11.2 Outcomes of Independent Events

***MathLinks 8*, pages 419–425**

Suggested Timing
80–100 minutes

Materials
- ruler
- nine small pieces of paper
- coloured pencils
- coloured counters
- manipulatives including coins, six-sided dice, four-sided dice, spinners, marbles, bags, and playing cards (optional)
- craft sticks

Blackline Masters
BLM 11–3 Chapter 11 Warm-Up
BLM 11–7 Section 11.2 Extra Practice
BLM 11–8 Section 11.2 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

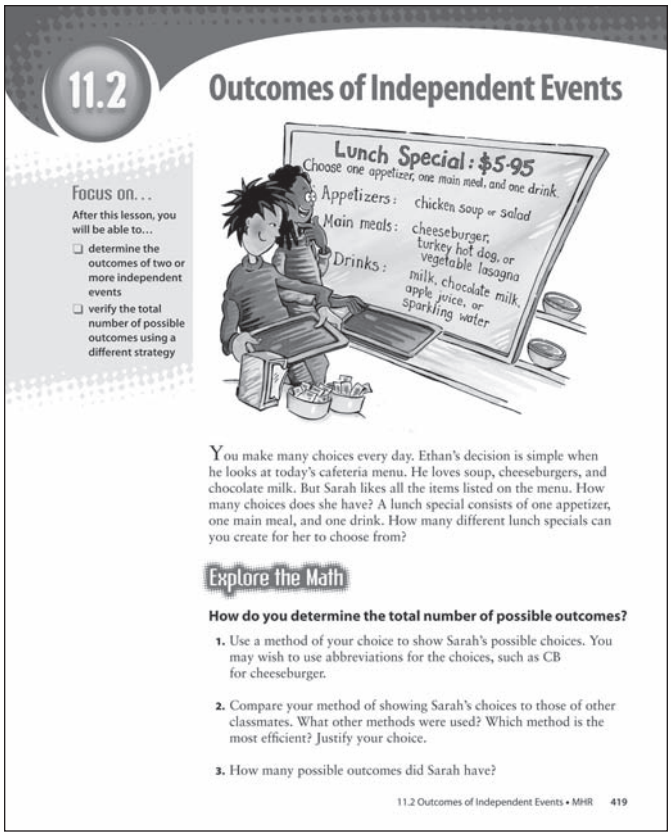
Specific Outcomes
SP2 Solve problems involving the probability of independent events.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–3, 5, 7, 10, Math Link
Typical	1–3, 5, 7–12, Math Link
Extension/Enrichment	1, 2, 10–16, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 11–3 Chapter 11 Warm-Up** to reinforce material learned in previous sections.

As a class, read and discuss the information about lunch specials as an introduction to the Explore the Math.



11.2 Outcomes of Independent Events

Focus on...
After this lesson, you will be able to...
- determine the outcomes of two or more independent events
- verify the total number of possible outcomes using a different strategy

Lunch Special: $5.95
Choose one appetizer, one main meal, and one drink.
Appetizers: chicken soup or salad
Main meals: cheeseburger, turkey hot dog, or vegetable lasagna
Drinks: milk, chocolate milk, apple juice, or sparkling water

You make many choices every day. Ethan's decision is simple when he looks at today's cafeteria menu. He loves soup, cheeseburgers, and chocolate milk. But Sarah likes all the items listed on the menu. How many choices does she have? A lunch special consists of one appetizer, one main meal, and one drink. How many different lunch specials can you create for her to choose from?

Explore the Math

How do you determine the total number of possible outcomes?

1. Use a method of your choice to show Sarah's possible choices. You may wish to use abbreviations for the choices, such as CB for cheeseburger.
2. Compare your method of showing Sarah's choices to those of other classmates. What other methods were used? Which method is the most efficient? Justify your choice.
3. How many possible outcomes did Sarah have?

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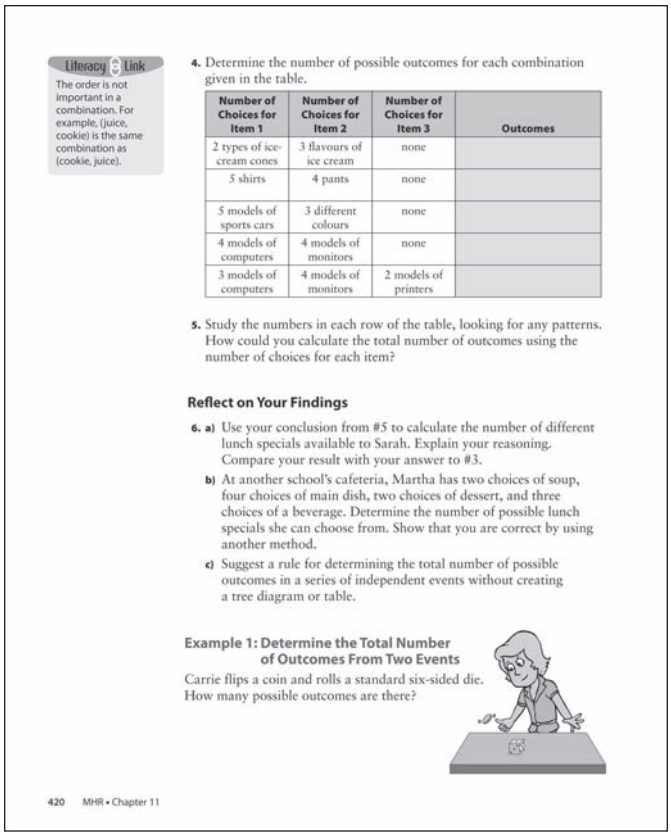
Literacy Link Direct students to the Literacy Link on page 420, which explains that order is not important in a combination.

Explore the Math

In this exploration, students use multiplication to determine the total number of possible outcomes when two or more independent events occur. Focus on counting the number of possible outcomes for each event, and on how the product relates to the total number of possible combinations. Tree diagrams are effective visual tools to show how all of the combinations can be counted.

Throughout this section, students are asked to use two or three methods (i.e., tree diagrams, tables, multiplication) to determine the number of possible outcomes.



Literacy Link
The order is not important in a combination. For example, (juice, cookie) is the same combination as (cookie, juice).

4. Determine the number of possible outcomes for each combination given in the table.

Number of Choices for Item 1	Number of Choices for Item 2	Number of Choices for Item 3	Outcomes
2 types of ice-cream cones	3 flavours of ice cream	none	
5 shirts	4 pants	none	
5 models of sports cars	3 different colours	none	
4 models of computers	4 models of monitors	none	
3 models of computers	4 models of monitors	2 models of printers	

5. Study the numbers in each row of the table, looking for any patterns. How could you calculate the total number of outcomes using the number of choices for each item?

Reflect on Your Findings

6. a) Use your conclusion from #5 to calculate the number of different lunch specials available to Sarah. Explain your reasoning. Compare your result with your answer to #3.
b) At another school's cafeteria, Martha has two choices of soup, four choices of main dish, two choices of dessert, and three choices of a beverage. Determine the number of possible lunch specials she can choose from. Show that you are correct by using another method.
c) Suggest a rule for determining the total number of possible outcomes in a series of independent events without creating a tree diagram or table.

Example 1: Determine the Total Number of Outcomes From Two Events

Carrie flips a coin and rolls a standard six-sided die. How many possible outcomes are there?

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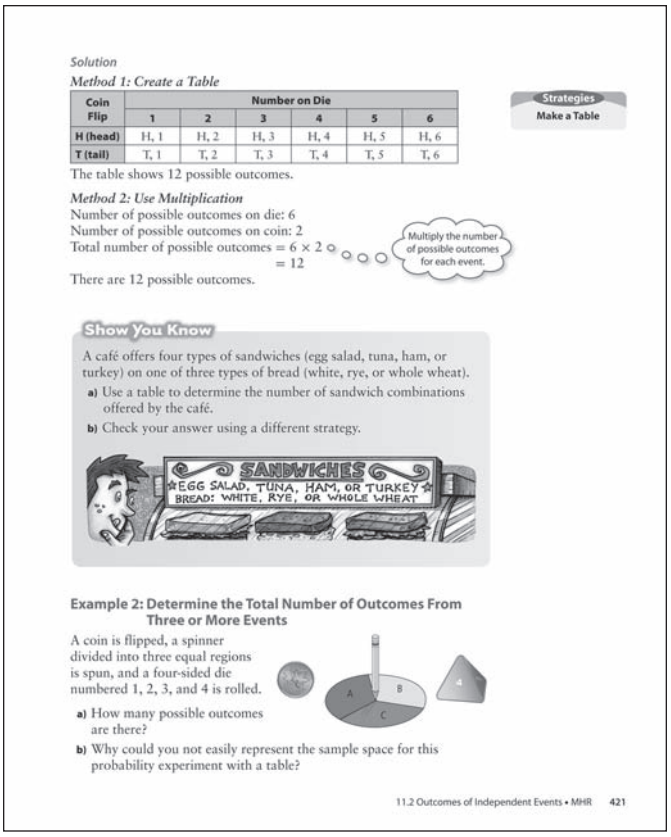

Solution

Method 1: Create a Table

Coin Flip	Number on Die					
	1	2	3	4	5	6
H (head)	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
T (tail)	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

Strategies
Make a Table

The table shows 12 possible outcomes.

Method 2: Use Multiplication

Number of possible outcomes on die: 6
Number of possible outcomes on coin: 2
Total number of possible outcomes = 6 × 2
= 12

Multiply the number of possible outcomes for each event.

There are 12 possible outcomes.

Show You Know

A café offers four types of sandwiches (egg salad, tuna, ham, or turkey) on one of three types of bread (white, rye, or whole wheat).

a) Use a table to determine the number of sandwich combinations offered by the café.
b) Check your answer using a different strategy.

Example 2: Determine the Total Number of Outcomes From Three or More Events

A coin is flipped, a spinner divided into three equal regions is spun, and a four-sided die numbered 1, 2, 3, and 4 is rolled.

a) How many possible outcomes are there?
b) Why could you not easily represent the sample space for this probability experiment with a table?

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Method 1 Have students work in pairs or small groups. Ask them to consider what methods they could use to show the possible choices, choose the one that they think works best, and try it. Have students present their methods to the class Ask:

- What are the advantages of this method of showing possible choices?
- What are the disadvantages?
- Are there other methods? (Discuss them.)
- Which method is the most efficient for this type of problem? Justify your thinking.

Suggest that students use the method that the class decided was the most efficient, or a method of their choice, to answer #4. Alternatively, they may wish to use different methods of showing possible outcomes for each row.

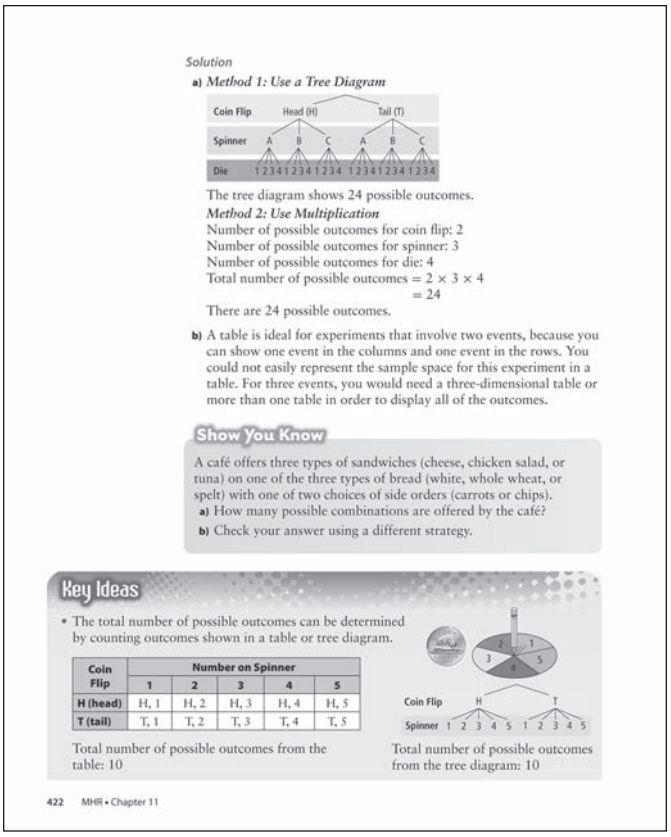
Challenge students to organize the data they are developing so that they can easily identify any patterns. You may wish to ask some students how the table on page 420 could help them organize their data.

Working with small groups and then the large group, have students develop generalizations about how they could calculate the number of outcomes in any experiment without writing down every possible outcome. Discuss their ideas. Have them test each one. Which ideas work for all of the situations in the table? Which do not?

Method 2 Have concrete and kinesthetic learners work in pairs to write each menu choice on separate pieces of paper and then use the choices to help them determine the possible outcomes on a tree diagram or in a table. Visual learners may benefit from using different coloured pencils to set up the sample space so they can quickly see the different combinations. For #4, allow students to use different coloured counters to represent each situation to help them visualize the answers.

Example 1

This example illustrates two methods (creating a table and using multiplication) to determine the number of outcomes from two events. Work through the solution with the class. Ensure that students understand that the numbers they multiply are the number of possible outcomes from each event. On an overhead copy of the table, consider displaying two coins and six dice (one in each corresponding cell of the table) that show the number of possible outcomes to help students' understanding. Have students consider that calculating the total number of outcomes from a table is similar to using an area model to calculate area; i.e., multiply the number of rows (length) by the number of columns (width).



Solution

a) *Method 1: Use a Tree Diagram*

The tree diagram shows 24 possible outcomes.

Method 2: Use Multiplication

Number of possible outcomes for coin flip: 2
Number of possible outcomes for spinner: 3
Number of possible outcomes for die: 4
Total number of possible outcomes = $2 \times 3 \times 4$
$= 24$

There are 24 possible outcomes.

b) A table is ideal for experiments that involve two events, because you can show one event in the columns and one event in the rows. You could not easily represent the sample space for this experiment in a table. For three events, you would need a three-dimensional table or more than one table in order to display all of the outcomes.

Show You Know

A café offers three types of sandwiches (cheese, chicken salad, or tuna) on one of the three types of bread (white, whole wheat, or spelt) with one of two choices of side orders (carrots or chips).

a) How many possible combinations are offered by the café?

b) Check your answer using a different strategy.

Key Ideas

- The total number of possible outcomes can be determined by counting outcomes shown in a table or tree diagram.

Coin Flip	Number on Spinner				
	1	2	3	4	5
H (head)	H, 1	H, 2	H, 3	H, 4	H, 5
T (tail)	T, 1	T, 2	T, 3	T, 4	T, 5

Total number of possible outcomes from the table: 10

Total number of possible outcomes from the tree diagram: 10

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Example 2

This example illustrates using multiplication to determine the number of outcomes from three or more events. Work through the solution as a class. For part a), you might ask students to create a table to verify the answer. They should quickly notice that it is impossible to represent three events using a table unless you use a three-dimensional table. For more than two independent events, multiplying the number of possible outcomes for each event is the best way to quickly calculate the total number of possible outcomes.

Meeting Student Needs

- You may wish to allow students who have trouble drawing tree diagrams to use virtual manipulatives.
- Allow concrete and kinesthetic learners to use manipulatives to help them visualize the possible outcomes.
- Some students may benefit from using a virtual manipulative such as the one described in the Web Link on this page to practise determining the number of possible outcomes.
- Some students may benefit from completing additional problems to reinforce their learning. Consider using scenarios relevant to students where they live. For example: Tamara and her family went to purchase a meal for $5.95. Tamara's choices for the main meal were a taco, bannock burger, or salmon. She had a choice of either a baked potato or a salad, and for dessert she had a choice of blueberries, Saskatoon berries, or raspberries. In addition, she had a choice of mint tea, milk, or cranberry juice. How many different combinations are possible? ($3 \times 2 \times 3 \times 3 = 54$)

ELL

- Ensure that English language learners understand the following terms: *appetizer*, *combinations*, *verify*, *efficient*, *randomly*, and *chosen*. Have students add any new terms to their dictionary.
- This lesson is very text dense. Give English language learners the opportunity to answer #6 in their own language or using visuals. Then, have them use the visuals to explain their thoughts orally in English.

Common Errors

- Some students may confuse favourable outcomes with possible outcomes.

R_x Remind students that favourable outcomes are successful results in a probability experiment. The number of possible outcomes is the total number of outcomes for two or more independent events. Use an example such as the table in Example 1. There are 12 possible outcomes. Determining the probability of getting a head and a number six, which is expressed as $P(H, 6)$, is an example of a favourable outcome. Refer students to the Key Ideas on page 415 in section 11.1.

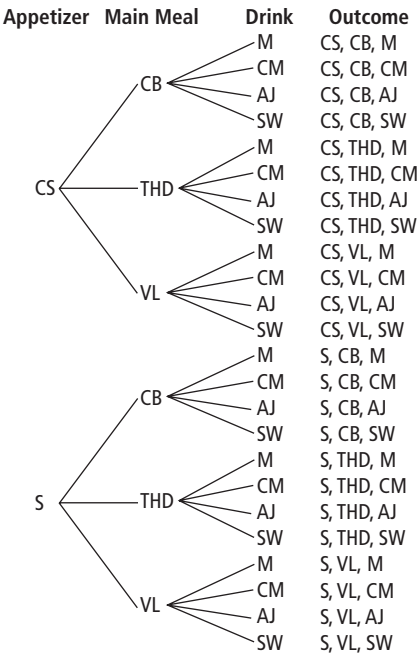
Web Link

For an interactive activity in which students practise using multiplication to determine the number of outcomes from two or more events, go to www.mathlinks8.ca and follow the links. Explain that the term *counting principle* means using multiplication to determine possible outcomes.

Answers

Explore the Math

1. Answers will vary. Students may choose some type of organizer (e.g., table, tree diagram). Example:



2. Answers may vary depending on the methods used. Example: A tree diagram is the most efficient. When there are more than two events it is difficult to display the possible outcomes in a table.

3. The total number of possible outcomes is 24.

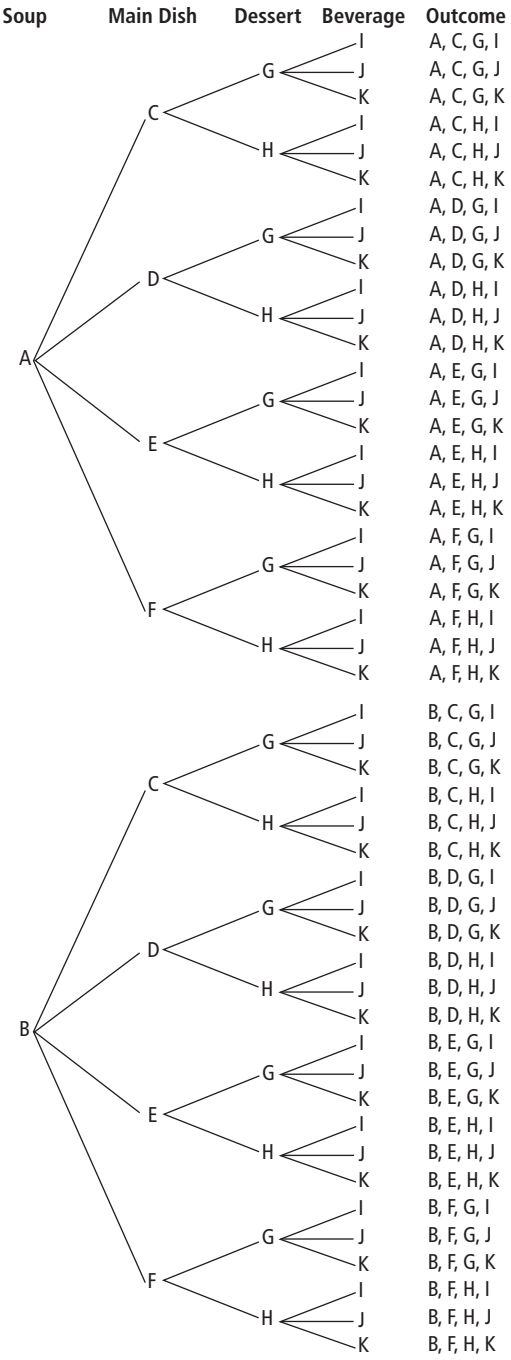
4. Students may use different strategies to determine the following possible outcomes:
- 6 cone-flavour combinations
- 20 shirt-pant combinations
- 15 sports car-colour combinations
- 16 computer-monitor combinations
- 24 computer-monitor-printer combinations

5. Answers may vary. Example: Multiply the number of choices for each item together to calculate the total number of outcomes.

6. a) The total number of possible outcomes is $2 \times 3 \times 4 = 24$. Answers may vary. Example: Determine the product of the number of appetizers, two, by the number of main meals, three, by the number of drinks, four.

b) Answers will vary. Students should use two methods (a tree diagram or multiplication), one to determine the possible outcomes and the other to check them. Example:
- Using multiplication: The total number of possible outcomes is $2 \times 4 \times 2 \times 3 = 48$.
- Using a tree diagram: There are 48 possible outcomes.

Students should use a tree diagram or multiplication to verify the answer.



c) Answers may vary. Example: Determine the total number of possible outcomes by multiplying the number of possible outcomes for each event.

Answers

Show You Know: Example 1

a) The table shows 12 possible sandwich combinations.

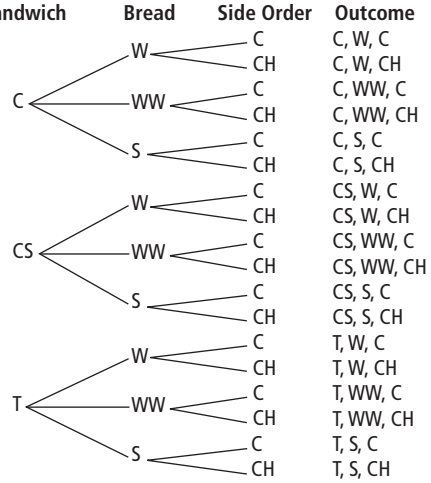
		Sandwich			
		Egg (E)	Tuna (T)	Ham (H)	Turkey (TY)
Bread	White (W)	W, E	W, T	W, H	W, TY
	Rye (R)	R, E	R, T	R, H	R, TY
	Whole wheat (WW)	WW, E	WW, T	WW, H	WW, TY

b) Answers may vary. Example: Multiply the number of possible outcomes of sandwiches, four, by the number of possible outcomes of bread, three. The total number of possible outcomes is $4 \times 3 = 12$.

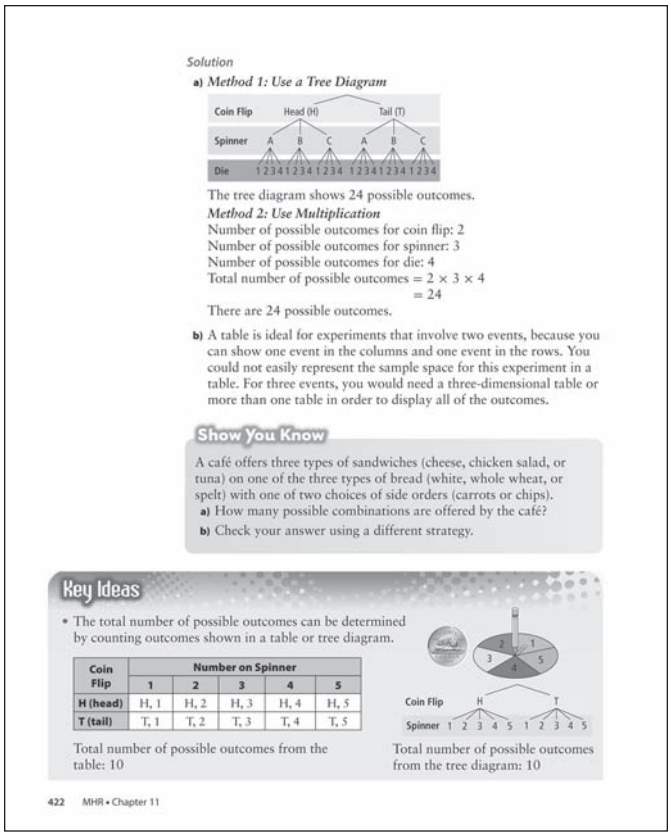
Show You Know: Example 2

a) 18 possible combinations

b) Answers may vary. Look for a tree diagram or using multiplication. Example:
- Draw a tree diagram to show 18 possible outcomes.
- Using multiplication, multiply the number of possible outcomes of sandwiches, three, by the number of possible outcomes of bread, three, by the number of possible outcomes of side orders, two. The total number of possible outcomes is $3 \times 3 \times 2 = 18$.



Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math, in particular as they discuss a response to #6c). For this question, students describe the multiplication method in their own words.	• Help students realize that the product of the possible outcomes for each independent event represents the total number of possible outcomes in a probability experiment. • Have students write the multiplication statement to determine the total number of possible outcomes for each situation in #5.
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize each sandwich combination. • You may wish to have students work with a partner. • Allow concrete and kinesthetic learners to use manipulatives such as different coloured counters, if necessary, to help them identify the possible outcomes. • Have students write the multiplication statement. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize each possible combination. • You may wish to have students work with a partner. • Allow concrete and kinesthetic learners to use manipulatives, if necessary, to help them identify the possible outcomes. • Coach students who need help to construct a tree diagram for three independent events. • Have students write the multiplication statement. • Discuss how this Show You Know is similar to and different from the Show You Know for Example 1. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.



Solution

a) *Method 1: Use a Tree Diagram*

The tree diagram shows 24 possible outcomes.

Method 2: Use Multiplication

Number of possible outcomes for coin flip: 2
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Number of possible outcomes for die: 4
Total number of possible outcomes $= 2 \times 3 \times 4$
$= 24$

There are 24 possible outcomes.

b) A table is ideal for experiments that involve two events, because you can show one event in the columns and one event in the rows. You could not easily represent the sample space for this experiment in a table. For three events, you would need a three-dimensional table or more than one table in order to display all of the outcomes.

Show You Know

A café offers three types of sandwiches (cheese, chicken salad, or tuna) on one of the three types of bread (white, whole wheat, or spelt) with one of two choices of side orders (carrots or chips).

a) How many possible combinations are offered by the café?

b) Check your answer using a different strategy.

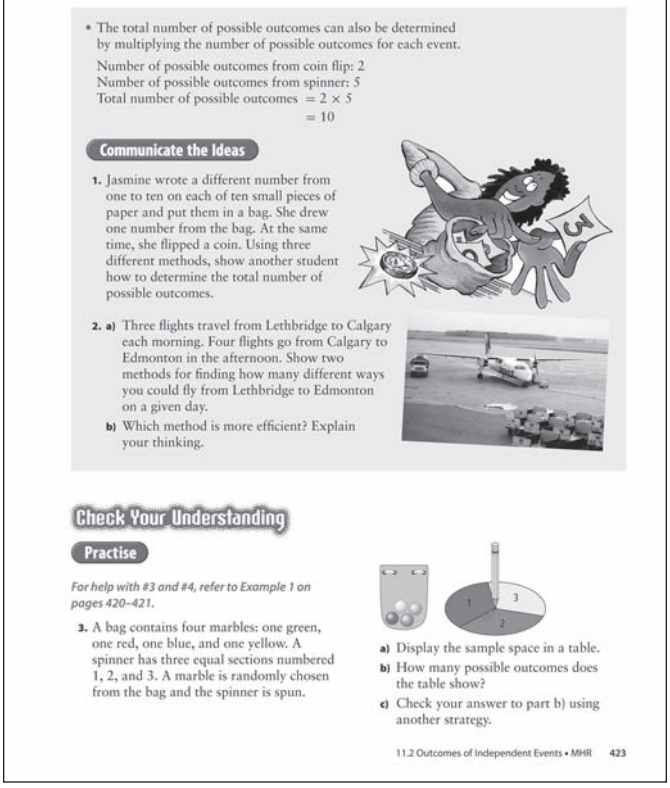
Key Ideas

- The total number of possible outcomes can be determined by counting outcomes shown in a table or tree diagram.

Coin Flip	Number on Spinner				
	1	2	3	4	5
H (head)	H, 1	H, 2	H, 3	H, 4	H, 5
T (tail)	T, 1	T, 2	T, 3	T, 4	T, 5

Total number of possible outcomes from the table: 10

Total number of possible outcomes from the tree diagram: 10



- The total number of possible outcomes can also be determined by multiplying the number of possible outcomes for each event.

Number of possible outcomes from coin flip: 2
Number of possible outcomes from spinner: 5
Total number of possible outcomes $= 2 \times 5$
$= 10$

Communicate the Ideas

1. Jasmine wrote a different number from one to ten on each of ten small pieces of paper and put them in a bag. She drew one number from the bag. At the same time, she flipped a coin. Using three different methods, show another student how to determine the total number of possible outcomes.

2. a) Three flights travel from Lethbridge to Calgary each morning. Four flights go from Calgary to Edmonton in the afternoon. Show two methods for finding how many different ways you could fly from Lethbridge to Edmonton on a given day.

b) Which method is more efficient? Explain your thinking.

Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on pages 420–421.

3. A bag contains four marbles: one green, one red, one blue, and one yellow. A spinner has three equal sections numbered 1, 2, and 3. A marble is randomly chosen from the bag and the spinner is spun.

a) Display the sample space in a table.

b) How many possible outcomes does the table show?

c) Check your answer to part b) using another strategy.

Key Ideas

The Key Ideas summarize using tables, tree diagrams, and multiplication to determine the number of possible outcomes in a probability experiment. Have students prepare their own summary of the Key Ideas, including an example for each. Have them record their summary in the section 11.2 booklet in their chapter Foldable.

Communicate the Ideas

These questions allow students to demonstrate their understanding of three methods (i.e., tree diagrams, tables, multiplication) used to determine the number of possible outcomes in a multi-event probability experiment. Help students understand how the multiplication method is derived from the table or tree diagram method. For #1 and #2, drawing the possible outcomes may help.

Meeting Student Needs

- Visual learners may find it difficult to move from the more concrete step of using a table or a tree diagram to multiplication. Referring back to the Key Ideas summary of the three methods may be helpful.

Gifted and Enrichment

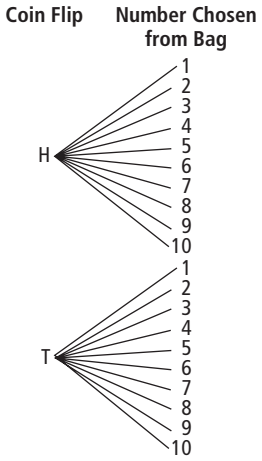
- Challenge students to solve the following problem: Bag A has three red and four black balls, Bag B has two red and five black balls. You reach into Bag A and grab a ball; without looking at it, you drop it into Bag B. Then, you reach into Bag B and grab a ball. What is the probability that this ball is red? black? Explain how the answer changes if you look at the ball before dropping it into Bag B.

Answer: $P(\text{red}) = \frac{3}{7} \times \frac{3}{8} + \frac{4}{7} \times \frac{2}{8} = \frac{17}{56}$;
$P(\text{black}) = \frac{3}{7} \times \frac{5}{8} + \frac{4}{7} \times \frac{6}{8} = \frac{39}{56}$

Answers

Communicate the Ideas

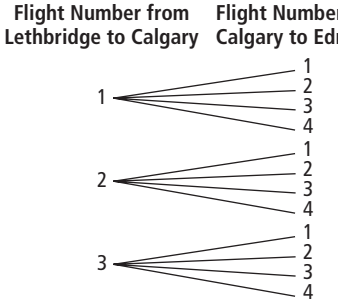
1. Look for three different methods. Example:
- Using multiplication, the total number of possible outcomes is $10 \times 2 = 20$.
- Using a tree diagram, count the number of branches, 20, in the final column.



- Using a table, count the total number of possible outcomes, 20.

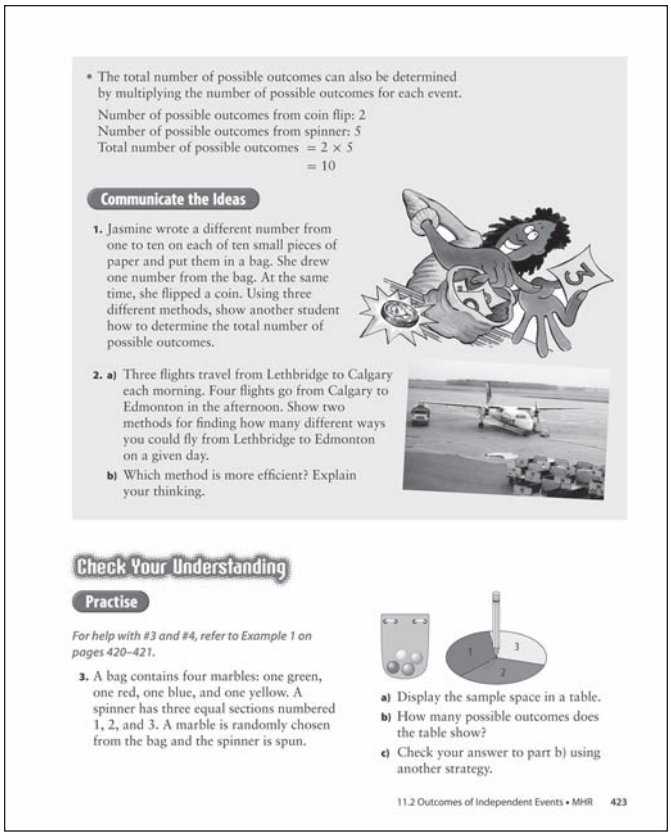
		Coin Flip	
		H	T
Number Chosen from Bag	1	1, H	1, T
	2	2, H	2, T
	3	3, H	3, T
	4	4, H	4, T
	5	5, H	5, T
	6	6, H	6, T
	7	7, H	7, T
	8	8, H	8, T
	9	9, H	9, T
	10	10, H	10, T

2. a) Answers may vary. Example:
- Using multiplication, the total number of possible outcomes is $3 \times 4 = 12$.
- Using a tree diagram, the total number of possible outcomes is 12.



b) Answers may vary. Example: Using the multiplication method requires less time and less writing and/or drawing.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete both questions. Have them share their answer to #1 with a partner and listen to each other's explanation.	• Allow students to use manipulatives to help them visualize possible outcomes. • As a class, discuss the response to #1 and demonstrate each method. • Ask students who need help with #2 to identify the two methods they are most familiar with and explain what the two methods share in common. • As a class, discuss the response to #2. Have students link the final statement in their table or tree diagram to a multiplication statement. • Have students who need help use the class responses as a springboard for their own answer. • Have students summarize the three methods for determining the total number of possible outcomes. Ask which method is the easiest to understand and why. Ask which method is the easiest to perform and why.



• The total number of possible outcomes can also be determined by multiplying the number of possible outcomes for each event.

Number of possible outcomes from coin flip: 2
Number of possible outcomes from spinner: 5
Total number of possible outcomes $= 2 \times 5$
$= 10$

Communicate the Ideas

1. Jasmine wrote a different number from one to ten on each of ten small pieces of paper and put them in a bag. She drew one number from the bag. At the same time, she flipped a coin. Using three different methods, show another student how to determine the total number of possible outcomes.

2. a) Three flights travel from Lethbridge to Calgary each morning. Four flights go from Calgary to Edmonton in the afternoon. Show two methods for finding how many different ways you could fly from Lethbridge to Edmonton on a given day.

b) Which method is more efficient? Explain your thinking.

Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on pages 420–421.

3. A bag contains four marbles: one green, one red, one blue, and one yellow. A spinner has three equal sections numbered 1, 2, and 3. A marble is randomly chosen from the bag and the spinner is spun.

a) Display the sample space in a table.

b) How many possible outcomes does the table show?

c) Check your answer to part b) using another strategy.

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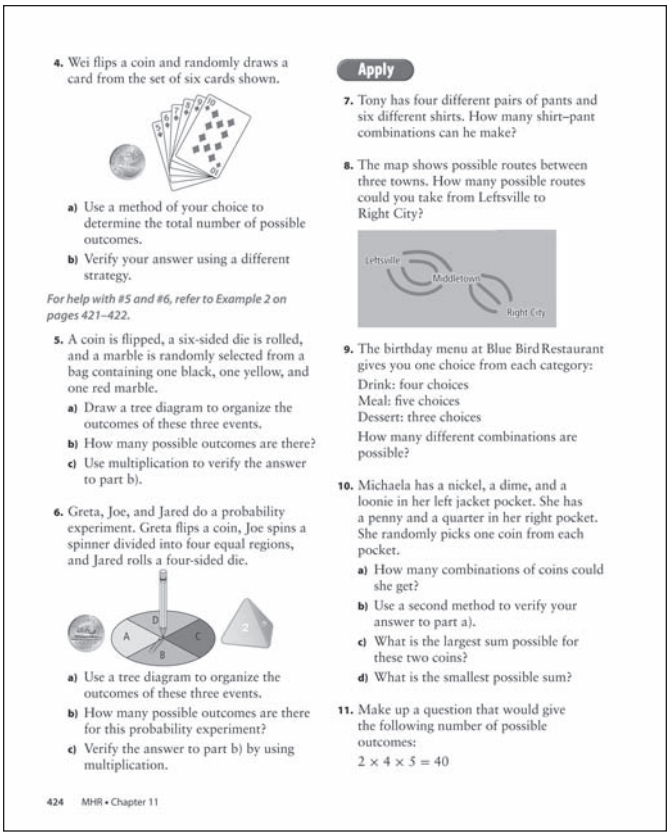

4. Wei flips a coin and randomly draws a card from the set of six cards shown.

a) Use a method of your choice to determine the total number of possible outcomes.

b) Verify your answer using a different strategy.

For help with #5 and #6, refer to Example 2 on pages 421–422.

5. A coin is flipped, a six-sided die is rolled, and a marble is randomly selected from a bag containing one black, one yellow, and one red marble.

a) Draw a tree diagram to organize the outcomes of these three events.

b) How many possible outcomes are there?

c) Use multiplication to verify the answer to part b).

6. Greta, Joe, and Jared do a probability experiment. Greta flips a coin, Joe spins a spinner divided into four equal regions, and Jared rolls a four-sided die.

a) Use a tree diagram to organize the outcomes of these three events.

b) How many possible outcomes are there for this probability experiment?

c) Verify the answer to part b) by using multiplication.

Apply

7. Tony has four different pairs of pants and six different shirts. How many shirt–pant combinations can he make?

8. The map shows possible routes between three towns. How many possible routes could you take from Leftsville to Right City?

9. The birthday menu at Blue Bird Restaurant gives you one choice from each category:

Drink: four choices
Meal: five choices
Dessert: three choices

How many different combinations are possible?

10. Michaela has a nickel, a dime, and a loonie in her left jacket pocket. She has a penny and a quarter in her right pocket. She randomly picks one coin from each pocket.

a) How many combinations of coins could she get?

b) Use a second method to verify your answer to part a).

c) What is the largest sum possible for these two coins?

d) What is the smallest possible sum?

11. Make up a question that would give the following number of possible outcomes:

$2 \times 4 \times 5 = 40$

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Check Your Understanding

Practise

For #5 and #6, the final row of the tree diagrams will include 36 and 32 branches respectively. Although you may wish to avoid assigning both of these questions, they do clearly demonstrate how time consuming it is to use a tree diagram to determine the possible outcomes for a large sample space.

Apply

Apart from #10, these questions require no tables or tree diagrams. However, consider allowing students who have difficulty using the multiplication method to use another method to verify their multiplication.

For #7, students may find it helpful to draw diagrams and link the diagrams to the corresponding multiplication statement.

Extend

Some of these questions require multiplication of several two-digit numbers.

Math Link

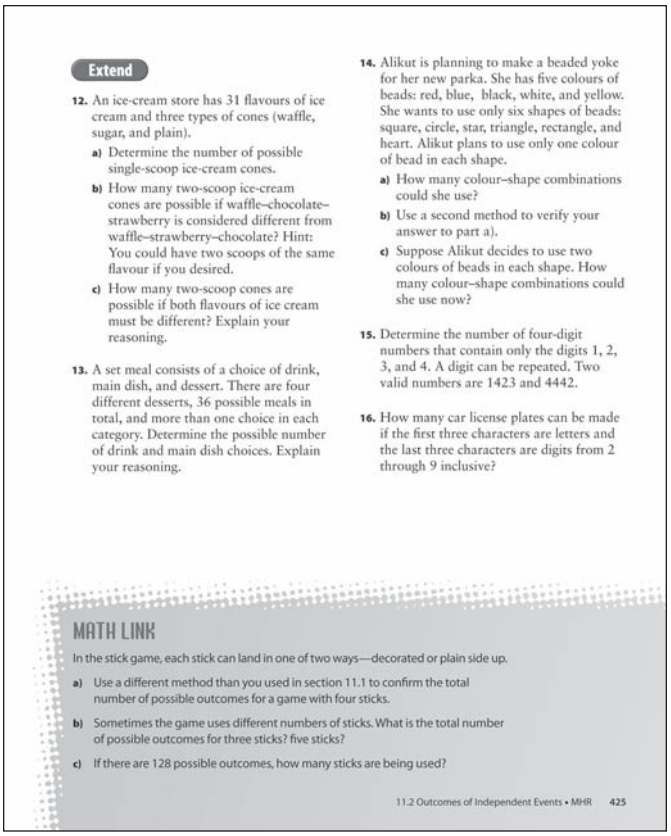
The Math Link provides students with an opportunity to verify the number of possible outcomes for a game.

Meeting Student Needs

- For #12, ensure students understand how the words *and* and *or* are used. The probability of choosing chocolate or strawberry is different from the probability of choosing chocolate and strawberry.
- Provide **BLM 11–7 Section 11.2 Extra Practice** to students who would benefit from more practice.

ELL

- Consider assigning fewer questions for English language learners.
- Give English language learners the opportunity to answer #11 in their own language. Alternatively, if several students share a common first language, consider having them listen to each other's answer for #11. After doing so, it may be easier for students to ask for the missing vocabulary to express their thinking in English.



Extend

12. An ice-cream store has 31 flavours of ice cream and three types of cones (waffle, sugar, and plain).
    a) Determine the number of possible single-scoop ice-cream cones.
    b) How many two-scoop ice-cream cones are possible if waffle–chocolate–strawberry is considered different from waffle–strawberry–chocolate? Hint: You could have two scoops of the same flavour if you desired.
    c) How many two-scoop cones are possible if both flavours of ice cream must be different? Explain your reasoning.

13. A set meal consists of a choice of drink, main dish, and dessert. There are four different desserts, 36 possible meals in total, and more than one choice in each category. Determine the possible number of drink and main dish choices. Explain your reasoning.

14. Alikut is planning to make a beaded yoke for her new parka. She has five colours of beads: red, blue, black, white, and yellow. She wants to use only six shapes of beads: square, circle, star, triangle, rectangle, and heart. Alikut plans to use only one colour of bead in each shape.
    a) How many colour–shape combinations could she use?
    b) Use a second method to verify your answer to part a).
    c) Suppose Alikut decides to use two colours of beads in each shape. How many colour–shape combinations could she use now?

15. Determine the number of four-digit numbers that contain only the digits 1, 2, 3, and 4. A digit can be repeated. Two valid numbers are 1423 and 4442.

16. How many car license plates can be made if the first three characters are letters and the last three characters are digits from 2 through 9 inclusive?

MATH LINK

In the stick game, each stick can land in one of two ways—decorated or plain side up.

a) Use a different method than you used in section 11.1 to confirm the total number of possible outcomes for a game with four sticks.

b) Sometimes the game uses different numbers of sticks. What is the total number of possible outcomes for three sticks? five sticks?

c) If there are 128 possible outcomes, how many sticks are being used?

11.2 Outcomes of Independent Events • MHR 425

Gifted and Enrichment

- Have students find an easier way to answer #12c). They could subtract the number of two-scoop cones of the same flavour (31) from the answer to #12b).

Answers

Math Link

a) Answers may vary. Students should use a different method than the one they used in section 11.1 Math Link. Example: Using multiplication, the total number of possible outcomes for a game with four sticks is $2 \times 2 \times 2 \times 2 = 16$.

b) Total number of possible outcomes for a game with three sticks: 8; Total number of possible outcomes for a game with five sticks: 32.

c) Number of sticks: 7

Assessment	Supporting Learning
Assessment *for* Learning	
Practise Have students do #3 and #5. Students who can readily complete these questions can go on to the Apply questions.	• Allow students to use manipulatives to help them visualize possible outcomes. • Provide additional coaching with Example 1 to students who need help with #3. Coach students through corrections to their answers, and then have them complete #4 on their own. • Provide additional coaching with Example 2 to students who need help with #5. Coach students through corrections to their answers, and then have them complete #6 on their own. • Have students refer back to the worked examples.
Math Link The Math Link on page 425 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 439.	• Some students may incorrectly multiply the number of possible outcomes for one stick (two) by the number of sticks (four) to determine the total possible outcomes. Help students realize that each stick has two outcomes and that they need to multiply the possible outcomes for each of four sticks ($2 \times 2 \times 2 \times 2 = 16$) by allowing them to use craft sticks or a tree diagram to check the total number of outcomes. • To help them get started, some students may benefit from using **BLM 11–8 Section 11.2 Math Link**, which provides scaffolding for this activity.
Assessment *as* Learning	
Math Learning Log Have students answer the following questions: • How does using multiplication differ from creating a tree diagram? How are the two methods the same?	• Help students understand that the multiplication method is useful for determining the total number of possible outcomes. Tables and tree diagrams are useful organizers for determining favourable outcomes as well as total possible outcomes. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.