11 Probability

General Outcomes

- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Outcomes

SP2 Solve problems involving the probability of independent events.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
11.1	✓ determine the sample space of a probability experiment with two independent events
	✓ represent the sample space in the form of a tree diagram or table
	✓ express the probability of an event as a fraction, a decimal, and a percent
11.2	✓ determine the outcomes of two or more independent events
	✓ verify the total number of possible outcomes using a different strategy
11.3	✓ solve probability problems
	✓ verify their answers using a different method

Assessment	Supporting Learning
Assessment *for* Learning	
Method 1: Use the Math Link introduction on page 409 in *MathLinks 8* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal to explain what they personally know about probability, sample space, and tree diagrams and tables in relation to probability experiments.	• **BLM 11–1 Chapter 11 Math Link Introduction** provides scaffolding for the Math Link introduction. • Have students use the What I Need to Work On section of their chapter Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete the Get Ready materials available on **BLM 11–2 Chapter 11 Get Ready**, in the *MathLinks 8 Practice and Homework Book*, and at the www.mathlinks8.ca book site.
Assessment *as* Learning	
Literacy Link (page 407) Before starting the chapter, have students develop a concept map showing what they currently understand about probability, and providing examples of probability and calculations they know about.	• Encourage students to use the glossary on pages 517–521 to help them. • Brainstorm definitions, comparisons, facts, and examples. Encourage students to use the brainstormed ideas to develop some of their own. • Use the concept maps to identify any misconceptions students may have. Deal with these misconceptions during the appropriate section of the chapter.
Chapter 11 Foldable As students work on each section in Chapter 11, have them keep track of any problems they are having in the What I Need to Work On section of their chapter Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled.
Assessment *for* Learning	
BLM 11–3 Chapter 11 Warm-Up This BLM includes three warm-ups, one to be used at the beginning of each section. Each warm-up provides cumulative review questions for the entire student resource to that point, as well as mental math practice.	• As students complete questions from previous chapters, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing mental math calculations.

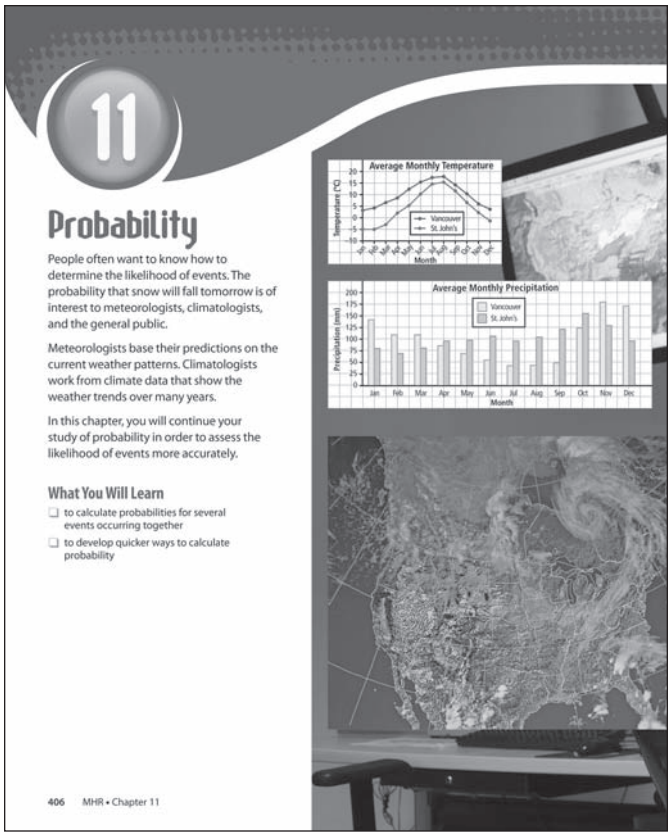
Problems of the Week

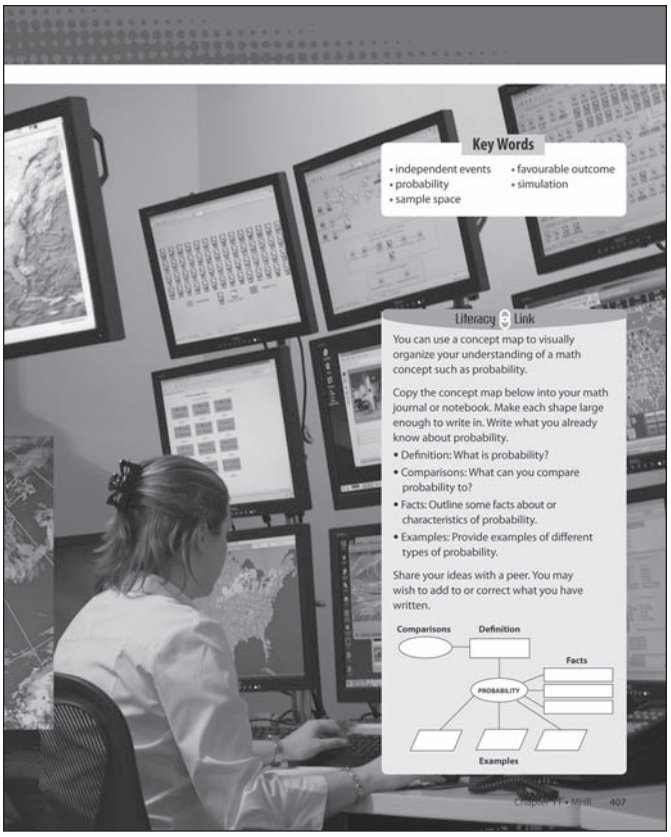
Have all students try at least one of the problems on **BLM 11–4 Chapter 11 Problems of the Week**. Many of these problems require students to think outside the box and experiment with a variety of approaches. Some have definitive answers; others can be answered in more than one way.

Students can take the problems home and consult with parents or guardians, work with other students when their work is completed, or try the problems on their own. The questions take a varying amount of time to solve, depending on the particular student and the problem itself. You may wish to give out these problems at the beginning of the chapter and discuss the solutions at appropriate times throughout your work on the chapter.

Chapter 11 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Extra Support	Assessment		
						Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 40–50 minutes (TR page 549)	Students should be familiar with • comparing the likelihood of two events occurring • identifying all possible outcomes of a probability experiment • expressing probabilities as decimals, fractions, and percents • identifying the sample space for a probability experiment involving two independent events	• 11 × 17 sheet of paper • two sheets of notebook paper • scissors • stapler	Master 18 Concept Map BLM 11–1 Chapter 11 Math Link Introduction BLM 11–2 Chapter 11 Get Ready BLM 11–4 Chapter 11 Problems of the Week		Online Learning Centre	TR page 548 Chapter 11 Foldable, TR page 548	TR page 548	
11.1 Determining Probabilities Using Tree Diagrams and Tables • 80–100 minutes (TR page 553)	Students should be familiar with • converting between fractions, decimals, and percents • reducing fractions • representing sample spaces using tree diagrams, tables, and lists	• compass or circular object to trace around (optional) • coloured pencils • paper clip (optional) • ruler • calculator • four-sided die (optional) • computer and spreadsheet software (optional) • craft sticks	BLM 11–3 Chapter 11 Warm-Up BLM 11–5 Section 11.1 Extra Practice BLM 11–6 Section 11.1 Math Link	**Essential:** 1, 3, 5, 7, Math Link **Typical:** 1, 3, 5, 7–10, Math Link **Extension/Enrichment:** 1, 2, 9–13, Math Link	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 557, 559 Math Learning Log, TR page 561 Chapter 11 Foldable, TR page 561	TR pages 557, 561	
11.2 Outcomes of Independent Events • 80–100 minutes (TR page 562)	Students should be familiar with • multiplication of decimals	• ruler • nine small pieces of paper • coloured pencils • coloured counters • craft sticks • manipulatives including coins, six-sided dice, four-sided dice, spinners, marbles, bags, and playing cards (optional)	BLM 11–3 Chapter 11 Warm-Up BLM 11–7 Section 11.2 Extra Practice BLM 11–8 Section 11.2 Math Link	**Essential:** 1–3, 5, 7, 10, Math Link **Typical:** 1–3, 5, 7–12, Math Link **Extension/Enrichment:** 1, 2, 10–16, Math Link	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 566, 568 Math Learning Log, TR page 570 Chapter 11 Foldable, TR page 570	TR pages 566, 570	
11.3 Determining Probabilities Using Fractions • 80–100 minutes (TR page 571)	Students should be familiar with • multiplication of common fractions	• ruler • red and yellow coloured pencils or markers • computer and spreadsheet software (optional) • calculator (optional) • manipulatives including coins, six-sided dice, four-sided dice, spinners, marbles, bags, and playing cards (optional) • craft sticks	Master 2 Two Stars and One Wish BLM 11–3 Chapter 11 Warm-Up BLM 11–9 Section 11.3 Explore the Math BLM 11–10 Section 11.3 Extra Practice BLM 11–11 Section 11.3 Math Link	**Essential:** 1, 2, 4, 6, 9, Math Link **Typical:** 1, 2, 4, 6, 8–14, Math Link **Extension/Enrichment:** 1, 2, 12–17, Math Link	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	Master 2 Two Stars and One Wish TR pages 574, 576 Math Learning Log, TR page 578 Chapter 11 Foldable, TR page 578	TR pages 574, 578	
Chapter 11 Review • 40–50 minutes (TR page 579)		• ruler	Master 18 Concept Map BLM 11–5 Section 11.1 Extra Practice BLM 11–7 Section 11.2 Extra Practice BLM 11–10 Section 11.3 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	*MathLinks 8 Practice and Homework Book* *MathLinks 8 CAB*	Chapter 11 Foldable, TR page 579	TR page 580	
Chapter 11 Practice Test • 40–50 minutes (TR page 581)		• ruler	BLM 11–12 Chapter 11 Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1, 2, 5–8	*MathLinks 8 CAB*	TR page 582		TR page 582 BLM 11–12 Chapter 11 Test
Chapter 11 Wrap It Up! • 60–90 minutes (TR page 583)		• tongue depressors or craft sticks • coloured pencils or markers	Master 1 Project Rubric BLM 11–1 Chapter 11 Math Link Introduction BLM 11–6 Section 11.1 Math Link BLM 11–8 Section 11.2 Math Link BLM 11–11 Section 11.3 Math Link BLM 11–13 Chapter 11 Wrap It Up!		Online Learning Centre			TR page 583 Master 1 Project Rubric
Chapter 11 Math Games • 30–40 minutes (TR page 585)		• two six-sided dice of different colours per pair of students • coin per pair of students • ruler					TR page 585	
Chapter 11 Challenge in Real Life • 40–50 minutes (TR page 586)		• centimetre grid paper • yellow and brown coloured pencils • red marker • ruler	Master 1 Project Rubric Master 8 Centimetre Grid Paper BLM 11–14 Theoretical Probability BLM 11–15 Chapter 11 BLM Answers		Online Learning Centre		TR page 588	TR page 588 Master 1 Project Rubric





***MathLinks 8*, pages 406–409**

Suggested Timing
40–50 minutes

Materials
- 11 × 17 sheet of paper
- two sheets of notebook paper
- scissors
- stapler

Blackline Masters
Master 18 Concept Map
BLM 11–1 Chapter 11 Math Link Introduction
BLM 11–2 Chapter 11 Get Ready
BLM 11–4 Chapter 11 Problems of the Week

Key Words
independent events
probability
sample space
favourable outcome
simulation

What's the Math?

In this chapter, students continue their work from grade 7 with probability experiments involving two independent events. Students begin by recalling how to determine probabilities using tree diagrams and tables. They then determine the number of outcomes from two or more events using multiplication. Multiplying allows students to quickly calculate the total number of possible outcomes. Students then practise solving probability problems, including simulations, using multiplication, tables, and tree diagrams, and verify their answers using a different method.

Planning Notes

Before starting Chapter 11, help students recall what they learned about probability in grade 7. Ask students what probability is and how it is used in daily life. Some possible responses might include lotteries, weather forecasting, and insurance. Use the text and the visual about weather patterns in the chapter opener to encourage discussion about probability.

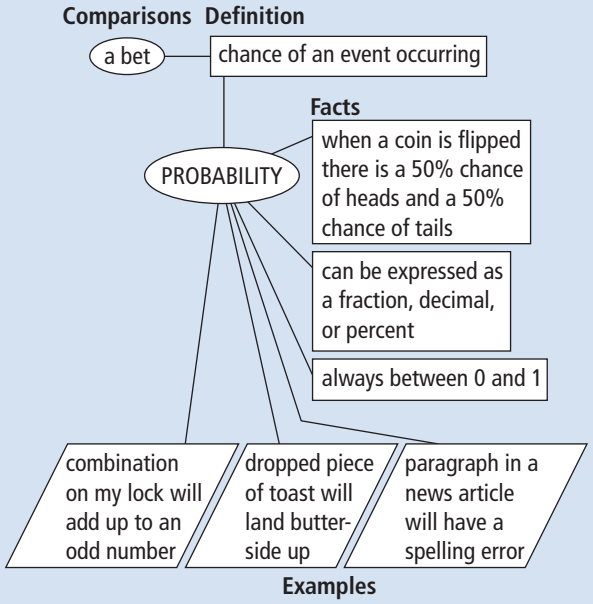
Encourage students to talk about what they know and to give real-world examples. Try to elicit ideas from all class members.

Literacy Link Concept maps are graphic organizers that help students to understand essential characteristics of a concept and to make connections that show how the information is related. This form of mind map provides an alternative to traditional outlining and note-taking, and it supports all students by providing both visual and written information. If students have not completed a concept map earlier, refer to the Chapter 4 notes for assistance.

Work with the class to start developing a concept map on probability at the beginning of Chapter 11. Brainstorm and discuss as a class the information needed to partially complete the map. You might use sticky notes to identify some details and add them to a copy of the map on chart paper.

Have students complete the concept map using their own definitions, comparisons, facts, and examples.

Have students revisit the concept map just before they do the practice test. They could develop a second concept map about probability showing what they know at the end of the chapter and compare it with their earlier map.

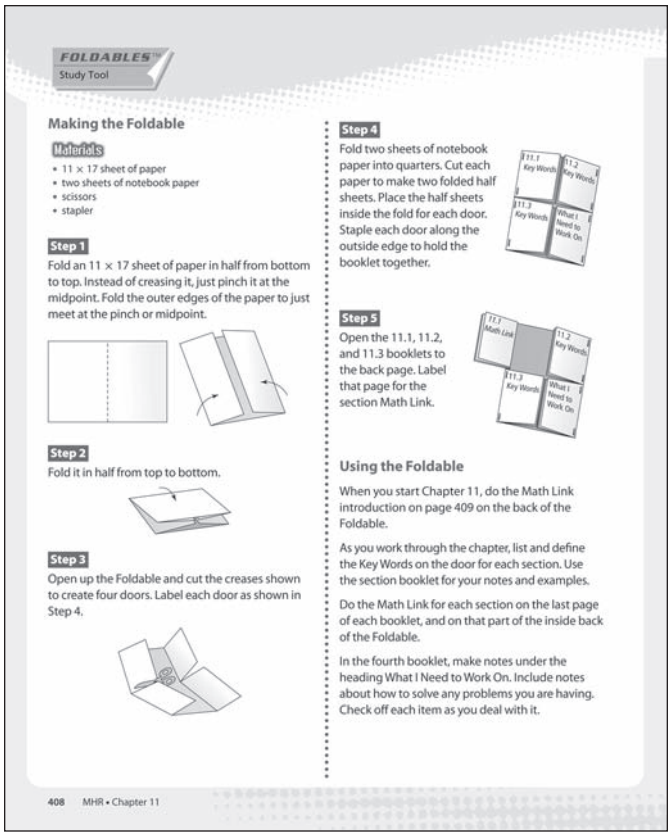


Meeting Student Needs

- Some students may need help to recall what they know about probability before beginning this chapter. Consider having them complete the questions on **BLM 11–2 Chapter 11 Get Ready** to activate the prerequisite skills for this chapter.
- Consider inviting a community Elder to demonstrate how to play games of chance, such as hand games. Historically, adults in some communities encouraged games of chance because they required players to develop their skills. You may also wish to have students research how probability was used in the past.
- Consider helping students recall how to convert among decimals, percents, and fractions.
- Consider having students design a space in your classroom where they can post questions and comments on sticky notes using headings such as Questions I Still Have and Things I'm Not Sure About. Use the notes to help you address issues as they arise.
- Students may benefit from using **Master 18 Concept Map** to create their concept map.
- Encourage students who have additional information to enlarge or change the concept map outline. Alternatively, they can create their own template.

ELL

- English language learners may have difficulty with terms such as *likelihood, meteorologists, climatologists,* and *precipitation*. Have students add any new terms to their dictionary.
- Very recently arrived Canadians in your class may never have seen snow. Ask volunteers to explain what snow is and how often it falls in winter.
- Consider displaying Key Words on a math word wall. Encourage students to create their own vocabulary/picture dictionary. Matching a picture with a key word and its definition helps reinforce students' understanding of vocabulary.



FOLDABLES™
Study Tool

Making the Foldable

Materials
- 11 × 17 sheet of paper
- two sheets of notebook paper
- scissors
- stapler

Step 1
Fold an 11 × 17 sheet of paper in half from bottom to top. Instead of creasing it, just pinch it at the midpoint. Fold the outer edges of the paper to just meet at the pinch or midpoint.

Step 2
Fold it in half from top to bottom.

Step 3
Open up the Foldable and cut the creases shown to create four doors. Label each door as shown in Step 4.

Step 4
Fold two sheets of notebook paper into quarters. Cut each paper to make two folded half sheets. Place the half sheets inside the fold for each door. Staple each door along the outside edge to hold the booklet together.

Step 5
Open the 11.1, 11.2, and 11.3 booklets to the back page. Label that page for the section Math Link.

Using the Foldable

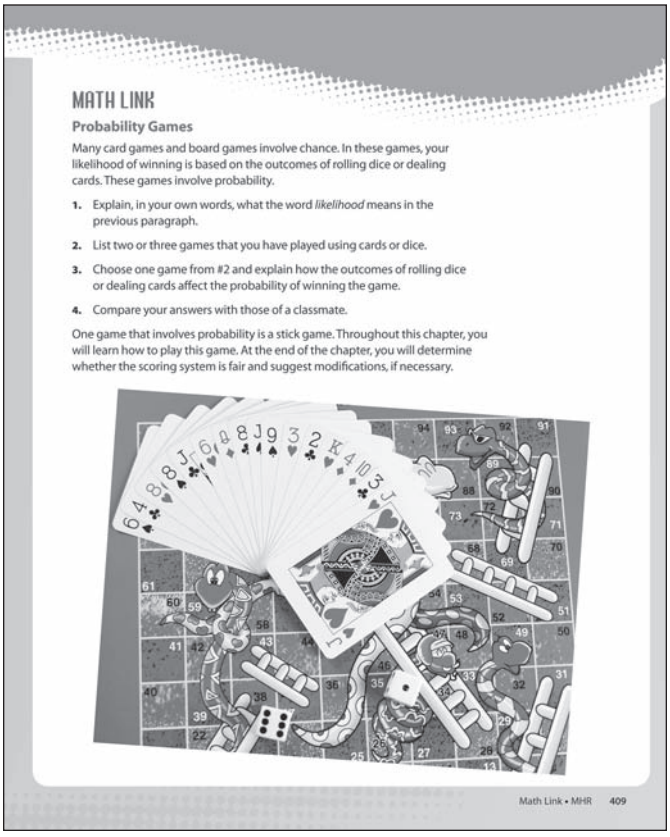
When you start Chapter 11, do the Math Link introduction on page 409 on the back of the Foldable.

As you work through the chapter, list and define the Key Words on the door for each section. Use the section booklet for your notes and examples.

Do the Math Link for each section on the last page of each booklet, and on that part of the inside back of the Foldable.

In the fourth booklet, make notes under the heading What I Need to Work On. Include notes about how to solve any problems you are having. Check off each item as you deal with it.

408 MHR • Chapter 11



MATH LINK

Probability Games

Many card games and board games involve chance. In these games, your likelihood of winning is based on the outcomes of rolling dice or dealing cards. These games involve probability.

1. Explain, in your own words, what the word *likelihood* means in the previous paragraph.
2. List two or three games that you have played using cards or dice.
3. Choose one game from #2 and explain how the outcomes of rolling dice or dealing cards affect the probability of winning the game.
4. Compare your answers with those of a classmate.

One game that involves probability is a stick game. Throughout this chapter, you will learn how to play this game. At the end of the chapter, you will determine whether the scoring system is fair and suggest modifications, if necessary.

Math Link • MHR 409

Foldables Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter. Some students may need more pages in each booklet. Others may not need as many.

Have students record their work for the Math Link introduction on the back of the Foldable. As students work through the chapter, have them list and define the Key Words on the door for each section, and use the section booklet for their notes and examples about the Key Ideas. Have students do the Math Link for each section on the last page of each booklet, and on that part of the inside back of the Foldable. Have students use the fourth booklet to keep track of what they need to work on and for notes about how to solve any problems they might have. Have students check off each item as they deal with it.

Have students store the Foldable in a binder. You may wish to provide students with a plastic envelope that fits into their binder.

Math Link

Read the Math Link introduction on page 409 as a class and ask students to list and describe games of chance using cards and/or dice that they are familiar with (e.g., Yahtzee®, Game of Life®, Go Fish, Crazy Eights). Have students work in pairs to answer the questions. During the class discussion afterward, check that students are using the words *likelihood* and *outcomes* appropriately.

The Math Links for this chapter allow students to explore probability associated with a stick game that is based on chance. You might introduce how to play a hand game of chance using the information in the related Web Link on TR page 552.

Have students read the Wrap It Up! problem on page 439 to give them a sense of where the Math Link is heading. The Wrap It Up! problem is a summative assessment. As students work through the chapter, they need to complete the related Math Links in sections 11.1, 11.2, and 11.3. These Math Links will assist them in doing the Wrap It Up! problem.

Meeting Student Needs

- Consider creating the chapter Foldable ahead of time to use as a model.
- To help them to get started, some students may benefit from using **BLM 11–1 Chapter 11 Math Link Introduction**, which provides scaffolding for this activity.

Web Link

For a description of and directions for playing an Aboriginal hand game, go to www.mathlinks8.ca and follow the links.

Answers

Math Link

1. Answers may vary. Example: Likelihood refers to the chance or probability that you will win the game.
2. Answers will vary. Card games may include Go Fish and Crazy Eights. Games that involve dice may include Yahtzee® and Game of Life®.
3. Answers will vary. Example: In the Game of Life®, the outcome of rolling the dice determines what type of job (and salary) you will have and how many kids you will have in your family. These outcomes will affect the amount of money you earn during the game and your likelihood of retiring in style and winning the game.
4. Answers will vary depending on the games selected.