Determining Probabilities Using Tree Diagrams and Tables

11.1

***MathLinks 8*, pages 410–418**

Suggested Timing
80–100 minutes

Materials
- compass or circular object to trace around (optional)
- coloured pencils
- paper clip (optional)
- ruler
- four-sided die (optional)
- calculator
- computer and spreadsheet software (optional)
- craft sticks

Blackline Masters
BLM 11–3 Chapter 11 Warm-Up
BLM 11–5 Section 11.1 Extra Practice
BLM 11–6 Section 11.1 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☐ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

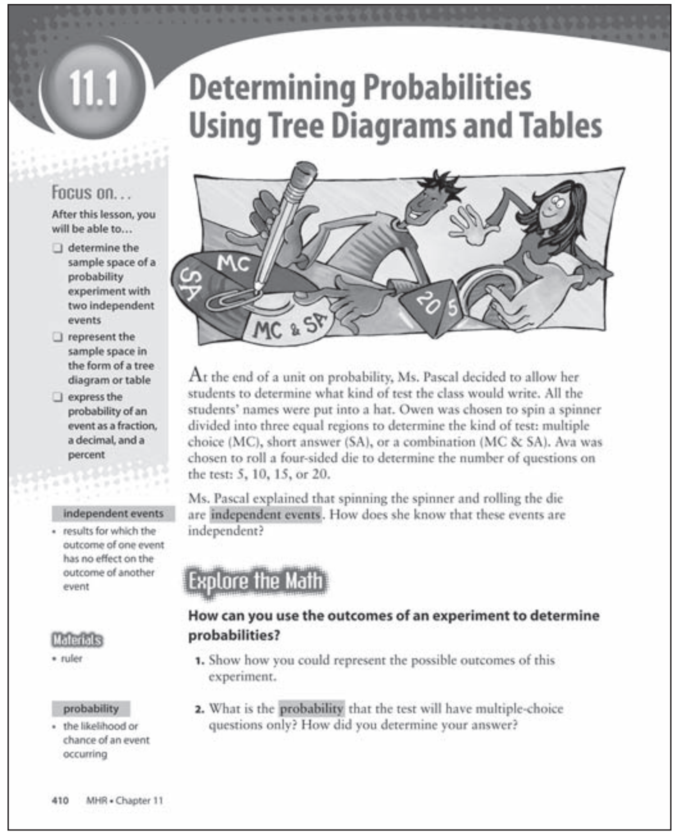
Specific Outcomes
SP2 Solve problems involving the probability of independent events.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 3, 5, 7, Math Link
Typical	1, 3, 5, 7–10, Math Link
Extension/Enrichment	1, 2, 9–13, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 11–3 Chapter 11 Warm-Up** to reinforce material learned in previous sections.

Use the opening text as an introduction to the Explore the Math. Consider having volunteers model using a spinner divided into three regions (constructed in advance) and a four-sided die to conduct a probability experiment ten times using the scenario in the opening text.



11.1

Determining Probabilities Using Tree Diagrams and Tables

Focus on…

After this lesson, you will be able to…
- ☐ determine the sample space of a probability experiment with two independent events
- ☐ represent the sample space in the form of a tree diagram or table
- ☐ express the probability of an event as a fraction, a decimal, and a percent

At the end of a unit on probability, Ms. Pascal decided to allow her students to determine what kind of test the class would write. All the students' names were put into a hat. Owen was chosen to spin a spinner divided into three equal regions to determine the kind of test: multiple choice (MC), short answer (SA), or a combination (MC & SA). Ava was chosen to roll a four-sided die to determine the number of questions on the test: 5, 10, 15, or 20.

Ms. Pascal explained that spinning the spinner and rolling the die are independent events. How does she know that these events are independent?

independent events
- results for which the outcome of one event has no effect on the outcome of another event

Explore the Math

How can you use the outcomes of an experiment to determine probabilities?

Materials
- ruler

1. Show how you could represent the possible outcomes of this experiment.

2. What is the probability that the test will have multiple-choice questions only? How did you determine your answer?

probability
- the likelihood or chance of an event occurring

410 MHR • Chapter 11

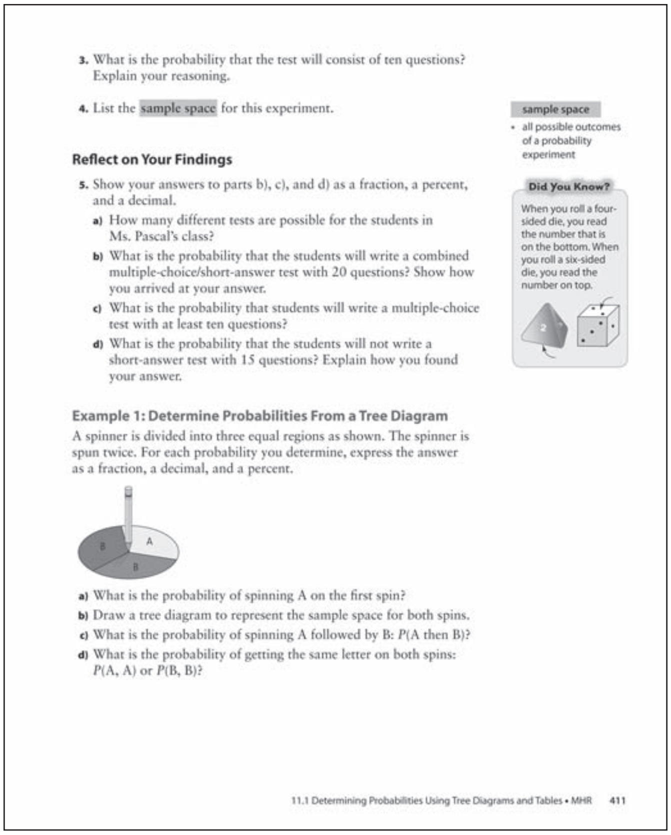
Direct students to the Did You Know? on page 411, which explains how to read a four-sided die. Unlike the six-sided die, you read the number on the bottom surface of the four-sided die.

Explore the Math

In this exploration, students review using tree diagrams and tables to represent the sample space of a probability experiment. Students will continue to use these visual organizers throughout their high school experience with probability.

Have students recall using short forms of words in probability diagrams and tables, and using the full words in their final answers.

Method 1 Have students work together in pairs or small groups. As students work, circulate and ask questions such as the following:
- Why did you use this method for recording the outcomes?
- What short forms are you using?



3. What is the probability that the test will consist of ten questions? Explain your reasoning.

4. List the sample space for this experiment.

sample space
- all possible outcomes of a probability experiment

Reflect on Your Findings

5. Show your answers to parts b), c), and d) as a fraction, a percent, and a decimal.
   a) How many different tests are possible for the students in Ms. Pascal's class?
   b) What is the probability that the students will write a combined multiple-choice/short-answer test with 20 questions? Show how you arrived at your answer.
   c) What is the probability that students will write a multiple-choice test with at least ten questions?
   d) What is the probability that the students will not write a short-answer test with 15 questions? Explain how you found your answer.

Did You Know?

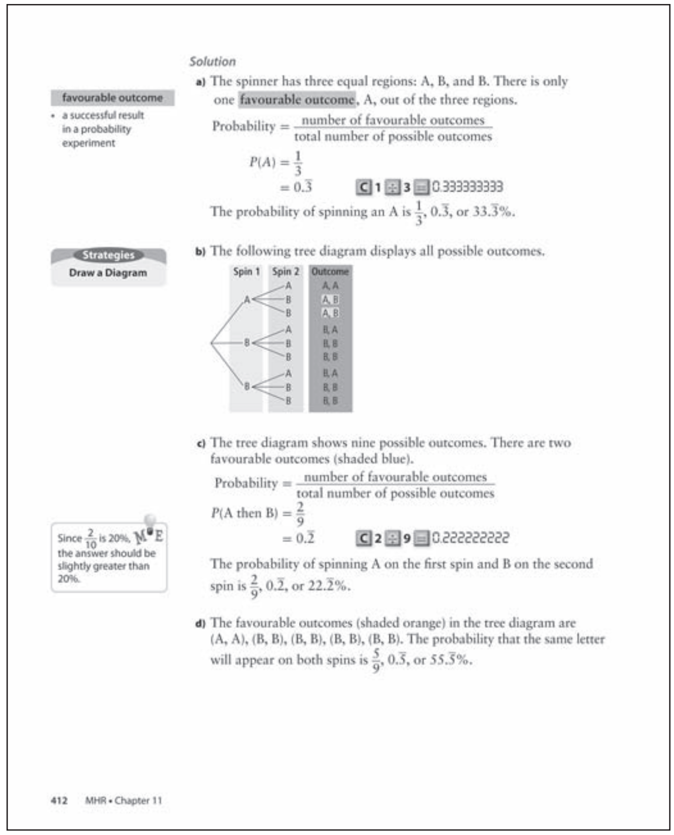
When you roll a four-sided die, you read the number that is on the bottom. When you roll a six-sided die, you read the number on top.

Example 1: Determine Probabilities From a Tree Diagram

A spinner is divided into three equal regions as shown. The spinner is spun twice. For each probability you determine, express the answer as a fraction, a decimal, and a percent.

a) What is the probability of spinning A on the first spin?
b) Draw a tree diagram to represent the sample space for both spins.
c) What is the probability of spinning A followed by B: P(A then B)?
d) What is the probability of getting the same letter on both spins: P(A, A) or P(B, B)?

11.1 Determining Probabilities Using Tree Diagrams and Tables • MHR 411



Solution

a) The spinner has three equal regions: A, B, and B. There is only one favourable outcome, A, out of the three regions.

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

$$P(A) = \frac{1}{3} = 0.\overline{3}$$

C 1 ÷ 3 = 0.333333333

The probability of spinning an A is $\frac{1}{3}$, $0.\overline{3}$, or $33.\overline{3}$%.

favourable outcome
- a successful result in a probability experiment

Strategies
Draw a Diagram

b) The following tree diagram displays all possible outcomes.

Spin 1	Spin 2	Outcome
A	A	A, A
	B	A, B
	B	A, B
B	A	B, A
	B	B, B
	B	B, B
B	A	B, A
	B	B, B
	B	B, B

c) The tree diagram shows nine possible outcomes. There are two favourable outcomes (shaded blue).

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

$$P(\text{A then B}) = \frac{2}{9} = 0.\overline{2}$$

C 2 ÷ 9 = 0.222222222

Since $\frac{2}{10}$ is 20%, the answer should be slightly greater than 20%.

The probability of spinning A on the first spin and B on the second spin is $\frac{2}{9}$, $0.\overline{2}$, or $22.\overline{2}$%.

d) The favourable outcomes (shaded orange) in the tree diagram are (A, A), (B, B), (B, B), (B, B), (B, B). The probability that the same letter will appear on both spins is $\frac{5}{9}$, $0.\overline{5}$, or $55.\overline{5}$%.

412 MHR • Chapter 11

- What short forms might you use to make this easier to read?
- What is the sample space for this experiment?
- How do you know what the sample space is?

Have different groups present their representations to the class and explain their conclusions. Ask:
- How did you use this tree diagram (or table) to come to this conclusion?
- Are there other conclusions you could have made? Explain.
- How might you record your conclusion using probability notation?
- Are there other ways to show this using probability notation?

Discuss the similarities and differences between students' methods. Post student examples of tree diagrams and tables.

Method 2 Have concrete and kinesthetic learners work in pairs to construct a spinner with three equal sections using a compass or a round object to trace around, two coloured pencils, a pencil, and a paper clip. Prompt students to determine the central angle for each sector (360° ÷ 3 = 120°). Have students conduct a probability experiment ten times before moving on to complete the Explore the Math. Have them use the spinner and the die to help them determine all outcomes, then discuss their answers, as in Method 1.

Note: Students may use different coins, dice, and spinners during this chapter. Discuss that these need to be fair. The term *fair* describes an item for a probability experiment in which each part has the same probability of appearing (e.g., the face of the coin or die, or the sector of the spinner). All coins, dice, and spinners used and referred to in this chapter are understood to be fair.

Example 1

In Example 1, students determine probabilities from a tree diagram. Explain that students will need to convert fractions to decimals and percents. Remind students that tree diagrams can be arranged vertically or horizontally. Point out a branch and explain that there is a branch for each possible outcome of an event. You might point out that the branches give form to the tree (diagram).

Show You Know

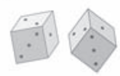
Ellen flips a coin and rolls a four-sided die numbered 1, 2, 3, and 4.

a) What is the sample space? Use a tree diagram to show how you got your answer.

b) What is $P(\text{H}, 4)$?

Example 2: Determine Probabilities From a Table

Two standard six-sided dice are rolled. One die is blue and the other is red. For each probability you determine, express the answer as a fraction, a decimal, and a percent.



a) Create a table to represent the sample space.

b) What is the probability of rolling a sum greater than ten?

c) What is the probability that the number on the red die is one larger than the number on the blue die?

d) What is the probability that the sum of the two numbers is less than 11?

Solution

a) The following table represents the sample space. The numbers from the red die are shown in red and the numbers from the blue die are shown in blue.

Strategies
Make a Table

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

b) The probability of rolling a sum greater than ten can be found by adding the two numbers in each cell of the table. There are three cells in the table with a sum greater than ten. So, there are three favourable outcomes.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

$$P(\text{sum} > 10) = \frac{3}{36}$$
$$= 0.08\overline{3}$$

C 3 ÷ 36 = 0.083333333

The probability of a sum greater than ten is $\frac{3}{36}$, $0.08\overline{3}$, or $8.\overline{3}\%$.

c) The probability that the number on the red die will be one larger than the number on the blue die can be found by counting favourable outcomes in the table.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

$$P(\text{number on red die is one larger than number on blue die}) = \frac{5}{36}$$
$$= 0.13\overline{8}$$

C 5 ÷ 36 = 0.138888889

The probability that the number on the red die is one larger than the number on the blue die is $\frac{5}{36}$, $0.13\overline{8}$, or $13.\overline{8}\%$.

Example 2

In Example 2, students determine probabilities from a table. Explain that a table can be a more compact, neat way to organize data for a probability experiment with two independent events. Generally, tables are more appropriate than tree diagrams when there are more than 20 possible outcomes for a single event. Conversely, the sample space for a probability experiment with more than two events is usually represented in a tree diagram.

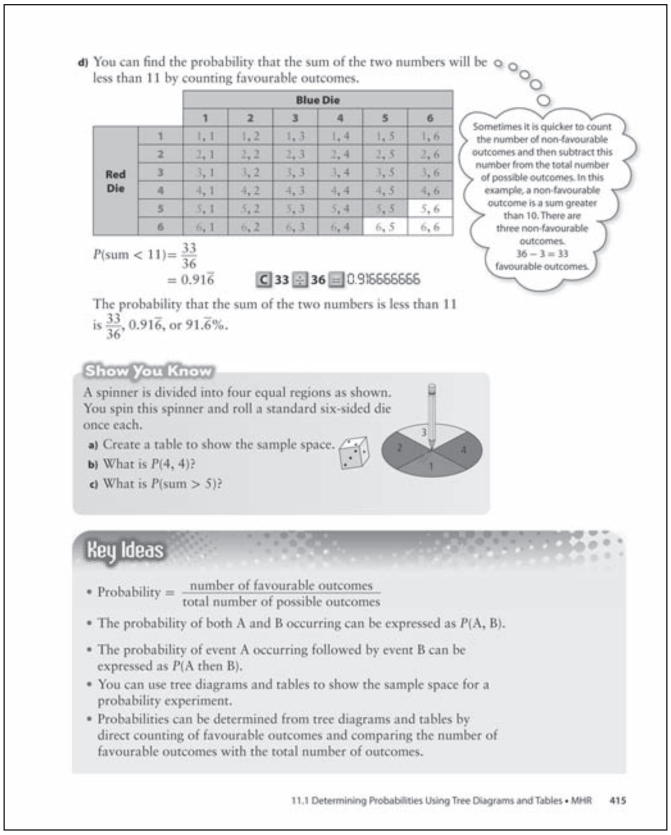
Ensure that students use a ruler to draw tables.

Meeting Student Needs

- Some students who need to use a spinner may have difficulty constructing one. Consider allowing these students to use a virtual spinner such as the one in the Web Link on TR page 556.
- Help students recall how to read a vertical tree diagram from left to right. Using the tree diagram in part b) on page 412, the branches on the left of the tree show the outcomes for the first spin. The branches on the right show the outcomes for the second spin. The column on the far right shows the combined outcomes.
- Allow concrete and kinesthetic learners to use coins, dice, and spinners, if necessary, to help them record possible outcomes.
- You may wish to allow students who have trouble drawing tree diagrams to use virtual manipulatives.
- The buttons on most calculators are too small and close together for students with motor difficulties to use accurately. Students may benefit from using a calculator with oversized keys.
- Some students may need to reactivate their knowledge of probability. Have these students complete similar problems to those in Examples 1 and 2, one as a class and the second with a partner, before completing the Show You Know.

ELL

- Make sure that English language learners understand the following terms: *spin/spun*, *spinner*, *spinning the spinner*, *rolling the die*, *outcomes*, *possible outcomes*, and *flips a coin*. Have students add new terms to their dictionary.



d) You can find the probability that the sum of the two numbers will be less than 11 by counting favourable outcomes.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Sometimes it is quicker to count the number of non-favourable outcomes and then subtract this number from the total number of possible outcomes. In this example, a non-favourable outcome is a sum greater than 10. There are three non-favourable outcomes.
36 − 3 = 33 favourable outcomes.

$P(\text{sum} < 11) = \frac{33}{36}$
$= 0.91\overline{6}$

C 33 ÷ 36 = 0.916666666

The probability that the sum of the two numbers is less than 11 is $\frac{33}{36}$, $0.91\overline{6}$, or $91.\overline{6}\%$.

Show You Know

A spinner is divided into four equal regions as shown. You spin this spinner and roll a standard six-sided die once each.

a) Create a table to show the sample space.
b) What is $P(4, 4)$?
c) What is $P(\text{sum} > 5)$?

Key Ideas

- Probability = $\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$
- The probability of both A and B occurring can be expressed as $P(\text{A, B})$.
- The probability of event A occurring followed by event B can be expressed as $P(\text{A then B})$.
- You can use tree diagrams and tables to show the sample space for a probability experiment.
- Probabilities can be determined from tree diagrams and tables by direct counting of favourable outcomes and comparing the number of favourable outcomes with the total number of outcomes.

11.1 Determining Probabilities Using Tree Diagrams and Tables • MHR 415

Common Errors

- Some students may have difficulty using a systematic approach to listing the sample space in a tree diagram or a table.

R_x Model a systematic approach. For example, if a coin is flipped and a die is rolled, list all of the flips with heads first and then the flips with tails: (H, 1), (H, 2), (H, 3), (H, 4), (T, 1), (T, 2), (T, 3), (T, 4). You might have students conduct a simple probability experiment, such as flipping a coin and rolling a four-sided die, and have them draw the tree diagram or the table for the sample space. For a table, consider having them draw a symbol for head or tail in each cell to remind them.

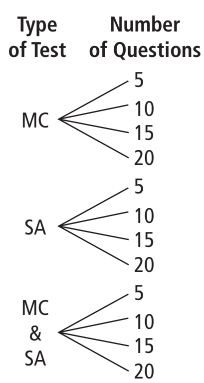
WWW Web Link

For a virtual manipulative that allows students to learn about chance and random choices using a spinner, go to www.mathlinks8.ca and follow the links. Students can spin the spinner, change spinner regions (name, colour, and size), and record results from multiple spins.

Answers

Explore the Math

1. Answers will vary. Students may use some type of organizer (e.g., table, tree diagram). Example:



2. $P(\text{MC}) = \frac{1}{3}$. Answers may vary. Example: There are three equally likely outcomes for the type of test. One of the outcomes is multiple choice.

3. $P(10 \text{ questions}) = \frac{1}{4}$. Answers may vary. Example: There are four equally likely outcomes for the number of questions. One of the outcomes is ten questions.

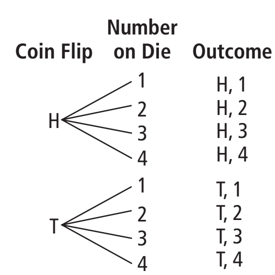
4. (MC, 5), (MC, 10), (MC, 15), (MC, 20), (SA, 5), (SA, 10), (SA, 15), (SA, 20), (MC & SA, 5), (MC & SA, 10), (MC & SA, 15), (MC & SA, 20)

5. a) 12
b) $P(\text{MC \& SA, 20}) = \frac{1}{12}$, $8.\overline{3}\%$, or $0.08\overline{3}$. Explanations may vary.
Example: There are 12 different equally likely possible outcomes. One of the outcomes is the favourable outcome that students will write a combined multiple choice–short answer test with 20 questions. The probability is equal to the number of favourable outcomes divided by the total number of possible outcomes.
c) $P(\text{MC, at least 10 questions}) = \frac{1}{4}$, 25%, or 0.25.
d) $P(\text{not SA, 15}) = \frac{11}{12}$, $91.\overline{6}\%$ or $0.91\overline{6}$. Expanations may vary.
Example: There are 11 favourable outcomes out of 12 possible outcomes.

Show You Know: Example 1

a) (H, 1), (H, 2), (H, 3), (H, 4), (T, 1), (T, 2), (T, 3), (T, 4)

b) $P(\text{H}, 4) = \frac{1}{8}$



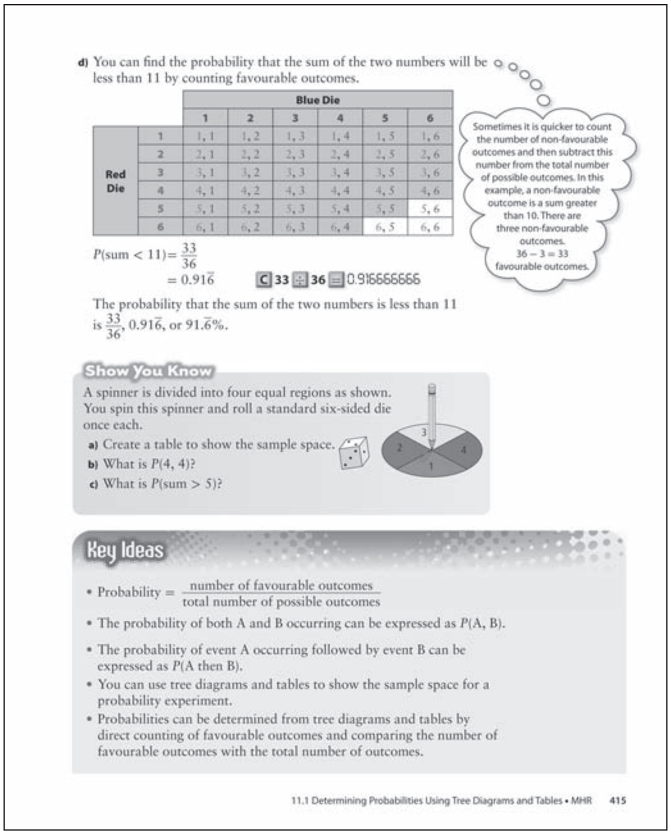
Show You Know: Example 2

a)

		Die					
		1	2	3	4	5	6
Spinner	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6

b) $P(4, 4) = \frac{1}{24}$ c) $P(\text{sum} > 5) = \frac{7}{12}$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusions about their findings. Check that students are able to convert fractions, decimals, and percents.	• Encourage students to use a different coloured pencil to highlight the favourable outcomes for #2 and #3. Ensure that they understand the difference between all possible outcomes and favourable outcomes. Remind students that favourable outcomes cannot be greater than the total of the possible outcomes. • You may wish to clarify the meaning of *least* and *not* as they apply to probability in #5c) and d). • Help students recall how to convert between fractions, decimals, and percents with and without using technology.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize the process for setting up the sample space. • You may wish to have students work with a partner. • Coach students to determine all the possible outcomes first and then determine the number of favourable outcomes. • Encourage visual learners to use two different coloured pencils (one for the coin and another for the die) in the sample space so they can readily see the different combinations.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students to use a ruler for creating the table. • Ensure that students draw a table large enough to record the data legibly. • Consider allowing students to use a computer and spreadsheet software to create the table. • Encourage visual learners to use two different coloured pencils (one for the spinner and another for the die) in the sample space so they can readily see the different combinations.



d) You can find the probability that the sum of the two numbers will be less than 11 by counting favourable outcomes.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Sometimes it is quicker to count the number of non-favourable outcomes and then subtract this number from the total number of possible outcomes. In this example, a non-favourable outcome is a sum greater than 10. There are three non-favourable outcomes. $36 - 3 = 33$ favourable outcomes.

$P(\text{sum} < 11) = \frac{33}{36}$
$= 0.91\overline{6}$

C 33 ÷ 36 = 0.916666666

The probability that the sum of the two numbers is less than 11 is $\frac{33}{36}$, $0.91\overline{6}$, or $91.\overline{6}\%$.

Show You Know

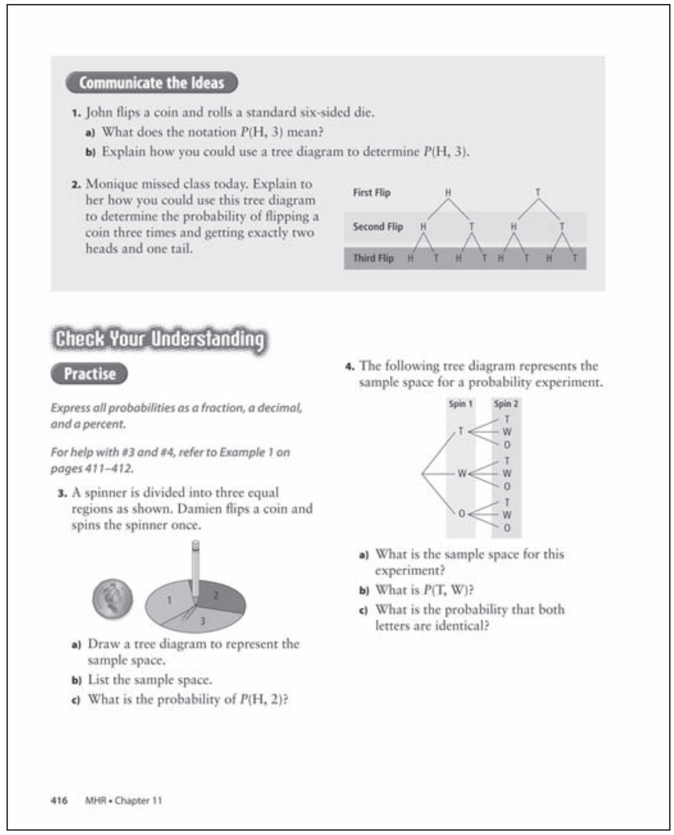
A spinner is divided into four equal regions as shown. You spin this spinner and roll a standard six-sided die once each.

a) Create a table to show the sample space.
b) What is $P(4, 4)$?
c) What is $P(\text{sum} > 5)$?

Key Ideas

- Probability $= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$
- The probability of both A and B occurring can be expressed as P(A, B).
- The probability of event A occurring followed by event B can be expressed as P(A then B).
- You can use tree diagrams and tables to show the sample space for a probability experiment.
- Probabilities can be determined from tree diagrams and tables by direct counting of favourable outcomes and comparing the number of favourable outcomes with the total number of outcomes.

11.1 Determining Probabilities Using Tree Diagrams and Tables • MHR 415



Communicate the Ideas

1. John flips a coin and rolls a standard six-sided die.
   a) What does the notation P(H, 3) mean?
   b) Explain how you could use a tree diagram to determine P(H, 3).
2. Monique missed class today. Explain to her how you could use this tree diagram to determine the probability of flipping a coin three times and getting exactly two heads and one tail.

Check Your Understanding

Practise

Express all probabilities as a fraction, a decimal, and a percent.

For help with #3 and #4, refer to Example 1 on pages 411–412.

3. A spinner is divided into three equal regions as shown. Damien flips a coin and spins the spinner once.

   a) Draw a tree diagram to represent the sample space.
   b) List the sample space.
   c) What is the probability of P(H, 2)?
4. The following tree diagram represents the sample space for a probability experiment.

   a) What is the sample space for this experiment?
   b) What is P(T, W)?
   c) What is the probability that both letters are identical?

416 MHR • Chapter 11

Key Ideas

The Key Ideas summarize the definition of probability and how to use tree diagrams and tables to show the sample space for a probability experiment. Ensure that students are clear on the definition of probability and using probability notation, P(A, B). In the student resource, the P is italicized, but the events are not. Have students prepare their own summary of the Key Ideas and record them in the section 11.1 booklet in their chapter Foldable.

Communicate the Ideas

Have students work on their own or with a partner to answer the questions. The questions allow students to apply their understanding of probability notation and the use of tree diagrams. Have students share their answers for #2 in a class discussion.

Meeting Student Needs

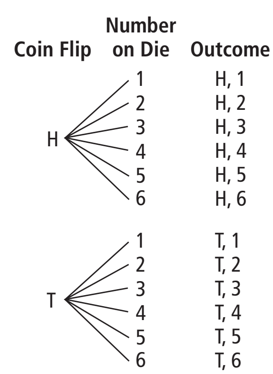
- Consider allowing concrete and kinesthetic learners to use a coin and a six-sided die, if necessary, to help them record possible outcomes.

Answers

Communicate the Ideas

1. a) Answers may vary. Example: P(H, 3) refers to the probability that the flip of the coin results in a head and the roll of the die results in a three.

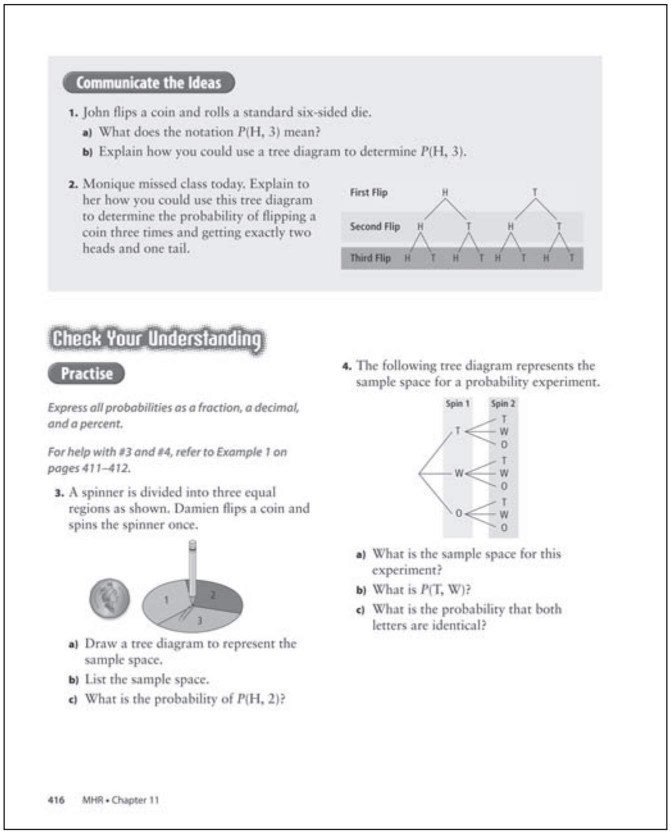
b) Answers may vary. Example:



The final column lists the possible outcomes. The probability will be the ratio of the number of favourable outcomes, 1, to the number of possible outcomes, 12, which is $P(\text{H}, 3) = \frac{1}{12}$.

2. Answers may vary. Example: Count the number of favourable outcomes, three, by looking down each of the eight possible pathways. The number of possible outcomes is eight. The probability P(2H and 1T in any order) $= \frac{3}{8}$.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1.	• Check that students understand the meaning of *P*(H, 3). • Have students verbalize how to set up a sample space for #1. • For #1, visual learners may benefit from using two different coloured pencils to set up a sample space so they can quickly see the different combinations. • Some students may benefit from developing a response to #2, sharing their response orally with a partner, and listening to each other's explanation.



Communicate the Ideas

1. John flips a coin and rolls a standard six-sided die.
   a) What does the notation *P*(H, 3) mean?
   b) Explain how you could use a tree diagram to determine *P*(H, 3).

2. Monique missed class today. Explain to her how you could use this tree diagram to determine the probability of flipping a coin three times and getting exactly two heads and one tail.

Check Your Understanding

Practise

Express all probabilities as a fraction, a decimal, and a percent.

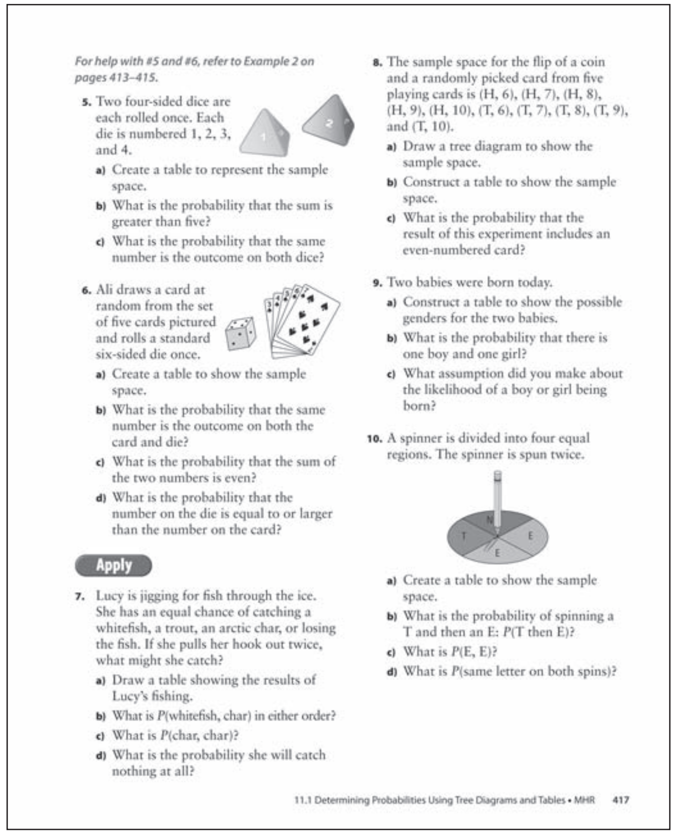
For help with #3 and #4, refer to Example 1 on pages 411–412.

3. A spinner is divided into three equal regions as shown. Damien flips a coin and spins the spinner once.
   a) Draw a tree diagram to represent the sample space.
   b) List the sample space.
   c) What is the probability of *P*(H, 2)?

4. The following tree diagram represents the sample space for a probability experiment.

   a) What is the sample space for this experiment?
   b) What is *P*(T, W)?
   c) What is the probability that both letters are identical?

416 MHR • Chapter 11



For help with #5 and #6, refer to Example 2 on pages 413–415.

5. Two four-sided dice are each rolled once. Each die is numbered 1, 2, 3, and 4.
   a) Create a table to represent the sample space.
   b) What is the probability that the sum is greater than five?
   c) What is the probability that the same number is the outcome on both dice?

6. Ali draws a card at random from the set of five cards pictured and rolls a standard six-sided die once.
   a) Create a table to show the sample space.
   b) What is the probability that the same number is the outcome on both the card and die?
   c) What is the probability that the sum of the two numbers is even?
   d) What is the probability that the number on the die is equal to or larger than the number on the card?

Apply

7. Lucy is jigging for fish through the ice. She has an equal chance of catching a whitefish, a trout, an arctic char, or losing the fish. If she pulls her hook out twice, what might she catch?
   a) Draw a table showing the results of Lucy's fishing.
   b) What is *P*(whitefish, char) in either order?
   c) What is *P*(char, char)?
   d) What is the probability she will catch nothing at all?

8. The sample space for the flip of a coin and a randomly picked card from five playing cards is (H, 6), (H, 7), (H, 8), (H, 9), (H, 10), (T, 6), (T, 7), (T, 8), (T, 9), and (T, 10).
   a) Draw a tree diagram to show the sample space.
   b) Construct a table to show the sample space.
   c) What is the probability that the result of this experiment includes an even-numbered card?

9. Two babies were born today.
   a) Construct a table to show the possible genders for the two babies.
   b) What is the probability that there is one boy and one girl?
   c) What assumption did you make about the likelihood of a boy or girl being born?

10. A spinner is divided into four equal regions. The spinner is spun twice.

   a) Create a table to show the sample space.
   b) What is the probability of spinning a T and then an E: *P*(T then E)?
   c) What is *P*(E, E)?
   d) What is *P*(same letter on both spins)?

11.1 Determining Probabilities Using Tree Diagrams and Tables • MHR 417

Check Your Understanding

Practise

For #5b), ensure that students understand the meaning of *sum* and the associated operation, and remind them not to include sums equal to five.

Apply

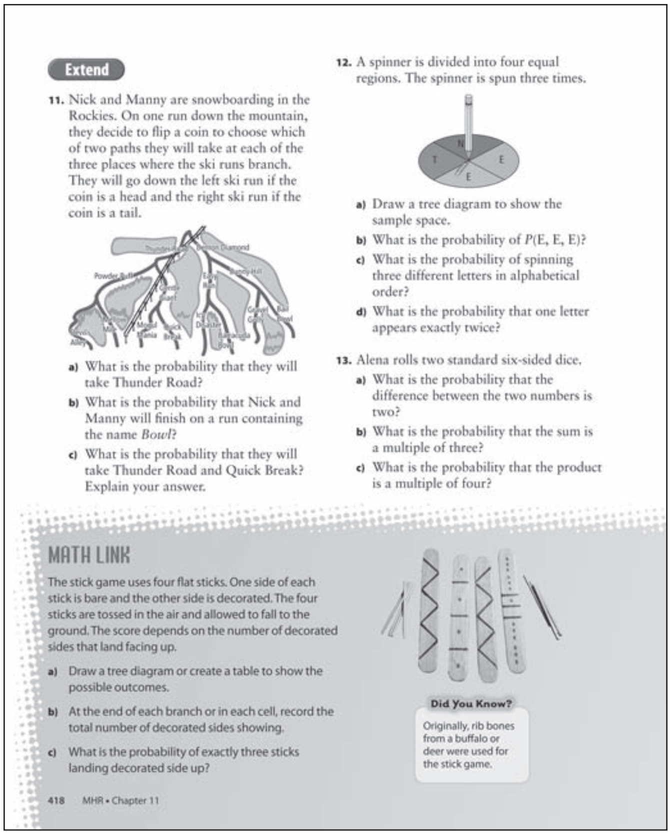
These questions provide a variety of contexts for students to determine and represent the sample space for different situations using tree diagrams and tables.

Extend

In #11, prompt students to notice that the snowboard trails resemble a tree diagram. This question does not represent independent events. After the first set of two branches, the likelihood for remaining ski runs depends on the previously chosen run. For example, if Thunder Road is chosen, then the probability of skiing Easy Run is zero. However, if Demon Diamond is chosen, then there is a 50% probability of skiing Easy Run. These events are therefore dependent, not independent. Students will explore this type of probability in high school.

Math Link

The Math Link provides students with an opportunity to use tree diagrams and tables to represent possible outcomes. Prompt students to realize that there should be four sets of branches, one for each stick. Some students may benefit from considering how the game might be played using a single stick (i.e., flip it four times).



Extend

11. Nick and Manny are snowboarding in the Rockies. On one run down the mountain, they decide to flip a coin to choose which of two paths they will take at each of the three places where the ski runs branch. They will go down the left ski run if the coin is a head and the right ski run if the coin is a tail.

a) What is the probability that they will take Thunder Road?
b) What is the probability that Nick and Manny will finish on a run containing the name *Bowl*?
c) What is the probability that they will take Thunder Road and Quick Break? Explain your answer.

12. A spinner is divided into four equal regions. The spinner is spun three times.

a) Draw a tree diagram to show the sample space.
b) What is the probability of *P*(E, E, E)?
c) What is the probability of spinning three different letters in alphabetical order?
d) What is the probability that one letter appears exactly twice?

13. Alena rolls two standard six-sided dice.

a) What is the probability that the difference between the two numbers is two?
b) What is the probability that the sum is a multiple of three?
c) What is the probability that the product is a multiple of four?

MATH LINK

The stick game uses four flat sticks. One side of each stick is bare and the other side is decorated. The four sticks are tossed in the air and allowed to fall to the ground. The score depends on the number of decorated sides that land facing up.

a) Draw a tree diagram or create a table to show the possible outcomes.
b) At the end of each branch or in each cell, record the total number of decorated sides showing.
c) What is the probability of exactly three sticks landing decorated side up?

Did You Know?

Originally, rib bones from a buffalo or deer were used for the stick game.

418 MHR • Chapter 11

Meeting Student Needs

- Some students may struggle with the text-dense questions in the Practice and Apply sections. Help students extract the information they need to answer each question. You may wish to have them underline the key words in pencil so that they can erase the marks later. Alternatively, allow students to work in pairs.
- Provide **BLM 11–5 Section 11.1 Extra Practice** to students who would benefit from more practice.

ELL

- For #11, explain snowboarding to new Canadians, including how one gets up the hill, the types of maps snowboarders use, and how to get down the hill.
- While doing the Math Link, have English language learners add the word *decorated* to their dictionary.

Gifted and Enrichment

- Challenge students to use the Web Link on this page to help determine what percent of babies are born male and female in Canada.
- Encourage students to research traditional stick games by addressing some or all of the following questions: What types of decorations are found on the authentic Aboriginal sticks? What variations of the stick game exist? Who plays the stick game?
- As an extension to the Math Link, consider having students research other traditional games of chance. For example, students could research a First Nations game of chance such as Sllekmew'es (Lahal). Students might learn to play the game and then create a tree diagram or table to represent the sample space.

Common Errors

- Some students may want to skip drawing tree diagrams and tables because they know some of the probabilities.

R_x Require students to draw tree diagrams and tables accurately to represent a sample space. In later sections and later math courses, students will realize that tree diagrams and tables are essential in solving more difficult probability problems.

WWW Web Link

For data about birth rates in Canada by gender, go to www.mathlinks8.ca and follow the links.

Answers

Math Link

a), b)

Stick 1	Stick 2	Stick 3	Stick 4	Outcome	Number of Decorated Sides
D	D	D	D	D, D, D, D	4
			B	D, D, D, B	3
		B	D	D, D, B, D	3
			B	D, D, B, B	2
	B	D	D	D, B, D, D	3
			B	D, B, D, B	2
		B	D	D, B, B, D	2
			B	D, B, B, B	1
B	D	D	D	B, D, D, D	3
			B	B, D, D, B	2
		B	D	B, D, B, D	2
			B	B, D, B, B	1
	B	D	D	B, B, D, D	2
			B	B, B, D, B	1
		B	D	B, B, B, D	1
			B	B, B, B, B	0

c) $\frac{1}{4}$

Assessment	Supporting Learning
Assessment *for* Learning	
Practise Have students do #3 and #5. Students who have no problems with these questions can go on to the Apply questions.	• Encourage students who have difficulties with #3 and #5 to refer back to the examples. • Provide additional coaching with Example 1 to students who need help with #3. You might have students use a systematic approach and verbalize each possible outcome before listing it as part of the sample space. Work with them to correct #3 and then have them do #4. • Provide additional coaching with Example 2 to students who need help with #5. Work with them to correct #5, and then have them do #6. • Allow students to use manipulatives to help them visualize possible outcomes.
Math Link The Math Link on page 418 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 439.	• Have students verbalize their thinking. • Provide coaching to students who have difficulties constructing the tree diagram. Each of the four sticks will need two branches—one for the decorated side (D) and one for the bare side (B). • Have students highlight the favourable outcomes so they are easy to see. • You may wish to have students use four craft sticks and colour one side of each one to help them visualize the possible outcomes. Letting students try playing the game may help concrete, kinesthetic, and visual learners link the concepts. • To help them get started, some students may benefit from using **BLM 11–6 Section 11.1 Math Link**, which provides scaffolding for this activity.
Assessment *as* Learning	
Math Learning Log Have students answer the following questions: • Define sample space. How is a sample space helpful? • Design a probability experiment that requires a table with two rows and six columns. Develop a probability problem that can be solved by using the table.	• Many students may choose a coin flip and roll of a six-sided die for their probability experiment. • Encourage concrete and kinesthetic learners to use manipulatives, and then to draw the table to show the possible outcomes. • For some students, you may need to highlight how the number of rows and columns in the table relates to the possible outcomes for a single event. Make sure that each outcome in a single event is equally likely. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.