12 Tessellations

General Outcomes

- Describe and analyze position and motion of objects and shapes.

Specific Outcomes

SS6 Demonstrate an understanding of tessellation by:
- explaining the properties of shapes that make tessellating possible
- creating tessellations
- identifying tessellations in the environment.

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
12.1	✓ identify regular and irregular polygons that can be used to create tessellations
	✓ describe why certain regular and irregular polygons can be used to tessellate the plane
	✓ create simple tessellating patterns using polygons
12.2	✓ identify how translations and reflections can be used to create a tessellation
	✓ create tessellating patterns using two or more polygons
12.3	✓ identify how rotations can be used to create a tessellation
	✓ create tessellating patterns using two or more polygons
12.4	✓ create tessellations from combinations of regular and irregular polygons
	✓ describe the tessellations in terms of the transformation used to create them

Assessment	Supporting Learning
Assessment *for* Learning	
Method 1: Use the Math Link introduction on page 445 in *MathLinks 8* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about regular and irregular polygons, characteristics of congruent polygons, and how to use transformations (including translations, reflections, and rotations) to make different designs.	• **BLM 12–1 Chapter 12 Math Link Introduction** provides scaffolding for the Math Link introduction. • The What I Need to Work On sections of the chapter Foldable provide a place for students to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete the Get Ready materials available on **BLM 12–2 Chapter 12 Get Ready**, in the *MathLinks 8 Practice and Homework Book*, and at the www.mathlinks8.ca book site.
Assessment *as* Learning	
Literacy Link (page 443) At the beginning of the chapter, work with students to model the use of a Frayer model for the term *tessellation*.	• Encourage students to use the glossary on pages 517–521 to help them. • Students who computerize their model may wish to access the *MathLinks 8* online glossary by going to www.mathlinks8.ca and following the links. • At the end of section 12.1, have students revisit their Frayer model for tessellation and make additions and improvements.
Chapter 12 Foldable As students work on each section in Chapter 12, have them keep track of any difficulties they are having in the space underneath the Math Link grid for each section.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled.
Assessment *for* Learning	
BLM 12–3 Chapter 12 Warm-Up This BLM includes four warm-ups, one to be used at the beginning of each section. Each warm-up provides cumulative review questions for the entire student resource to that point, as well as mental math practice.	• As students complete questions from previous chapters, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing mental math calculations.

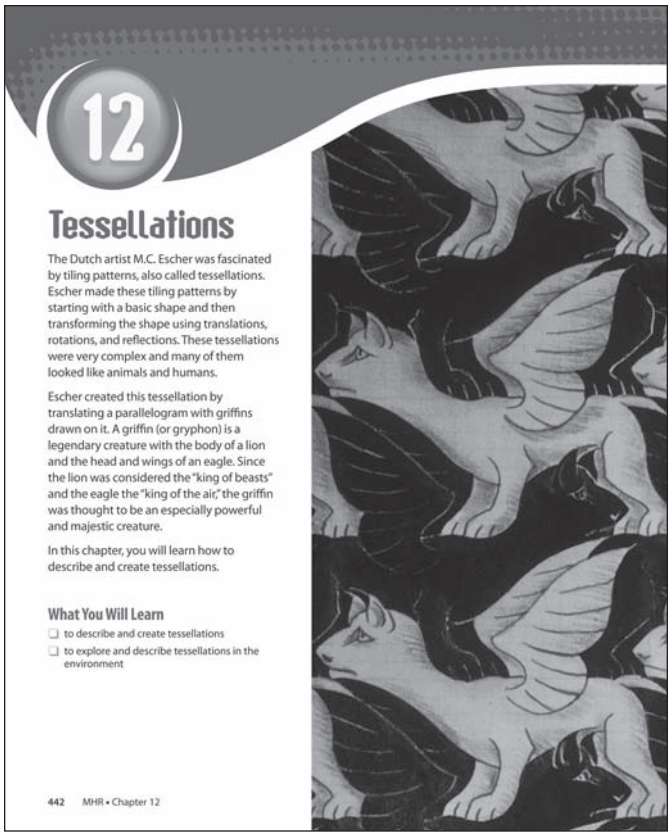
Problems of the Week

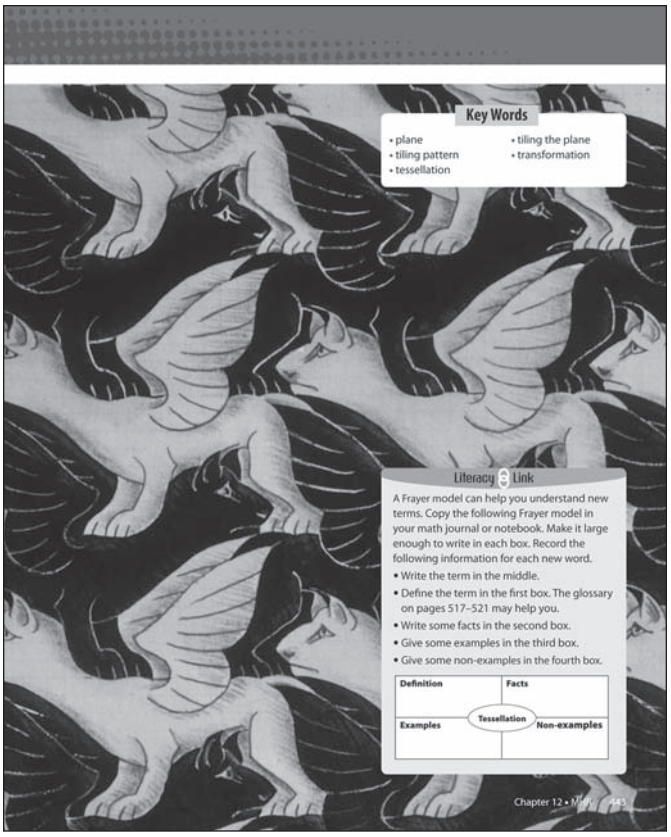
Have all students try at least one of the problems on **BLM 12–4 Chapter 12 Problems of the Week**. Many of these problems require students to think outside the box and experiment with a variety of approaches. Some have definitive answers; others can be answered in more than one way.

Students can take the problems home and consult with parents or guardians, work with other students when their work is completed, or try them on their own. The questions take a varying amount of time to solve, depending on the particular student and the problem itself. You may wish to give out these problems at the beginning of the chapter and discuss the solutions at appropriate times throughout your work on the chapter.

Chapter 12 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Extra Support	Assessment		
						Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 40–50 minutes (TR page 595)	Students should be familiar with • characteristics of regular and irregular polygons • describing why two triangles are or are not congruent • using transformations (translations, reflections, rotations) to create 2-D patterns	• 11 × 17 sheet of paper • three sheets of 0.5-cm grid paper • scissors • examples of art that show transformations and tessellations (optional) • coloured transparent shapes (optional) • cardboard or construction paper • coloured pencils • ruler • index cards (optional) • stapler • glue (optional)	Master 9 0.5 Centimetre Grid Paper Master 17 Frayer Model BLM 12–1 Chapter 12 Math Link Introduction BLM 12–2 Chapter 12 Get Ready BLM 12–4 Chapter 12 Problems of the Week		Online Learning Centre	TR page 594 Chapter 12 Foldable, TR page 594	TR page 594	
12.1 Exploring Tessellations with Regular and Irregular Polygons • 50–60 minutes (TR page 599)	Students should be familiar with • measuring interior angles of a polygon • angle properties of regular polygons	• cardboard cutouts of an isosceles triangle, a square, a hexagon, a regular pentagon, and a regular octagon • cardboard • scissors • ruler • set of pattern blocks or cardboard cutouts of pattern block shapes • transparent shapes • tracing paper (optional) • protractor	Master 2 Two Stars and One Wish Master 9 0.5 Centimetre Grid Paper Master 7 Isometric Dot Paper BLM 12–3 Chapter 12 Warm-Up BLM 12–5 Section 12.1 Explore the Math BLM 12–6 Irregular Polygons BLM 12–7 Section 12.1 Extra Practice BLM 12–8 Section 12.1 Math Link BLM 12–9 Design Templates	**Essential:** 1, 2, 4, 6, Math Link **Typical:** 1, 2, 4, 6, 8–10, Math Link **Extension/Enrichment:** 1, 2, 7, 8, 11–14	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 602, 603 Math Learning Log, TR page 605 Chapter 12 Foldable, TR page 605 Master 2 Two Stars and One Wish	TR pages 602, 605	
12.2 Constructing Tessellations Using Translations and Reflections • 50–60 minutes (TR page 606)	Students should be familiar with • describing translations and reflections of 2-D shapes • characteristics of equilateral triangles	• set of pattern blocks or cardboard cutouts of pattern block shapes • ruler • scissors • glue stick • transparent tape • cardboard or construction paper • protractor • transparent shapes (optional)	Master 9 0.5 Centimetre Grid Paper BLM 12–3 Chapter 12 Warm-Up BLM 12–10 Shapes BLM 12–11 Section 12.2 Extra Practice BLM 12–12 Section 12.2 Math Link	**Essential:** 1, 3, 4, 6, Math Link **Typical:** 1–4, 6–8, Math Link **Extension/Enrichment:** 1, 6–9	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 608, 610 Math Learning Log, TR page 611 Chapter 12 Foldable, TR page 611	TR pages 608, 611	
12.3 Constructing Tessellations Using Rotations • 50–60 minutes (TR page 612)	Students should be familiar with • describing rotations of 2-D shapes • characteristics of equilateral triangles • angle measures	• tracing paper • scissors • glue stick • transparent tape • cardboard or construction paper • coloured pencils • ruler • pattern blocks of equilateral triangles (optional) • transparent shapes (optional) • protractor	Master 7 Isometric Dot Paper Master 9 0.5 Centimetre Grid Paper BLM 12–3 Chapter 12 Warm-Up BLM 12–10 Shapes BLM 12–13 Extend Chart BLM 12–14 Section 12.3 Extra Practice BLM 12–15 Section 12.3 Math Link	**Essential:** 1–4, Math Link **Typical:** 1–4, 5 *or* 6, 7, Math Link **Extension/Enrichment:** 1, 2, 6–9	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 614, 615 Math Learning Log, TR page 617 Chapter 12 Foldable, TR page 617	TR pages 614, 617	
12.4 Creating Escher-Style Tessellations • 50–60 minutes (TR page 618)	Students should be familiar with • describing translations, reflections, and rotations of 2-D shapes • characteristics of equilateral triangles and parallelograms • angle measures	• ruler • scissors • glue stick • cardboard or construction paper • transparent tape • coloured pencils • pattern blocks (optional) • protractor • tracing paper (optional)	Master 7 Isometric Dot Paper Master 9 0.5 Centimetre Grid Paper BLM 12–3 Chapter 12 Warm-Up BLM 12–16 Section 12.4 Extra Practice BLM 12–17 Section 12.4 Math Link	**Essential:** 1, 3–5, 8, Math Link **Typical:** 1, 3–5, 6 *or* 7, 8 *or* 9 *or* 10, Math Link **Extension/Enrichment:** 1, 3, 6 *or* 7, 8 *or* 9 *or* 10, 11	*MathLinks 8 Practice and Homework Book* *MathLinks 8 Solutions Manual*	TR pages 620, 621 Math Learning Log, TR page 623 Chapter 12 Foldable, TR page 623	TR pages 620, 623	
Chapter 12 Review • 40–50 minutes (TR page 624)		• pattern blocks (optional) • ruler	Master 9 0.5 Centimetre Grid Paper BLM 12–7 Section 12.1 Extra Practice BLM 12–11 Section 12.2 Extra Practice BLM 12–14 Section 12.3 Extra Practice BLM 12–16 Section 12.4 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	*MathLinks 8 Practice and Homework Book* *MathLinks 8 CAB*	Chapter 12 Foldable, TR page 625	TR page 625	
Chapter 12 Practice Test • 40–50 minutes (TR page 626)		• pattern tiles (optional) • protractor • ruler	Master 9 0.5 Centimetre Grid Paper BLM 12–18 Chapter 12 Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1, 2, 7–9	*MathLinks 8 CAB*	TR page 627		TR page 627 BLM 12–18 Chapter 12 Test
Chapter 12 Wrap It Up! • 80–100 minutes (TR page 628)		• construction paper • coloured transparencies • tile pieces • paint • ruler • scissors • glue	Master 1 Project Rubric BLM 12–1 Chapter 12 Math Link Introduction BLM 12–8 Section 12.1 Math Link BLM 12–12 Section 12.2 Math Link BLM 12–15 Section 12.3 Math Link BLM 12–17 Section 12.4 Math Link BLM 12–19 Chapter 12 Wrap It Up!		Online Learning Centre			TR page 628 Master 1 Project Rubric
Chapter 12 Math Games • 40–50 minutes (TR page 630)		• two 6-sided dice per pair of students or small group • one coloured counter per student • ruler	BLM 12–20 Playing at Tiling Game Board #1 BLM 12–21 Playing at Tiling Game Board #2 BLM 12–22 Playing at Tiling Game Board #3				TR page 630	
Chapter 12 Challenge in Real Life • 80–100 minutes (TR page 631)		• construction paper • scissors • ruler • coloured pencils or markers • glue	Master 1 Project Rubric		Online Learning Centre		TR page 632	TR page 632 Master 1 Project Rubric
Chapters 9–12 Review • 60–75 minutes (TR page 634)		• grid paper • ruler • algebra tiles • pattern blocks	Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper Master 15 Algebra Tiles	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 3–5, 7, 8 *or* 9, 10–12, 13 *or* 14, 15–17	*MathLinks 8 CAB*	TR page 636 Chapters 9, 10, 11, and 12 Foldables, TR page 636 Math Learning Log, TR, page 636	TR page 636	
Task • 100–120 minutes (TR page 637)		• shape to tessellate • ruler • coloured pencils (blue, orange, green) • modelling clay or bingo chips	Master 1 Project Rubric Master 9 0.5 Centimetre Grid Paper BLM 12–23 Shape to Tessellate BLM 12–24 Chapter 12 BLM Answers		Online Learning Centre			TR page 638 Master 1 Project Rubric





***MathLinks 8*, pages 442–445**

Suggested Timing
40–50 minutes

Materials
- 11 × 17 sheet of paper
- three sheets of 0.5-cm grid paper
- scissors
- stapler
- glue (optional)
- examples of art that show transformations and tessellations (optional)
- coloured transparent shapes (optional)
- cardboard or construction paper
- coloured pencils
- ruler
- index cards (optional)

Blackline Masters
Master 9 0.5 Centimetre Grid Paper
Master 17 Frayer Model
BLM 12–1 Chapter 12 Math Link Introduction
BLM 12–2 Chapter 12 Get Ready
BLM 12–4 Chapter 12 Problems of the Week

Key Words

plane	tiling pattern	tessellation
tiling the plane	transformation	

What's the Math?

In this chapter, students use their knowledge of regular polygons, irregular polygons, and geometric transformation to describe and create tessellations. Students identify what types of polygons can be used to tile or tessellate the plane and explain how they can determine which polygons tile or tessellate the plane. They also describe how translations, reflections, and rotations are applied to polygons to create patterns, including Escher-like tessellations.

Planning Notes

Begin this section by having a class discussion about the different polygons students are familiar with and the three types of transformations they have previously studied (translations, reflections, and rotations). Ask students to describe where in nature or their surroundings they have seen these shapes, and have them describe any repeating patterns they have observed. Show the class different types of art that use polygons and tiling patterns. Ask them to identify the shapes they recognize and what transformations they think have been used to make the patterns.

Literacy Link At the beginning of the chapter, model how to use a Frayer model using the word *tessellation*. Explain the purpose of each part of the model.

- Definition – Work with students to develop a clear definition. They may wish to check the *MathLinks 8* glossary.
- Facts – Ask students to record what they already know about tessellations. Discuss what students may already know about M.C. Escher and his work, and then have them consider how the opening visual on pages 442–443 was developed using tessellations.
- Examples – Have students look around the classroom and school to find examples of tiling patterns. Suggest that they draw an example of a simple tessellation such as they might find on a tile floor.
- Non-examples – Work with students to identify designs around the classroom or school that are not tessellations.

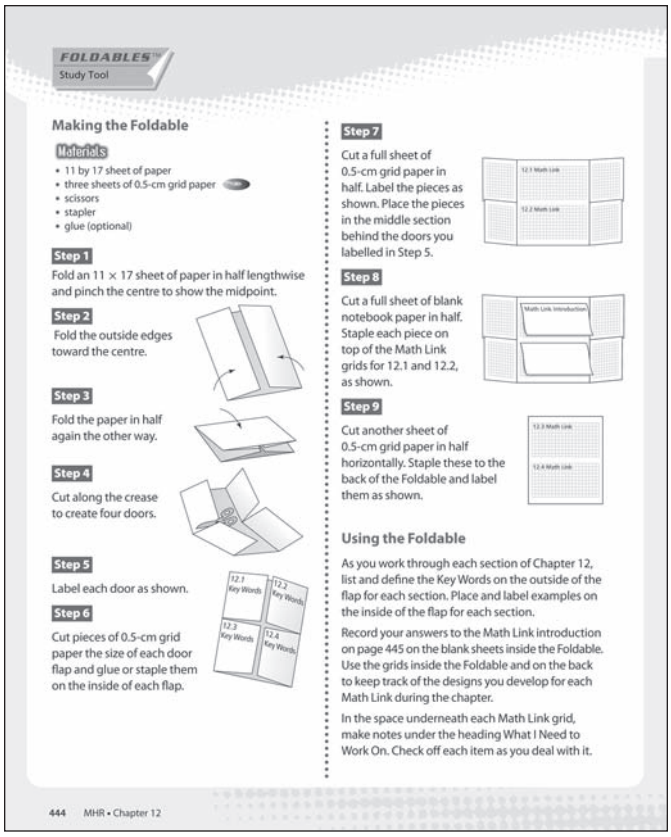
During the chapter, have students use the Frayer model to show their understanding of the terms *tiling pattern*, *tiling the plane*, and *transformation*. Encourage them to use visuals as examples.

Meeting Student Needs

- Consider having students complete the questions on **BLM 12–2 Chapter 12 Get Ready** to activate the prerequisite skills for this chapter.
- Some students may benefit from assistance in reactivating their skills and knowledge in the following areas:
  - geometry terminology such as *translations*, *rotations*, *reflections*, *polygons*, and names of common regular polygons
  - measuring angles
- You may wish to begin this section by demonstrating, with transparent pieces on an overhead, how tiling patterns can be made. Use different-coloured pieces that are easily discernable by students who may have partial colour blindness (e.g., do not use blue and green, or red and green, in the same pattern).
- Consider providing students with a variety of different polygon tiles to experiment with to help them visualize how a polygon can be used to tile the plane. Software programs are also very useful for giving students an opportunity to explore the relationships between angle measures of polygons and whether the polygon can be used to tile the plane.
- Show students patterns from different cultures that show tessellations. For example, some traditional First Nations weaving patterns show tessellations, as do quilts (e.g., Lakota Star Quilt) and Ukrainian pysanky (elaborately decorated eggs).
- Have students use small cards for their Frayer models. By making a Frayer model for each key term in the chapter, they can develop a deck of cards they can use for review. Provide students with **Master 17 Frayer Model** to use for this activity.

ELL

- During the introduction, provide examples of the meaning of *translations*, *reflections* and *rotations*. Have students add any new terms to their dictionary.
- Most of the introductory activities in this chapter have detailed instructions on how to make a tessellation or pattern. Consider going through these instructions and modelling the steps for students.



FOLDABLES™
Study Tool

Making the Foldable

Materials

- 11 by 17 sheet of paper
- three sheets of 0.5-cm grid paper
- scissors
- stapler
- glue (optional)

Step 1
Fold an 11 × 17 sheet of paper in half lengthwise and pinch the centre to show the midpoint.

Step 2
Fold the outside edges toward the centre.

Step 3
Fold the paper in half again the other way.

Step 4
Cut along the crease to create four doors.

Step 5
Label each door as shown.

Step 6
Cut pieces of 0.5-cm grid paper the size of each door flap and glue or staple them on the inside of each flap.

Step 7
Cut a full sheet of 0.5-cm grid paper in half. Label the pieces as shown. Place the pieces in the middle section behind the doors you labelled in Step 5.

Step 8
Cut a full sheet of blank notebook paper in half. Staple each piece on top of the Math Link grids for 12.1 and 12.2, as shown.

Step 9
Cut another sheet of 0.5-cm grid paper in half horizontally. Staple these to the back of the Foldable and label them as shown.

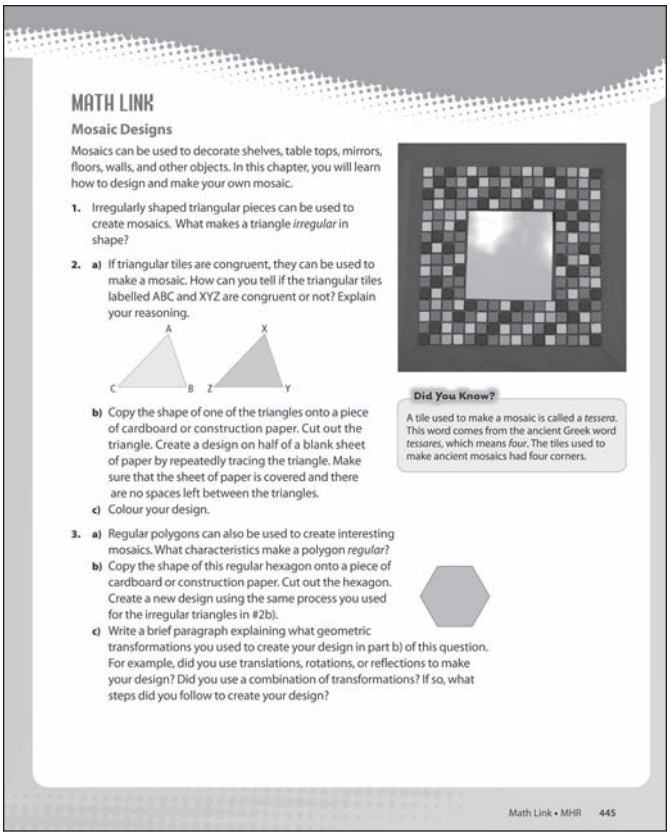
Using the Foldable

As you work through each section of Chapter 12, list and define the Key Words on the outside of the flap for each section. Place and label examples on the inside of the flap for each section.

Record your answers to the Math Link introduction on page 445 on the blank sheets inside the Foldable. Use the grids inside the Foldable and on the back to keep track of the designs you develop for each Math Link during the chapter.

In the space underneath each Math Link grid, make notes under the heading What I Need to Work On. Check off each item as you deal with it.

444 MHR • Chapter 12



MATH LINK

Mosaic Designs

Mosaics can be used to decorate shelves, table tops, mirrors, floors, walls, and other objects. In this chapter, you will learn how to design and make your own mosaic.

1. Irregularly shaped triangular pieces can be used to create mosaics. What makes a triangle *irregular* in shape?

2. a) If triangular tiles are congruent, they can be used to make a mosaic. How can you tell if the triangular tiles labelled ABC and XYZ are congruent or not? Explain your reasoning.

b) Copy the shape of one of the triangles onto a piece of cardboard or construction paper. Cut out the triangle. Create a design on half of a blank sheet of paper by repeatedly tracing the triangle. Make sure that the sheet of paper is covered and there are no spaces left between the triangles.

c) Colour your design.

3. a) Regular polygons can also be used to create interesting mosaics. What characteristics make a polygon *regular*?

b) Copy the shape of this regular hexagon onto a piece of cardboard or construction paper. Cut out the hexagon. Create a new design using the same process you used for the irregular triangles in #2b).

c) Write a brief paragraph explaining what geometric transformations you used to create your design in part b) of this question. For example, did you use translations, rotations, or reflections to make your design? Did you use a combination of transformations? If so, what steps did you follow to create your design?

Did You Know?

A tile used to make a mosaic is called a *tessera*. This word comes from the ancient Greek word *tessares*, which means *four*. The tiles used to make ancient mosaics had four corners.

Math Link • MHR 445

Foldables Study Tool

Have students make the Foldable in the student resource to keep track of the information in the chapter. Have students keep track of what they need to work on in the space underneath the Math Link for each section as they progress through the chapter; this will assist them in identifying and solving any difficulties with concepts, skills, and processes.

The Foldable allows students to keep track of their progress on the chapter problem during the Math Link introduction on page 445 and the section Math Links on pages 451, 456, 460, and 465.

Students may find it more useful to use half sheets of grid paper for this Foldable. They can add other pieces of grid paper to their Foldable if needed.

Math Link

Use #1 and #2a) in the Math Link introduction to give students an opportunity to recall and discuss their prior knowledge of regular and irregular shapes and congruency. You may wish to use transparent shapes on the overhead to demonstrate what makes shapes regular and irregular and what makes them congruent or not.

Have students work through parts b) and c) of #2 to create a tessellation using a triangle.

Then, discuss what makes a polygon regular, and have students work through #3 on their own or in pairs.

Have students read the Wrap It Up! problem on page 469 to give them a sense of where the Math Link is heading. As they work through the chapter, consider having students complete the related Math Links in sections 12.1, 12.2, 12.3, and 12.4. These Math Links are particularly useful for students who need assistance with the chapter, because they will help students in doing the Wrap It Up! problem. Alternatively, you may wish to assign only the Wrap It Up! problem when students have completed Chapter 12. This problem is a summative assessment.

Meeting Student Needs

- Have students work individually, in partners, or as a whole class to complete the chapter opener or Math Link introduction, depending on the needs of your class.
- Consider having students complete the questions on **BLM 12–2 Chapter 12 Get Ready** to activate the prerequisite skills for this chapter.
- To help them get started, some students may benefit from using **BLM 12–1 Chapter 12 Math Link Introduction**, which provides scaffolding for this activity.

ELL

- For the Math Link introduction, provide examples of the meaning of *congruent*, *polygons*, and *mosaic*. Have students add any new terms to their dictionary.

Gifted and Enrichment

- Provide additional challenge for students by having them increase the size of triangle ABC for their tessellation. Have them use a protractor and compass to draw the new triangle. Emphasize that all angles must be the same and that the sides must keep the same proportions as the original triangle.

Answers

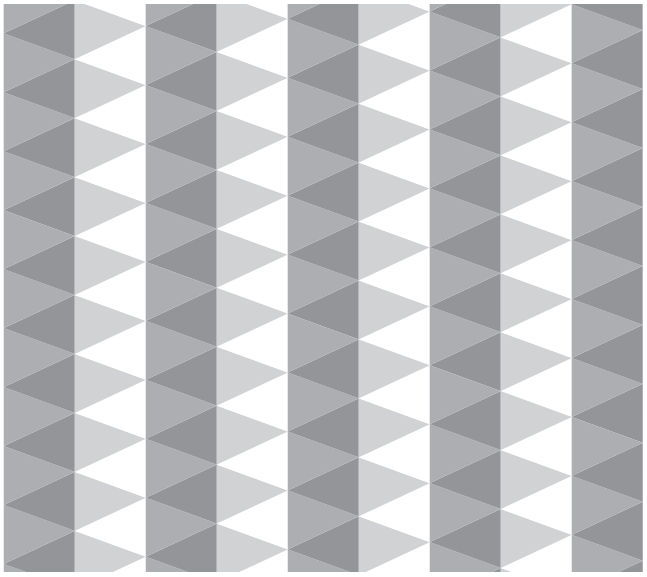
Math Link

1. Answers may vary. Example: The side lengths and the interior angles of an irregular shape are different.

2. a) Tiles ABC and XYZ are congruent if AB = XY, BC = YZ, and AC = XZ, and if ∠A = ∠X, ∠B = ∠Y, and ∠C = ∠Z.

b), c) Diagrams may vary. Look for the following:
- The design should be covered with no spaces between the triangles.
- The design should be coloured.

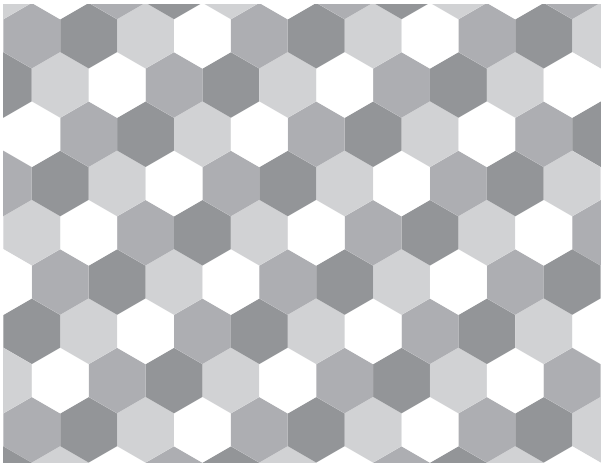
Example:



3. a) A regular polygon has equal side lengths and interior angles.

b) Diagrams may vary. Look for the following:
- The design should be covered with no spaces between the hexagons.
- The design should be created using a transformation.

Example:



c) Answers may vary. Example: The design can be made by repeating three translations on the given hexagon: translation of 1.8 cm to the right, translation of 3.2 cm down, and translation of 0.9 cm right and 1.6 cm down.