Determining Probabilities Using Fractions

11.3

***MathLinks 8*, pages 426–435**

Suggested Timing

80–100 minutes

Materials

- ruler
- red and yellow coloured pencils or markers
- computer and spreadsheet software (optional)
- manipulatives including coins, six-sided dice, four-sided dice, spinners, marbles, bags, and playing cards (optional)
- calculator (optional)
- craft sticks

Blackline Masters

Master 2 Two Stars and One Wish
BLM 11–3 Chapter 11 Warm-Up
BLM 11–9 Section 11.3 Explore the Math
BLM 11–10 Section 11.3 Extra Practice
BLM 11–11 Section 11.3 Math Link

Mathematical Processes

☑ Communication (C)
☑ Connections (CN)
☐ Mental Mathematics and Estimation (ME)
☑ Problem Solving (PS)
☑ Reasoning (R)
☐ Technology (T)
☑ Visualization (V)

Specific Outcomes

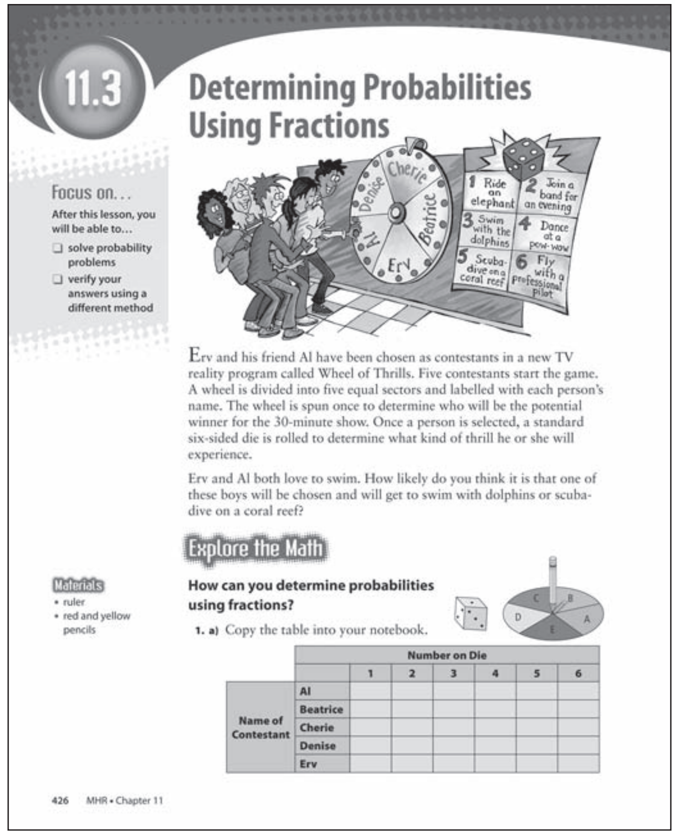
SP2 Solve problems involving the probability of independent events.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 2, 4, 6, 9, Math Link
Typical	1, 2, 4, 6, 8–14, Math Link
Extension/Enrichment	1, 2, 12–17, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 11–3 Chapter 11 Warm-Up** to reinforce material learned in previous sections.

As a class, read the opening text as an introduction to the Explore the Math.



11.3

Determining Probabilities Using Fractions

Focus on…

After this lesson, you will be able to…

- solve probability problems
- verify your answers using a different method

Erv and his friend Al have been chosen as contestants in a new TV reality program called Wheel of Thrills. Five contestants start the game. A wheel is divided into five equal sectors and labelled with each person's name. The wheel is spun once to determine who will be the potential winner for the 30-minute show. Once a person is selected, a standard six-sided die is rolled to determine what kind of thrill he or she will experience.

Erv and Al both love to swim. How likely do you think it is that one of these boys will be chosen and will get to swim with dolphins or scuba-dive on a coral reef?

Explore the Math

Materials
- ruler
- red and yellow pencils

How can you determine probabilities using fractions?

1. a) Copy the table into your notebook.

		Number on Die					
		1	2	3	4	5	6
Name of Contestant	Al						
	Beatrice						
	Cherie						
	Denise						
	Erv						

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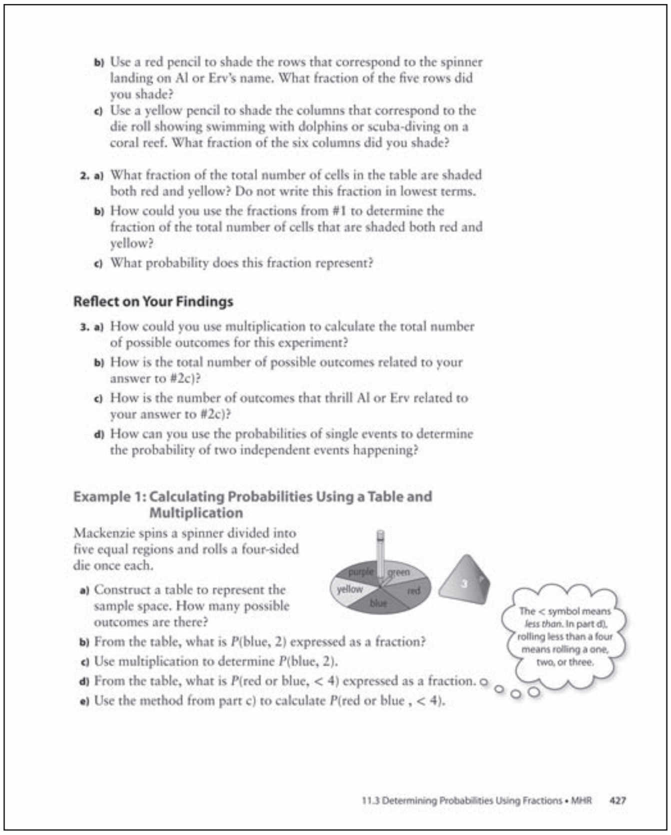
Explore the Math

In the previous section, students used multiplication to determine the total number of possible outcomes when two or more events happen. In this exploration, students use multiplication to determine the number of favourable outcomes of two or more events. The probability can be found by multiplying the probabilities of success for each single event.

The number of favourable outcomes for an event is always between zero and one. Therefore, the number of favourable outcomes can be expressed as a fraction, a decimal, or a percent. If the number of favourable outcomes is determined using a table or tree diagram, the probability is usually expressed as a fraction.

This exploration provides a visual approach for developing the concept of fraction multiplication. It is a very important activity for visual learners and students who struggle with fraction multiplication.



b) Use a red pencil to shade the rows that correspond to the spinner landing on Al or Erv's name. What fraction of the five rows did you shade?

c) Use a yellow pencil to shade the columns that correspond to the die roll showing swimming with dolphins or scuba-diving on a coral reef. What fraction of the six columns did you shade?

2. a) What fraction of the total number of cells in the table are shaded both red and yellow? Do not write this fraction in lowest terms.

b) How could you use the fractions from #1 to determine the fraction of the total number of cells that are shaded both red and yellow?

c) What probability does this fraction represent?

Reflect on Your Findings

3. a) How could you use multiplication to calculate the total number of possible outcomes for this experiment?

b) How is the total number of possible outcomes related to your answer to #2c)?

c) How is the number of outcomes that thrill Al or Erv related to your answer to #2c)?

d) How can you use the probabilities of single events to determine the probability of two independent events happening?

Example 1: Calculating Probabilities Using a Table and Multiplication

Mackenzie spins a spinner divided into five equal regions and rolls a four-sided die once each.

a) Construct a table to represent the sample space. How many possible outcomes are there?

b) From the table, what is *P*(blue, 2) expressed as a fraction?

c) Use multiplication to determine *P*(blue, 2).

d) From the table, what is *P*(red or blue, < 4) expressed as a fraction.

e) Use the method from part c) to calculate *P*(red or blue , < 4).

The $<$ symbol means *less than*. In part d), rolling less than a four means rolling a one, two, or three.

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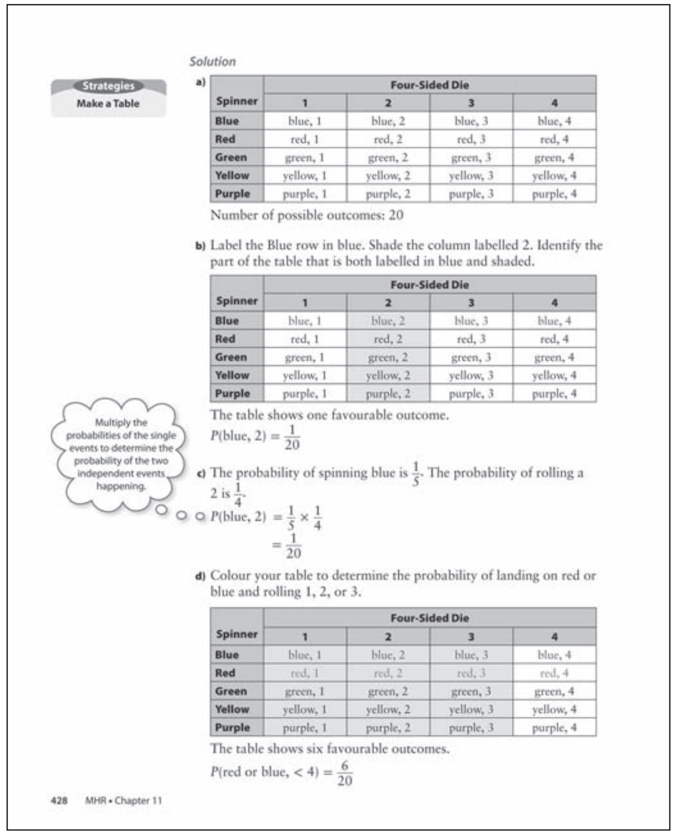

Solution

Strategies
Make a Table

a)

Spinner	Four-Sided Die 1	2	3	4
Blue	blue, 1	blue, 2	blue, 3	blue, 4
Red	red, 1	red, 2	red, 3	red, 4
Green	green, 1	green, 2	green, 3	green, 4
Yellow	yellow, 1	yellow, 2	yellow, 3	yellow, 4
Purple	purple, 1	purple, 2	purple, 3	purple, 4

Number of possible outcomes: 20

b) Label the Blue row in blue. Shade the column labelled 2. Identify the part of the table that is both labelled in blue and shaded.

Spinner	Four-Sided Die 1	2	3	4
Blue	blue, 1	blue, 2	blue, 3	blue, 4
Red	red, 1	red, 2	red, 3	red, 4
Green	green, 1	green, 2	green, 3	green, 4
Yellow	yellow, 1	yellow, 2	yellow, 3	yellow, 4
Purple	purple, 1	purple, 2	purple, 3	purple, 4

The table shows one favourable outcome.

$P(\text{blue}, 2) = \frac{1}{20}$

Multiply the probabilities of the single events to determine the probability of the two independent events happening.

c) The probability of spinning blue is $\frac{1}{5}$. The probability of rolling a 2 is $\frac{1}{4}$.

$$P(\text{blue}, 2) = \frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$$

d) Colour your table to determine the probability of landing on red or blue and rolling 1, 2, or 3.

Spinner	Four-Sided Die 1	2	3	4
Blue	blue, 1	blue, 2	blue, 3	blue, 4
Red	red, 1	red, 2	red, 3	red, 4
Green	green, 1	green, 2	green, 3	green, 4
Yellow	yellow, 1	yellow, 2	yellow, 3	yellow, 4
Purple	purple, 1	purple, 2	purple, 3	purple, 4

The table shows six favourable outcomes.

$P(\text{red or blue}, < 4) = \frac{6}{20}$

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Method 1

Have students copy the table on page 426 into their notebook. Have them answer the questions and then discuss their findings as a class. Encourage students to relate what they see on the table for this Explore the Math to what they learned about multiplying fractions in section 6.3, Example 1, on page 211.

Method 2

Have students copy the table into their notebook. Alternatively, provide **BLM 11–9 Section 11.3 Explore the Math**, which provides a copy of the table. On an overhead copy of the table, complete the table with student input. Have volunteers use a red and a yellow marker to shade the rows on the table that correspond to the favourable outcomes in #1b) and c). Have students work on their own or with a partner to answer the remaining questions and then discuss their findings as a class.

Example 1

This example illustrates calculating probabilities using a table and reinforces the fraction multiplication that was developed in the Explore the Math. Work through the solution as a class. Ensure that students understand that for part b) they need to determine the probability of each single event (i.e., blue, 2) before multiplying the probabilities of the single events.

Example 2

This example illustrates calculating probabilities using multiplication and then verifying using a tree diagram. Work through the solution as a class and determine the probability of each single event before multiplying the probabilities of the two events. In the final answer, prompt students to realize that the product numerator (before changing to lowest terms) represents the total number of favourable outcomes and the product denominator (before changing to lowest terms) represents the total number of possible outcomes.

Example 3

This example illustrates a simulation of a real situation. Simulations were introduced in grade 7. In a simulation, students compare experimental and theoretical probabilities. Explain that simulations are useful when determining probabilities that are difficult to compute theoretically. Explain that as the number of trials in a simulation increases, the results get closer to the theoretical probability.

e) The probability of spinning red or blue is $\frac{2}{5}$.

The probability of rolling a 1, 2, or 3 is $\frac{3}{4}$.

$$P(\text{red or blue}, < 4) = \frac{2}{5} \times \frac{3}{4}$$
$$= \frac{6}{20}$$

Example 2: Calculating Probabilities Using a Tree Diagram and Multiplication

Jason rolls a standard six-sided die and Rachel spins a spinner with three equal sections. What is the probability of rolling an even number and spinning a B? Verify your answer using another method.

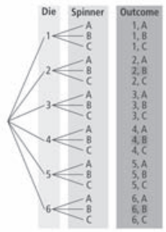
Solution

For the die: $P(\text{even number}) = \frac{3}{6}$

For the spinner: $P(\text{B}) = \frac{1}{3}$

$$P(\text{even number, B}) = P(\text{even number}) \times P(\text{B})$$
$$= \frac{3}{6} \times \frac{1}{3}$$
$$= \frac{3}{18}$$

Use a tree diagram to verify your answer.



The tree diagram shows that there are 18 possible outcomes and three favourable outcomes.

$P(\text{even number, B}) = \frac{3}{18}$

The tree diagram agrees with the result of the multiplication. The probability of rolling an even number and spinning a B is $\frac{3}{18}$ or $\frac{1}{6}$.

Show You Know

A blue, standard six-sided die and a red, four-sided die numbered 1, 2, 3, and 4 are each rolled once. Determine the following probabilities, and then verify your calculations using a second method.

a) $P(\text{blue} = 4, \text{red} = 4)$ b) $P(\text{blue} < 4, \text{red} < 4)$

c) $P(\text{blue} = 4, \text{red} < 4)$

Literacy Link

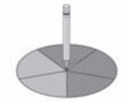
In a *simulation*, you model a real situation using an experiment.

Example 3: Simulations

Gina is planning the time needed to get to her soccer game. There are two traffic lights between her house and the soccer field. These lights are red (or yellow) 60% of the time. Gina wonders how likely it is that both lights will be red on her way to the game.

Model this situation by spinning a spinner divided into five equal regions twice. The table shows the results for ten trials.

Trial	Experimental Results		
	First Light (Green or Red)	Second Light (Red or Green)	Both Lights Red?
1	R	R	yes
2	G	G	no
3	R	G	no
4	G	R	no
5	R	R	yes
6	R	G	no
7	R	R	yes
8	G	G	no
9	G	R	no
10	G	G	no



Literacy Link

An *experimental probability* is the probability of an event occurring based on experimental results.

A *theoretical probability* is the calculated probability of an event occurring.

a) What is the experimental probability that both lights are red?

b) What is the theoretical probability that both lights are red?

c) Compare the experimental probability with the theoretical probability. How could Gina improve the accuracy of the experimental probability?

Solution

a) From the table, there are three favourable outcomes.

$$P(\text{both lights red}) = \frac{3}{10}$$
$$= 0.3$$

The experimental probability that both lights are red is $\frac{3}{10}$, 0.3, or 30%.

b) The probability that one traffic light is red is 60% or $\frac{3}{5}$.

$$P(\text{both lights red}) = \frac{3}{5} \times \frac{3}{5}$$
$$= \frac{9}{25}$$
$$= 0.36$$

The theoretical probability that both lights are red is $\frac{9}{25}$, 0.36, or 36%.

Literacy Link Direct students to the Literacy Link on page 430 that defines *simulation*.

Use the Literacy Link on the same page to clarify the meaning of experimental *probability* and *theoretical probability*. Help students remember the terms by explaining that experimental outcomes are usually collected and counted at the end of an experiment. Also highlight that experimental probability and theoretical probability are not always the same.

Meeting Student Needs

- Give students extra time and a ruler to copy the table in the Explore the Math. Alternatively, provide **BLM 11–9 Section 11.3 Explore the Math** or allow students to use a computer to create the table.
- Consider using an alternative scenario to the Wheel of Thrills for the Explore the Math. For example, students might work in groups of five and create a spinner with five equal sectors each labelled with a student's name. Have them choose six travel destinations. Have students use the scenario to develop a probability problem to solve.
- Have students who struggle with changing fractions to lowest terms verbalize the process with you.

ELL

- Ensure that English language learners understand the terms *contestants* and *TV reality program*.

Common Errors

- Some students may quickly multiply the probabilities without fully understanding the reasoning for the process.

R_x Ensure that students continue to develop their understanding of why the favourable results are fractions through activities like the Explore the Math and Example 1.

Answers

Explore the Math

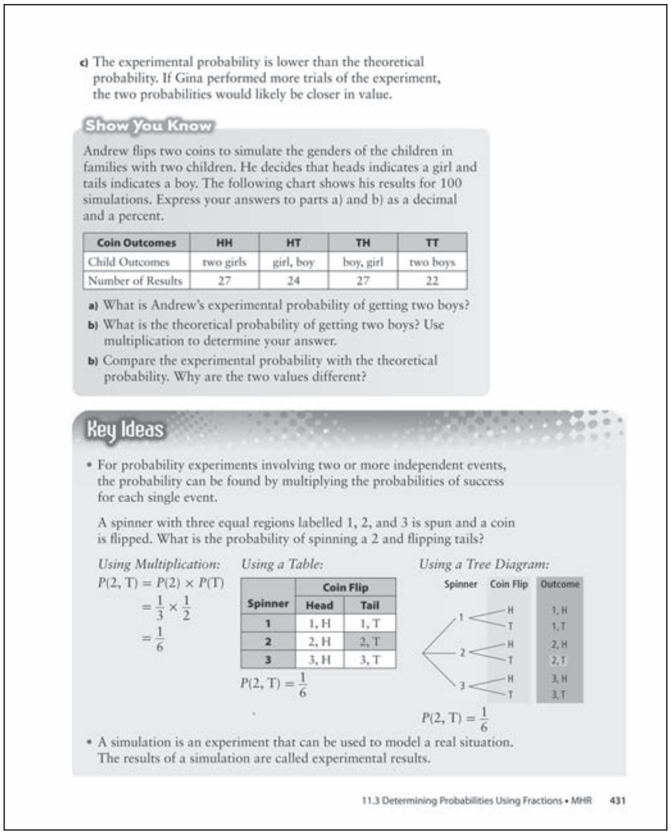
1. a) Answers will vary. **b)** $\frac{2}{5}$ **c)** $\frac{2}{6}$

2. a) Fraction of the total number of cells shaded red and yellow: $\frac{4}{30}$

b) Answers may vary. Example: Multiply the two probabilities from #1b) and #1c) to get the fraction of the total number of cells shaded red and yellow: $\frac{2}{5} \times \frac{2}{6} = \frac{4}{30}$.

c) This fraction represents the probability that Al or Erv will be chosen and will get to swim with dolphins or scuba dive on a coral reef.

3. a) Answers may vary. Example: Multiply the number of people, five, by the number of activities, six, to get the total number of possible outcomes: $5 \times 6 = 30$.



c) The experimental probability is lower than the theoretical probability. If Gina performed more trials of the experiment, the two probabilities would likely be closer in value.

Show You Know

Andrew flips two coins to simulate the genders of the children in families with two children. He decides that heads indicates a girl and tails indicates a boy. The following chart shows his results for 100 simulations. Express your answers to parts a) and b) as a decimal and a percent.

Coin Outcomes	HH	HT	TH	TT
Child Outcomes	two girls	girl, boy	boy, girl	two boys
Number of Results	27	24	27	22

a) What is Andrew's experimental probability of getting two boys?
b) What is the theoretical probability of getting two boys? Use multiplication to determine your answer.
b) Compare the experimental probability with the theoretical probability. Why are the two values different?

Key Ideas

- For probability experiments involving two or more independent events, the probability can be found by multiplying the probabilities of success for each single event.

  A spinner with three equal regions labelled 1, 2, and 3 is spun and a coin is flipped. What is the probability of spinning a 2 and flipping tails?

  Using Multiplication:
  $P(2, \text{T}) = P(2) \times P(\text{T})$
  $= \frac{1}{3} \times \frac{1}{2}$
  $= \frac{1}{6}$

  Using a Table:

Spinner	Coin Flip: Head	Coin Flip: Tail
1	1, H	1, T
2	2, H	2, T
3	3, H	3, T

  $P(2, \text{T}) = \frac{1}{6}$

  Using a Tree Diagram:

  Spinner / Coin Flip / Outcome: 1 – H: 1, H; 1 – T: 1, T; 2 – H: 2, H; 2 – T: 2, T; 3 – H: 3, H; 3 – T: 3, T

  $P(2, \text{T}) = \frac{1}{6}$

- A simulation is an experiment that can be used to model a real situation. The results of a simulation are called experimental results.

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Answers

Explore the Math

3. b) Answers may vary. Example: The total number of possible outcomes, 30, is equivalent to the denominator of the answer to #2b).

c) Answers may vary. Example: The total number of outcomes that thrill Al or Erv, four, is the numerator of the answer to #2b).

d) Answers may vary. Example: For two independent events, the product of the probabilities of the single events is equal to the probability of two independent events happening.

Show You Know: Examples 1 and 2

a) $P(\text{blue} = 4, \text{red} = 4) = \frac{1}{24}$

b) $P(\text{blue} < 4, \text{red} < 4) = \frac{9}{24} = \frac{3}{8}$

c) $P(\text{blue} = 4, \text{red} < 4) = \frac{3}{24} = \frac{1}{8}$

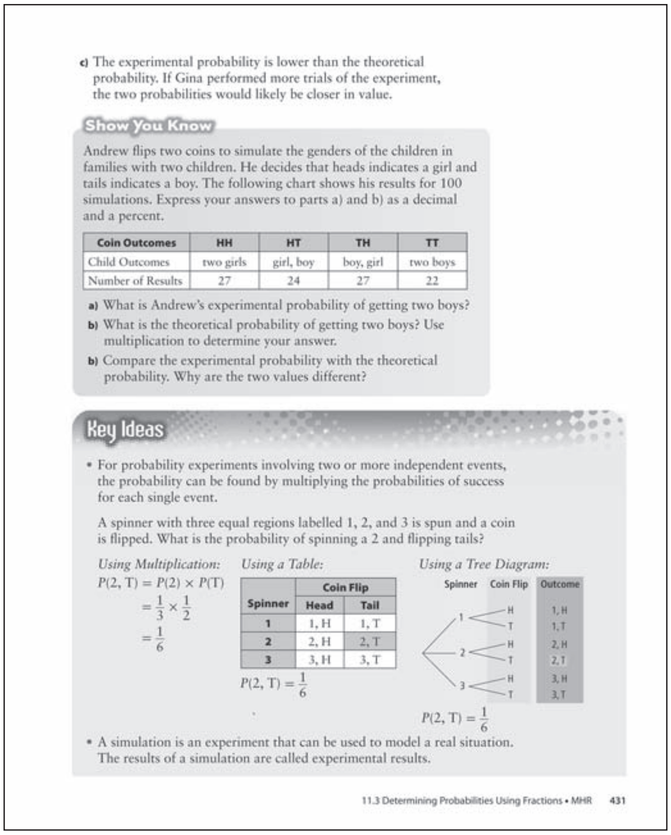
Show You Know: Example 3

a) Experimental probability of getting two boys: $P(\text{two boys}) = \frac{22}{100}$.
The experimental probability of getting two boys is 0.22 or 22%.

b) Answers may vary. Example: The theoretical probability of getting one boy: $P(\text{one boy}) = \frac{1}{2}$. The theoretical probability of getting two boys: $P(\text{two boys}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. The theoretical probability of getting two boys is 0.25 or 25%.

c) Answers may vary. Example: The experimental probability of getting two boys is lower than the theoretical probability. If Andrew had performed more trials of the experiment, the two probabilities would likely be closer in value.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.	• You may need to help reactivate students' knowledge of how to multiply fractions. • Check that students have made the connection that the answer to #3b) corresponds to the numerator in #2b), and that the answer to #3c) corresponds to the denominator in #2c). Use the opportunity to strengthen conceptual understanding of fraction multiplication. • Some students may benefit from using the class responses as springboards to prepare their own answers.
Assessment *for* Learning	
Examples 1 and 2 There is one Show You Know for Examples 1 and 2. Once you have discussed both Examples 1 and 2 with students, have them do the Show You Know and choose the method for answering the question.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Examples 1 and 2 provide three different methods for answering the type of question in the Show You Know: a table, a tree diagram, or multiplication. Allow students to choose any two methods for answering this question. They can use one method to obtain their answer and another to verify it. • Encourage students to use multiplication to determine the number of outcomes, and then to verify their answer using either a table or tree diagram. • Consider allowing students to use a computer and spreadsheet software to create the table. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to verbalize their understanding of the differences between experimental probability and theoretical probability. Clarify any misunderstandings. • Some students will benefit from continuing to use a tree diagram or a table to verify the sample space. Encourage them to use multiplication to determine the sample space. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.



c) The experimental probability is lower than the theoretical probability. If Gina performed more trials of the experiment, the two probabilities would likely be closer in value.

Show You Know

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b) Compare the experimental probability with the theoretical probability. Why are the two values different?

Key Ideas

- For probability experiments involving two or more independent events, the probability can be found by multiplying the probabilities of success for each single event.

  A spinner with three equal regions labelled 1, 2, and 3 is spun and a coin is flipped. What is the probability of spinning a 2 and flipping tails?

  Using Multiplication:

  $$P(2, \mathrm{T}) = P(2) \times P(\mathrm{T}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

  Using a Table:

Spinner	Coin Flip: Head	Coin Flip: Tail
1	1, H	1, T
2	2, H	2, T
3	3, H	3, T

  $P(2, \mathrm{T}) = \frac{1}{6}$

  Using a Tree Diagram:

  Spinner | Coin Flip | Outcome
  1 — H: 1, H; T: 1, T
  2 — H: 2, H; T: 2, T
  3 — H: 3, H; T: 3, T

  $P(2, \mathrm{T}) = \frac{1}{6}$

- A simulation is an experiment that can be used to model a real situation. The results of a simulation are called experimental results.

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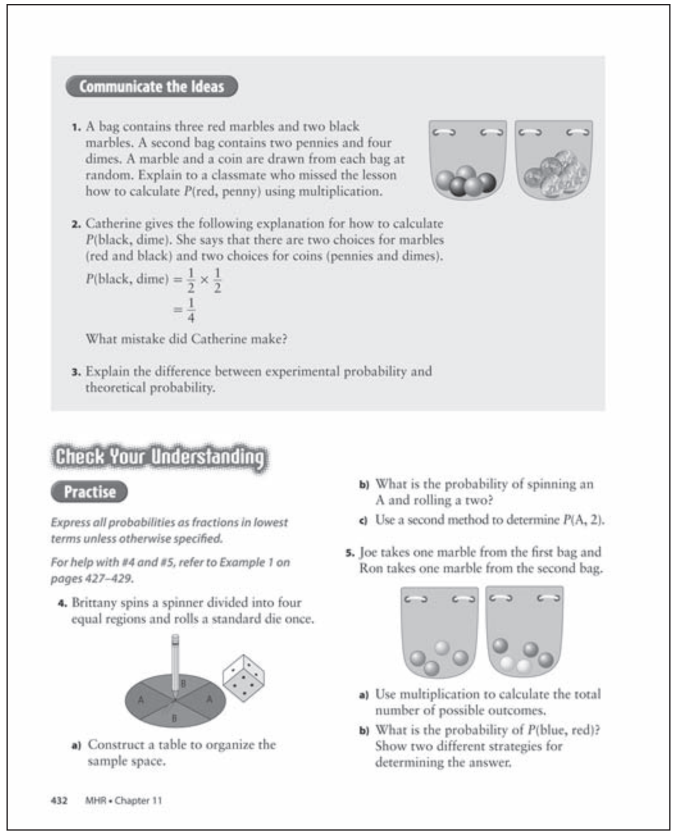

Communicate the Ideas

1. A bag contains three red marbles and two black marbles. A second bag contains two pennies and four dimes. A marble and a coin are drawn from each bag at random. Explain to a classmate who missed the lesson how to calculate *P*(red, penny) using multiplication.

2. Catherine gives the following explanation for how to calculate *P*(black, dime). She says that there are two choices for marbles (red and black) and two choices for coins (pennies and dimes).

   $$P(\text{black, dime}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

   What mistake did Catherine make?

3. Explain the difference between experimental probability and theoretical probability.

Check Your Understanding

Practise

Express all probabilities as fractions in lowest terms unless otherwise specified.

For help with #4 and #5, refer to Example 1 on pages 427–429.

4. Brittany spins a spinner divided into four equal regions and rolls a standard die once.

   a) Construct a table to organize the sample space.

   b) What is the probability of spinning an A and rolling a two?

   c) Use a second method to determine *P*(A, 2).

5. Joe takes one marble from the first bag and Ron takes one marble from the second bag.

   a) Use multiplication to calculate the total number of possible outcomes.

   b) What is the probability of *P*(blue, red)? Show two different strategies for determining the answer.

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Key Ideas

Ensure that students understand that the product of the numerators is equal to the total number of favourable (successful) outcomes, and that the product of the denominators is equal to the total number of possible outcomes. Have students prepare their own summary of the Key Ideas and record it in the section 11.3 booklet in their chapter Foldable.

Communicate the Ideas

Use #1 and #2 to gain insight into how well students understand the Key Ideas. For #1, ensure that students understand that the number of each item (three red and two black) means a total number of five possible outcomes. If students have difficulty with this, give them a bag with three red marbles and two black marbles, and suggest that they do a simulation. They will find after several trials that they take the red marble from the bag about $\frac{3}{5}$ of the time, not $\frac{1}{2}$ the time. For #2, students should realize that marble colour is only one property of the bag. The number of each marble is also important. For #3, have students explain the difference between experimental probability and theoretical probability to a partner and listen to each other's explanation.

Meeting Student Needs

- Some students may benefit from additional practice with multiplication of fractions. See the related Web Link on this page.
- Some students may benefit from exploring probability concepts using the virtual manipulative described in the Web Link on this page.

ELL

- Give English language learners the opportunity to answer the questions in their own language and illustrate their thoughts using diagrams. Alternatively, if several students share a common first language, consider having them listen to each other's answers. Then, have students use their diagrams to explain their thoughts orally to you.
- You may need to explain terms such as *nickels* and *dimes*.

WWW Web Link

For a virtual manipulative that allows students to practise multiplication of fractions, go to www.mathlinks8.ca and follow the links.

For a virtual manipulative that allows students to explore probability concepts by simulating repeated coin tosses, go to www.mathlinks8.ca and follow the links.

Answers

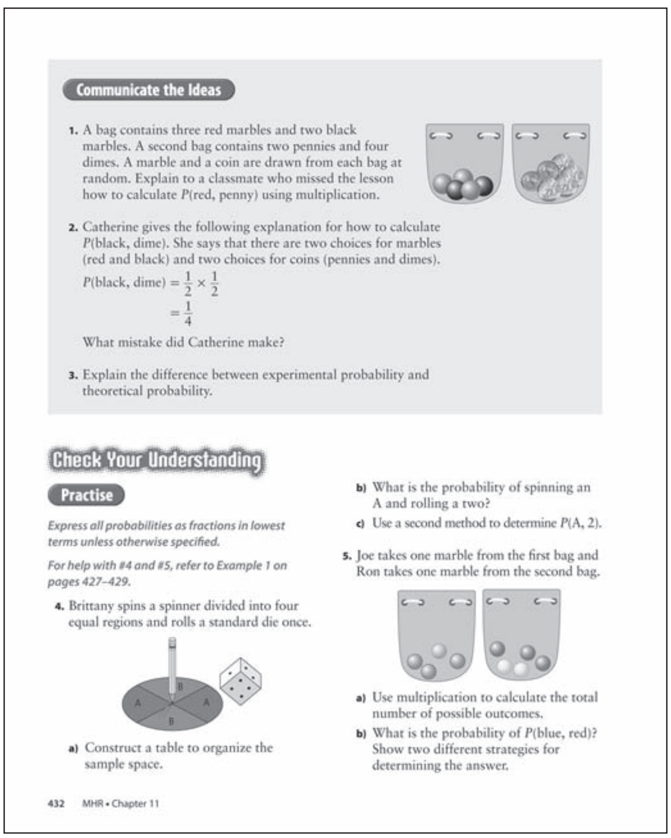
Communicate the Ideas

1. Answers may vary. Example: Using multiplication, multiply the probability of selecting a red marble by the probability of selecting a penny: $P(\text{red, penny}) = \frac{3}{5} \times \frac{2}{6} = \frac{6}{30} = \frac{1}{5}$.

2. Answers may vary. Example: There are more than one of each item in each bag and the numbers of items are different. Therefore, the probabilities for each single event is not $\frac{1}{2}$.

3. Answers may vary. Example:
- Experimental probability is found by conducting a probability experiment and using all of the data from each trial.
- Theoretical probability is the expected probability that might occur if a probability experiment was conducted. It is found by dividing the number of ways an event can occur by the total number of possible outcomes.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to #3.	• Consider having students work in pairs to answer the questions. • Check each student's response to #1 and #2. These are key questions: make sure students understand the concepts about probability before proceeding. • Coach students who need help to connect the concepts of probability multiplication with direct counting from tree diagrams or tables. • Some students may benefit from verbalizing the three methods they learned and explaining which method is easier for them to understand and why. Have them explain when they would use each method; clarify any misunderstandings. This process should assist students with #2. • Use **Master 2 Two Stars and One Wish** and have students critique other students' writing.



Communicate the Ideas

1. A bag contains three red marbles and two black marbles. A second bag contains two pennies and four dimes. A marble and a coin are drawn from each bag at random. Explain to a classmate who missed the lesson how to calculate *P*(red, penny) using multiplication.

2. Catherine gives the following explanation for how to calculate *P*(black, dime). She says that there are two choices for marbles (red and black) and two choices for coins (pennies and dimes).

$$P(\text{black, dime}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

What mistake did Catherine make?

3. Explain the difference between experimental probability and theoretical probability.

Check Your Understanding

Practise

Express all probabilities as fractions in lowest terms unless otherwise specified.

For help with #4 and #5, refer to Example 1 on pages 427–429.

4. Brittany spins a spinner divided into four equal regions and rolls a standard die once.

a) Construct a table to organize the sample space.

b) What is the probability of spinning an A and rolling a two?

c) Use a second method to determine *P*(A, 2).

5. Joe takes one marble from the first bag and Ron takes one marble from the second bag.

a) Use multiplication to calculate the total number of possible outcomes.

b) What is the probability of *P*(blue, red)? Show two different strategies for determining the answer.

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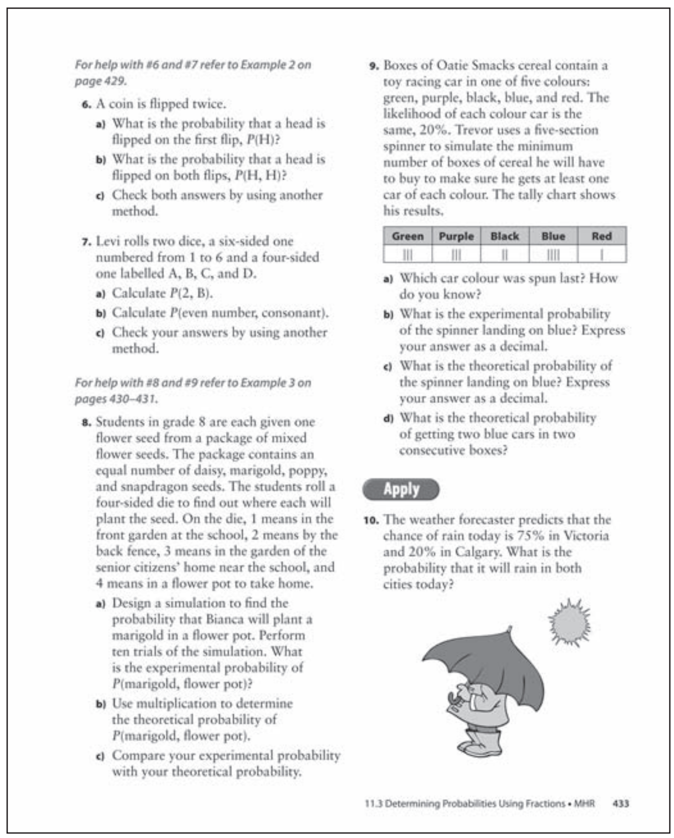

For help with #6 and #7 refer to Example 2 on page 429.

6. A coin is flipped twice.

a) What is the probability that a head is flipped on the first flip, *P*(H)?

b) What is the probability that a head is flipped on both flips, *P*(H, H)?

c) Check both answers by using another method.

7. Levi rolls two dice, a six-sided one numbered from 1 to 6 and a four-sided one labelled A, B, C, and D.

a) Calculate *P*(2, B).

b) Calculate *P*(even number, consonant).

c) Check your answers by using another method.

For help with #8 and #9 refer to Example 3 on pages 430–431.

8. Students in grade 8 are each given one flower seed from a package of mixed flower seeds. The package contains an equal number of daisy, marigold, poppy, and snapdragon seeds. The students roll a four-sided die to find out where each will plant the seed. On the die, 1 means in the front garden at the school, 2 means by the back fence, 3 means in the garden of the senior citizens' home near the school, and 4 means in a flower pot to take home.

a) Design a simulation to find the probability that Bianca will plant a marigold in a flower pot. Perform ten trials of the simulation. What is the experimental probability of *P*(marigold, flower pot)?

b) Use multiplication to determine the theoretical probability of *P*(marigold, flower pot).

c) Compare your experimental probability with your theoretical probability.

9. Boxes of Oatie Smacks cereal contain a toy racing car in one of five colours: green, purple, black, blue, and red. The likelihood of each colour car is the same, 20%. Trevor uses a five-section spinner to simulate the minimum number of boxes of cereal he will have to buy to make sure he gets at least one car of each colour. The tally chart shows his results.

Green	Purple	Black	Blue	Red
|||	|||	||	||||	|

a) Which car colour was spun last? How do you know?

b) What is the experimental probability of the spinner landing on blue? Express your answer as a decimal.

c) What is the theoretical probability of the spinner landing on blue? Express your answer as a decimal.

d) What is the theoretical probability of getting two blue cars in two consecutive boxes?

Apply

10. The weather forecaster predicts that the chance of rain today is 75% in Victoria and 20% in Calgary. What is the probability that it will rain in both cities today?

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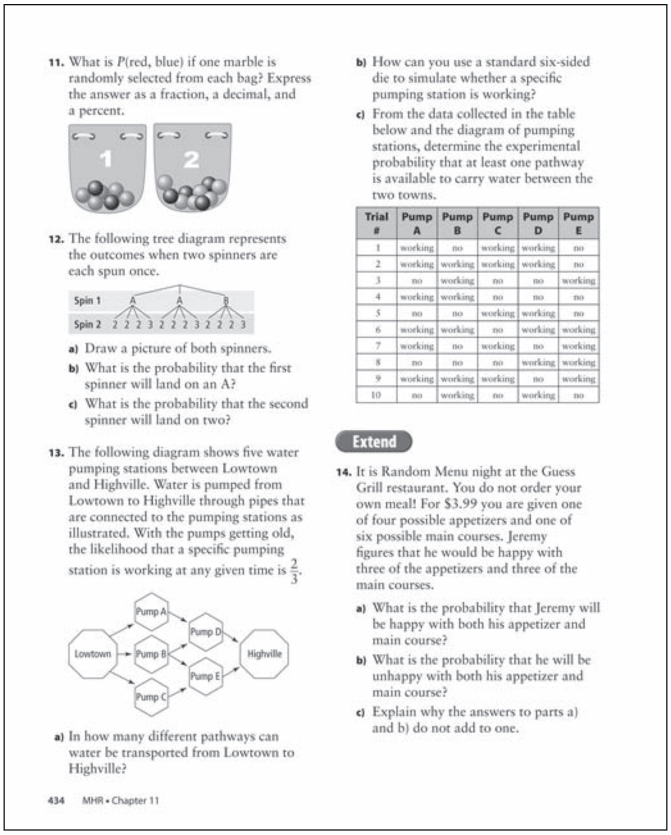
Check Your Understanding

Practise

Some students may be confused by #5. Clarify that they need to account for each individual marble, even if two or more are the same colour. For #6 and #7, students need to draw a table or a tree diagram to verify their answers.



11. What is *P*(red, blue) if one marble is randomly selected from each bag? Express the answer as a fraction, a decimal, and a percent.

1 2

12. The following tree diagram represents the outcomes when two spinners are each spun once.

Spin 1 A A B
Spin 2 2 2 2 3 2 2 2 3 2 2 2 3

a) Draw a picture of both spinners.

b) What is the probability that the first spinner will land on an A?

c) What is the probability that the second spinner will land on two?

13. The following diagram shows five water pumping stations between Lowtown and Highville. Water is pumped from Lowtown to Highville through pipes that are connected to the pumping stations as illustrated. With the pumps getting old, the likelihood that a specific pumping station is working at any given time is $\frac{2}{3}$.

Pump A
Pump D
Lowtown
Pump B
Highville
Pump E
Pump C

a) In how many different pathways can water be transported from Lowtown to Highville?

b) How can you use a standard six-sided die to simulate whether a specific pumping station is working?

c) From the data collected in the table below and the diagram of pumping stations, determine the experimental probability that at least one pathway is available to carry water between the two towns.

Trial #	Pump A	Pump B	Pump C	Pump D	Pump E
1	working	no	working	working	no
2	working	working	working	working	no
3	no	working	no	no	working
4	working	working	no	no	no
5	no	no	working	working	no
6	working	working	no	working	working
7	working	no	working	no	working
8	no	no	no	working	working
9	working	working	working	no	working
10	no	working	no	working	no

Extend

14. It is Random Menu night at the Guess Grill restaurant. You do not order your own meal! For $3.99 you are given one of four possible appetizers and one of six possible main courses. Jeremy figures that he would be happy with three of the appetizers and three of the main courses.

a) What is the probability that Jeremy will be happy with both his appetizer and main course?

b) What is the probability that he will be unhappy with both his appetizer and main course?

c) Explain why the answers to parts a) and b) do not add to one.

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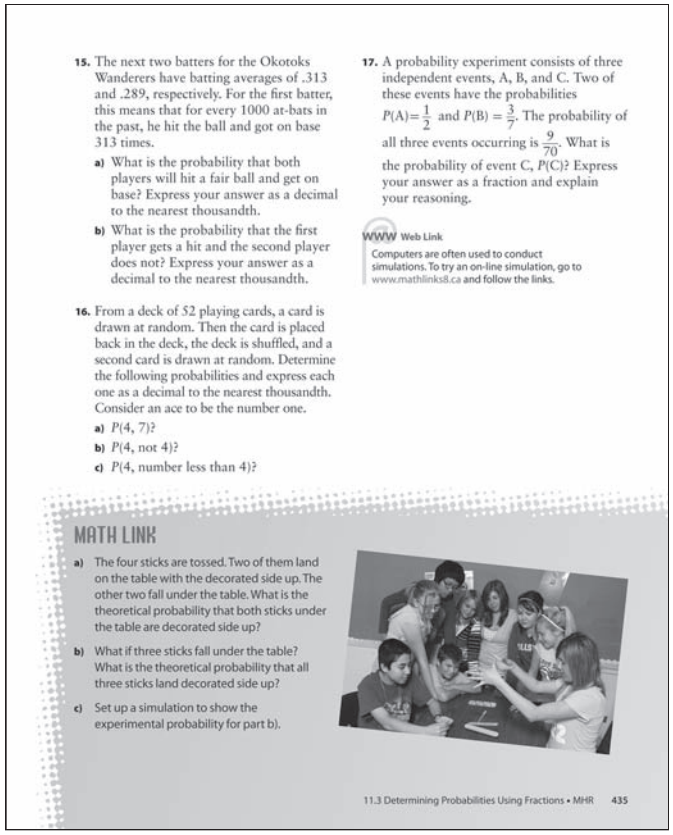

15. The next two batters for the Okotoks Wanderers have batting averages of .313 and .289, respectively. For the first batter, this means that for every 1000 at-bats in the past, he hit the ball and got on base 313 times.

a) What is the probability that both players will hit a fair ball and get on base? Express your answer as a decimal to the nearest thousandth.

b) What is the probability that the first player gets a hit and the second player does not? Express your answer as a decimal to the nearest thousandth.

16. From a deck of 52 playing cards, a card is drawn at random. Then the card is placed back in the deck, the deck is shuffled, and a second card is drawn at random. Determine the following probabilities and express each one as a decimal to the nearest thousandth. Consider an ace to be the number one.

a) *P*(4, 7)?

b) *P*(4, not 4)?

c) *P*(4, number less than 4)?

17. A probability experiment consists of three independent events, A, B, and C. Two of these events have the probabilities $P(A)=\frac{1}{2}$ and $P(B)=\frac{3}{7}$. The probability of all three events occurring is $\frac{9}{70}$. What is the probability of event C, *P*(C)? Express your answer as a fraction and explain your reasoning.

WWW Web Link

Computers are often used to conduct simulations. To try an on-line simulation, go to www.mathlinks8.ca and follow the links.

MATH LINK

a) The four sticks are tossed. Two of them land on the table with the decorated side up. The other two fall under the table. What is the theoretical probability that both sticks under the table are decorated side up?

b) What if three sticks fall under the table? What is the theoretical probability that all three sticks land decorated side up?

c) Set up a simulation to show the experimental probability for part b).

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Apply

The Apply questions provide a range of contexts in which probability is used. Consider giving students some choice in the questions they do.

Extend

For #15 and #16, students are required to work with decimals rounded to the nearest thousandth. Calculators may be appropriate for these questions.

Math Link

The Math Link allows students to solve probability problems using the stick game. Emphasize that the outcomes for the sticks under the table are independent of the outcomes for the visible sticks.

Meeting Student Needs

- Some students may struggle with the text-dense questions in the Practice and Apply sections. Help students extract the information they need.
- Consider allowing students to work in pairs. They might work on one question together and then work individually on the next one. Ensure that students complete a number of questions individually.
- Consider allowing students to use a computer and spreadsheet software to create tables.
- You may need to clarify the scenario in #14.
- Provide **BLM 11–10 Section 11.3 Extra Practice** to students who would benefit from more practice.

ELL

- In #8, English language learners may not be familiar with the following terms: *mixed flower seeds*, *daisy*, *marigold*, *poppy*, *snapdragon*, *front garden*, *back fence*, *garden of the senior citizens' home*, and *flower pot*. Use visuals and ask other students to help describe each of these terms.

Gifted and Enrichment

- Have students try some of the more challenging virtual simulations in the Web Link described below.

Common Errors

- In the Math Link, some students may think that because the two visible sticks land decorated side up, the two under the table will fall in the same way.

R_x Remind students that when the sticks are tossed, each stick is an independent event and has no influence on the outcome for the other sticks.

WWW Web Link

For additional practice with virtual simulations, go to www.mathlinks8.ca and follow the links.

Answers

Math Link

a) $P(\text{both sticks under the table decorated side up}) = \frac{1}{4}$

b) $P(\text{three sticks under the table decorated side up}) = \frac{1}{8}$

c) Answers will vary. Example: Using a coin, heads represent the decorated side and tails represent the plain side. Look for a reasonable number of trials and a record of results.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise Have students do #4, #6, and #9. Students who can readily complete these questions can go on to the Apply questions.	• Provide additional coaching with Example 1 to students who need help with #4. Have students verbalize their choice of a second method. Have them explain their thinking; clarify any misunderstandings. Then, have students complete #5 on their own. Check back with them several times to make sure that they understand the concepts. • Provide additional coaching with Example 2 to students who need help with #6. Encourage them to design a sample space to help visualize the possible outcomes. Have them verbalize the possible outcomes. Have them link the multiplication statement to their work. Then, have students complete #7 on their own. Check back with them several times to make sure that they understand the concepts. • Provide additional coaching with Example 3 to students who need help with #9. Ask questions such as the following: What does the chart tell you? What is the total of each? Discuss the answer to part a). Ensure that students have a clear understanding of the difference between theoretical probability and experimental probability. (Theoretical probability can be calculated immediately from the information given. Experimental probability can be obtained only by doing the test.)
Math Link The Math Link on page 435 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 439.	• For part a), ask students if the outcomes of the visible sticks influence the likelihood that the hidden sticks are decorated side up. • For part b), have students verbalize all the possible outcomes for the hidden sticks. Some students may benefit from listing a sample space or using a table to organize the outcomes for the hidden sticks. • To help them get started, some students may benefit from using **BLM 11–11 Section 11.3 Math Link**, which provides scaffolding for this activity.
Assessment *as* Learning	
Math Learning Log Have students complete the following statements: • The difference between experimental and theoretical probability is … • I get confused when …	• Depending on students' learning style, have them provide oral or written answers. • As an alternative, challenge Enrichment students to complete the following activity: Develop a probability problem that demonstrates theoretical probability and experimental probability. Explain why they are not the same. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.