chapter problem wrap-up.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Wrap It Up! and provides notes that specify how to identify the level of specific answers for the project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete response, with weak communication in part c) *or* • provides a complete response, with weak justification in part a) or b) *or* • provides a complete response with a calculation error that does not hinder the understanding of the problem
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a correct part a) *or* • provides correct parts b) and c) based on an incorrect part a) *or* • provides partially complete parts a), b), and c), showing a basic understanding *or* • provides a complete response to the question with answers only, with no justification, and communication is weak or absent
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides correct calculations for at least three theoretical probabilities for part a) but there are errors in the remaining calculations and/or the work is not justified *or* • provides a correct and complete part b) *or* • provides a correct interpretation of part c) based on incorrect calculations
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct initial start to part a) *or* • provides a correct start to part b), with a weak explanation

Math Games

***MathLinks 8*, page 440**

Suggested Timing
30–40 minutes

Materials
- two six-sided dice of different colours per pair of students
- coin per pair of students
- ruler

Specific Outcomes
SP2 Solve problems involving the probability of independent events.

Planning Notes

In these games, students create and use a table of possible outcomes values to determine equal probabilities and recognize unequal probabilities. Have students work in pairs and play each version of the game. Ensure that students realize that there are four versions of the game (i.e., one version in each of steps 1, 3, 4, and 5). In order to evaluate the fairness of each game, encourage students to use a table or a tree diagram to organize the sample space.

Meeting Student Needs

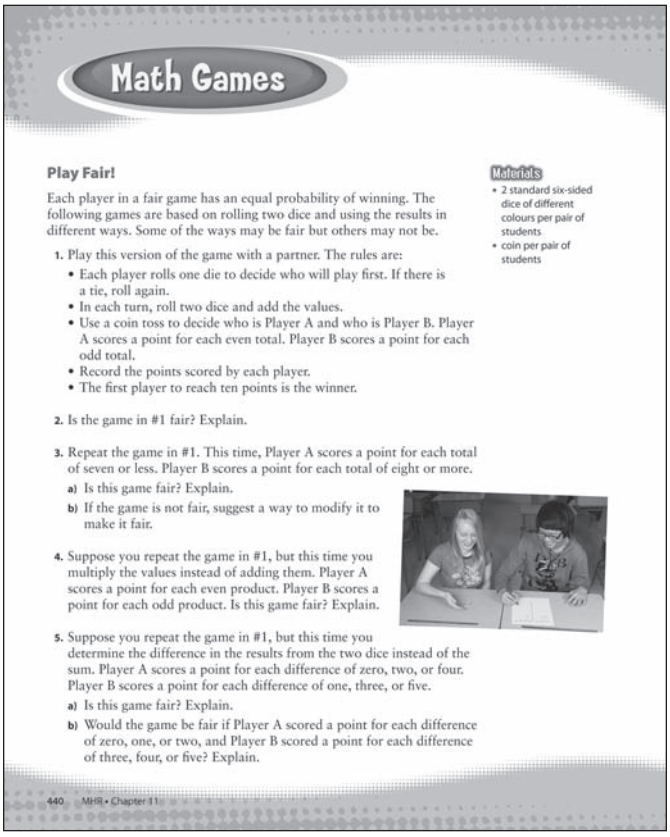
Gifted and Enrichment

- Have students use their findings from #1 to #5 to design a fair game based on rolling two dice. Answers will vary. Examples:
  - Modify previous versions and give a point to Player A for a difference of 0 or 1 and a point to Player B for a difference of 2, 3, or 5 with neither player getting a point for a difference of 4.
  - Work with two dice of different colours, such as red and blue. Give Player A a point for red greater than blue and Player B a point for blue greater than red (with no points for red equal to blue.)

Common Errors

- Some students may have difficulty determining if each game is fair.

R$_x$ Have students create a table of possible outcomes.



Math Games

Play Fair!

Each player in a fair game has an equal probability of winning. The following games are based on rolling two dice and using the results in different ways. Some of the ways may be fair but others may not be.

Materials
- 2 standard six-sided dice of different colours per pair of students
- coin per pair of students

1. Play this version of the game with a partner. The rules are:
   - Each player rolls one die to decide who will play first. If there is a tie, roll again.
   - In each turn, roll two dice and add the values.
   - Use a coin toss to decide who is Player A and who is Player B. Player A scores a point for each even total. Player B scores a point for each odd total.
   - Record the points scored by each player.
   - The first player to reach ten points is the winner.
2. Is the game in #1 fair? Explain.
3. Repeat the game in #1. This time, Player A scores a point for each total of seven or less. Player B scores a point for each total of eight or more.
   a) Is this game fair? Explain.
   b) If the game is not fair, suggest a way to modify it to make it fair.
4. Suppose you repeat the game in #1, but this time you multiply the values instead of adding them. Player A scores a point for each even product. Player B scores a point for each odd product. Is this game fair? Explain.
5. Suppose you repeat the game in #1, but this time you determine the difference in the results from the two dice instead of the sum. Player A scores a point for each difference of zero, two, or four. Player B scores a point for each difference of one, three, or five.
   a) Is this game fair? Explain.
   b) Would the game be fair if Player A scored a point for each difference of zero, one, or two, and Player B scored a point for each difference of three, four, or five? Explain.

Answers

Play Fair!

2. The game in #1 is fair. There are 18 even sums and 18 odd sums. The probability of each player winning a point in each turn is 50%.

3. a) The game is not fair. There are 21 totals of 7 or less, but only 15 totals of 8 or more, so Player A has a greater probability of winning a point than Player B ($\frac{7}{12}$ versus $\frac{5}{12}$).

b) Answers will vary. Example: Award no points for a total of 7, so that Player A gets a point for a total of 6 or less.

4. This game is not fair. There are only 9 odd products out of 36, so Player A has a greater probability of winning a point than Player B ($\frac{3}{4}$ versus $\frac{1}{4}$).

5. a) This game is fair. There are ten differences of 1, six differences of 3, and two differences of 5, giving a total of 18 of the 36 differences. There are six differences of 0, eight differences of 2, and four differences of 4, which also gives 18 of 36. The probability of each player winning is 50%.

b) This game is not fair, because 24 of the 36 possibilities or $\frac{2}{3}$ result in a point for Player A versus $\frac{1}{3}$ for Player B.

Assessment	Supporting Learning
Assessment *for* Learning	
Play Fair! Have students play the game with a partner.	• Encourage students to use a table to organize their outcomes.

Challenge in Real Life

***MathLinks 8*, page 441**

Suggested Timing
40–50 minutes

Materials
- centimetre grid paper
- yellow and brown coloured pencils
- red marker
- ruler

Blackline Masters
Master 1 Project Rubric
Master 8 Centimetre Grid Paper
BLM 11–14 Theoretical Probability

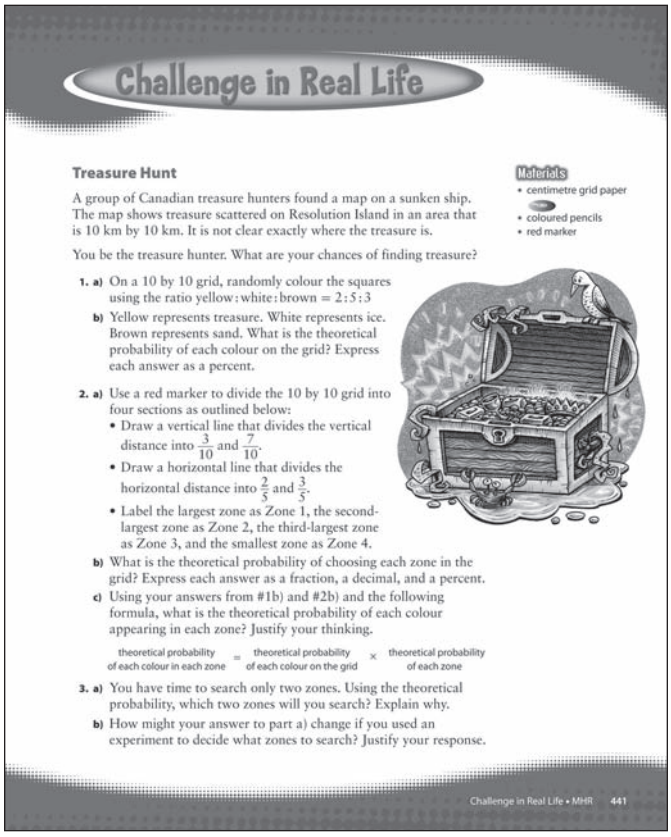
Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
SP2 Solve problems involving the probability of independent events.
N5 Solve problems that involve rates, ratios and proportional reasoning.
N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

Planning Notes

You may wish to use the following steps to introduce and complete this challenge:

1. Read the introduction to Treasure Hunt as a class. Discuss what might have led to the burying of treasure on the island.
2. Have students work in pairs. Distribute **Master 8 Centimetre Grid Paper** and have students mark a 10 by 10 grid according to the instructions in #1a) and #2a). For #2a), clarify that the four sections of the grid are not the quadrants of a coordinate plane, but used only to divide the hundred grid.
3. Help students recall how to calculate the probability of independent events by multiplying the probability of each event. Ask students to calculate the experimental probability of finding treasure in any one section by counting the number of yellow squares and dividing it by the total number of squares in that zone.
4. Clarify that the task is to
   - create a 10 by 10 grid showing yellow, white, or brown shading according to given proportions and calculate the theoretical probability of each colour
   - divide the hundred squares into four zones and calculate the theoretical probability of choosing each zone in the grid
   - calculate the theoretical probability of each colour appearing in each zone
   - decide which two zones to search for treasure and justify the decision
   - compare theoretical probability to experimental probability
5. Review the **Master 1 Project Rubric** with students so that they will know what is expected.



Challenge in Real Life

Treasure Hunt

A group of Canadian treasure hunters found a map on a sunken ship. The map shows treasure scattered on Resolution Island in an area that is 10 km by 10 km. It is not clear exactly where the treasure is.

Materials
- centimetre grid paper
- coloured pencils
- red marker

You be the treasure hunter. What are your chances of finding treasure?

1. a) On a 10 by 10 grid, randomly colour the squares using the ratio yellow : white : brown = 2 : 5 : 3

b) Yellow represents treasure. White represents ice. Brown represents sand. What is the theoretical probability of each colour on the grid? Express each answer as a percent.

2. a) Use a red marker to divide the 10 by 10 grid into four sections as outlined below:
- Draw a vertical line that divides the vertical distance into $\frac{3}{10}$ and $\frac{7}{10}$.
- Draw a horizontal line that divides the horizontal distance into $\frac{2}{5}$ and $\frac{3}{5}$.
- Label the largest zone as Zone 1, the second-largest zone as Zone 2, the third-largest zone as Zone 3, and the smallest zone as Zone 4.

b) What is the theoretical probability of choosing each zone in the grid? Express each answer as a fraction, a decimal, and a percent.

c) Using your answers from #1b) and #2b) and the following formula, what is the theoretical probability of each colour appearing in each zone? Justify your thinking.

theoretical probability of each colour in each zone = theoretical probability of each colour on the grid × theoretical probability of each zone

3. a) You have time to search only two zones. Using the theoretical probability, which two zones will you search? Explain why.

b) How might your answer to part a) change if you used an experiment to decide what zones to search? Justify your response.

Challenge in Real Life • MHR 441

Meeting Student Needs

- Some students may find it easier to use an enlarged hundred grid and markers.
- Some students may need assistance understanding how to use the three-term ratio (2:5:3) to colour the map (yellow:white:brown). Remind students that there are 10 × 10 or 100 cells on the grid. For every ten cells, there are two yellow cells, five white cells, and three brown cells. It may be helpful to display an example of a completed grid for students to refer to.
- For #2c), some students may benefit from using **BLM 11–14 Theoretical Probability**.

ELL

- Ensure that English language learners understand the following terms: *treasure hunters*, *sunken ship*, *scattered*, *vertical*, *horizontal*, and *zone*.

Gifted and Enrichment

- Have students develop a game similar to Battleship using a treasure search theme. Players could hide their gold and try to find other players' gold.
- Have students research Sable Island, also known as the graveyard of the Atlantic, and provide that background to introduce the challenge.

WWW Web Link

For information on the history of Sable Island, go to www.mathlinks8.ca and follow the links.

Answers

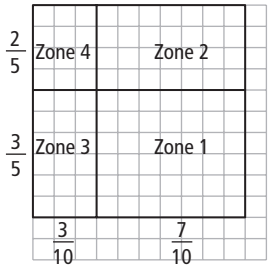
Treasure Hunt

1. b) $P(\text{yellow}) = \frac{2}{10} = 20\%$; $P(\text{white}) = \frac{5}{10} = 50\%$

$P(\text{brown}) = \frac{3}{10} = 30\%$

The theoretical probability of yellow is 20%.
The theoretical probability of white is 50%.
The theoretical probability of brown is 30%.

2. a) The grid should be divided as shown.



b) The theoretical probability of landing in Zone 1 is $\frac{42}{100}$, 0.42, or 42%.

The theoretical probability of landing in Zone 2 is $\frac{28}{100}$, 0.28, or 28%.

The theoretical probability of landing in Zone 3 is $\frac{18}{100}$, 0.18, or 18%.

The theoretical probability of landing in Zone 4 is $\frac{12}{100}$, 0.12, or 12%.

c) The table shows the probability of each colour by multiplying the probability of landing in each zone by the fraction of coloured squares.

Zone	Calculation	Probability
1	$P(\text{yellow}) = \frac{2}{10} \times \frac{42}{100} = \frac{84}{1000}$	0.084 or 8.4%
	$P(\text{white}) = \frac{5}{10} \times \frac{42}{100} = \frac{210}{1000}$	0.210 or 2.1%
	$P(\text{brown}) = \frac{3}{10} \times \frac{42}{100} = \frac{126}{1000}$	0.126 or 12.6%
2	$P(\text{yellow}) = \frac{2}{10} \times \frac{28}{100} = \frac{56}{1000}$	0.056 or 5.6%
	$P(\text{white}) = \frac{5}{10} \times \frac{28}{100} = \frac{140}{1000}$	0.140 or 14%
	$P(\text{brown}) = \frac{3}{10} \times \frac{28}{100} = \frac{84}{1000}$	0.084 or 8.4%
3	$P(\text{yellow}) = \frac{2}{10} \times \frac{18}{100} = \frac{36}{1000}$	0.036 or 3.6%
	$P(\text{white}) = \frac{5}{10} \times \frac{18}{100} = \frac{90}{1000}$	0.090 or 9%
	$P(\text{brown}) = \frac{3}{10} \times \frac{18}{100} = \frac{54}{1000}$	0.054 or 5.4%
4	$P(\text{yellow}) = \frac{2}{10} \times \frac{12}{100} = \frac{24}{1000}$	0.024 or 2.4%
	$P(\text{white}) = \frac{5}{10} \times \frac{12}{100} = \frac{60}{1000}$	0.060 or 6%
	$P(\text{brown}) = \frac{3}{10} \times \frac{12}{100} = \frac{36}{1000}$	0.036 or 3.6%

3. a) Zone 1 and Zone 2 because they have the largest area. Note: Students who go back and count the number of yellow shaded squares on the grid are drawing a conclusion without using probability and should not receive credit for their answer.

b) Answers will vary. Students should point out that experimental probability may differ from theoretical probability, depending on the number of times you repeat the experiment. In some cases it might be higher; in other cases it might be lower.

This challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment	Supporting Learning
Assessment *for* Learning	
Treasure Hunt Discuss the challenge as a class. Have students provide individual reports.	• Allow students to present their reports either in written form or orally. • For a second challenge, complete with teaching notes and student exemplars, go to www.mathlinks8.ca, access the online Teacher Centre, go to Assessment, and then follow the links.
Assessment *of* Learning	
Treasure Hunt Introduce the challenge to the class. Have students provide individual reports.	• **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this challenge. Page 589 provides notes on how to use the rubric for the challenge. • To view student exemplars, go to www.mathlinks8.ca, access the online Teacher Centre, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as the Challenge in Real Life and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution Note: The solution may contain a minor calculation error that does not change the final outcome or a weakness in communication that does not hinder understanding of the problem.
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete solution with weak communication *or* • provides a complete solution, which contains a calculation error that hinders the overall solution *or* • provides a complete and correct solution to all parts based on an incorrect #1a) *or* • provides a complete solution with no justification for #3a) or #3b)
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	• provides a correct and complete #1 and #2 *or* • provides a correct and complete #1 and #2a) and b) *or* • provides a complete response to #2 and #3 *or* • provides partial solutions to all parts
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a correct and complete #1 *or* • provides a correct and complete #2a) and b) *or* • provides a correct and complete #1 and #2c) based on an incorrect #2b) *or* • provides a correct and complete #2a), b), and #3a)
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct and complete #1a) or b)

For student exemplars, go to www.mathlinks8.ca and follow the links.