12.2 Constructing Tessellations Using Translations and Reflections

***MathLinks 8*, pages 452–456**

Suggested Timing
50–60 minutes

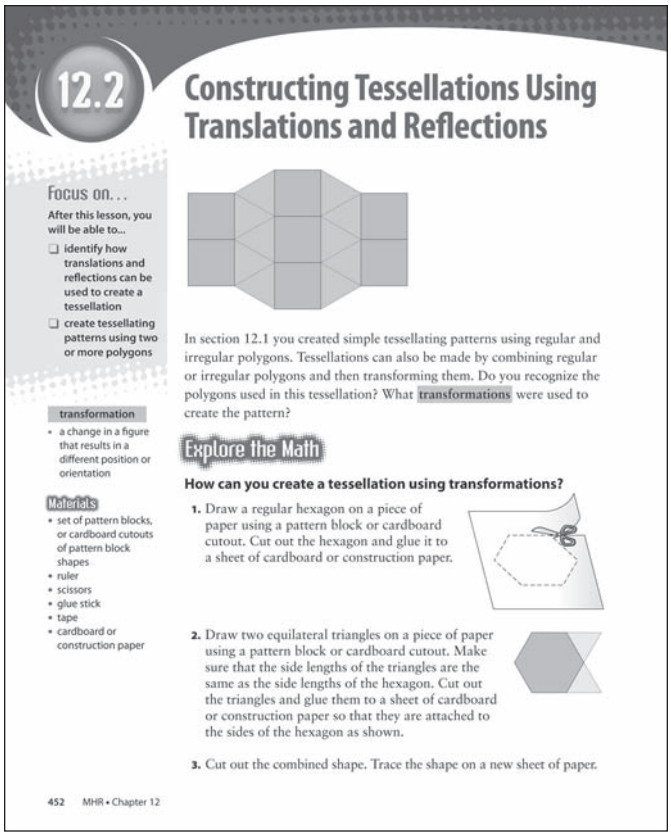
Materials
- set of pattern blocks or cardboard cutouts of pattern block shapes
- ruler
- scissors
- glue stick
- transparent tape
- cardboard or construction paper
- protractor
- transparent shapes (optional)

Blackline Masters
Master 9 0.5 Centimetre Grid Paper
BLM 12–3 Chapter 12 Warm-Up
BLM 12–10 Shapes
BLM 12–11 Section 12.2 Extra Practice
BLM 12–12 Section 12.2 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☐ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
SS6 Demonstrate an understanding of tessellation by:
- explaining the properties of shapes that make tessellating possible
- creating tessellations
- identifying tessellations in the environment.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 3, 4, 6, Math Link
Typical	1–4, 6–8, Math Link
Extension/Enrichment	1, 6–9


12.2 Constructing Tessellations Using Translations and Reflections

Focus on...
After this lesson, you will be able to...
- identify how translations and reflections can be used to create a tessellation
- create tessellating patterns using two or more polygons

transformation
- a change in a figure that results in a different position or orientation

Materials
- set of pattern blocks, or cardboard cutouts of pattern block shapes
- ruler
- scissors
- glue stick
- tape
- cardboard or construction paper

In section 12.1 you created simple tessellating patterns using regular and irregular polygons. Tessellations can also be made by combining regular or irregular polygons and then transforming them. Do you recognize the polygons used in this tessellation? What transformations were used to create the pattern?

Explore the Math

How can you create a tessellation using transformations?

1. Draw a regular hexagon on a piece of paper using a pattern block or cardboard cutout. Cut out the hexagon and glue it to a sheet of cardboard or construction paper.

2. Draw two equilateral triangles on a piece of paper using a pattern block or cardboard cutout. Make sure that the side lengths of the triangles are the same as the side lengths of the hexagon. Cut out the triangles and glue them to a sheet of cardboard or construction paper so that they are attached to the sides of the hexagon as shown.

3. Cut out the combined shape. Trace the shape on a new sheet of paper.

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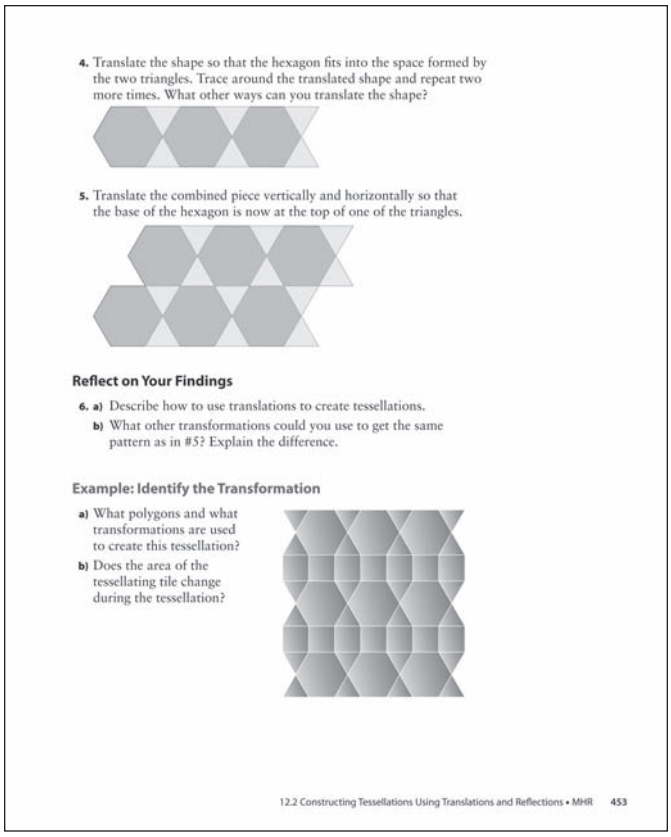
Planning Notes

Have students complete the warm-up questions on **BLM 12–3 Chapter 12 Warm-Up** to reinforce material learned in previous sections.

As a class, read and discuss the information about tessellations provided in the student resource. Discuss with students the characteristics of regular and irregular polygons that they have previously studied (triangles, rectangles, pentagons, hexagons, etc.). Ensure that students are able to correctly identify and describe translations and reflections.

Explore the Math

In this exploration, students follow directions to make a tessellation from two different shapes. Then, they experiment by transforming the same shapes in different ways and describe a different way the shapes could be transformed to make the same tessellation.



4. Translate the shape so that the hexagon fits into the space formed by the two triangles. Trace around the translated shape and repeat two more times. What other ways can you translate the shape?

5. Translate the combined piece vertically and horizontally so that the base of the hexagon is now at the top of one of the triangles.

Reflect on Your Findings

6. a) Describe how to use translations to create tessellations.
   b) What other transformations could you use to get the same pattern as in #5? Explain the difference.

Example: Identify the Transformation

a) What polygons and what transformations are used to create this tessellation?
b) Does the area of the tessellating tile change during the tessellation?

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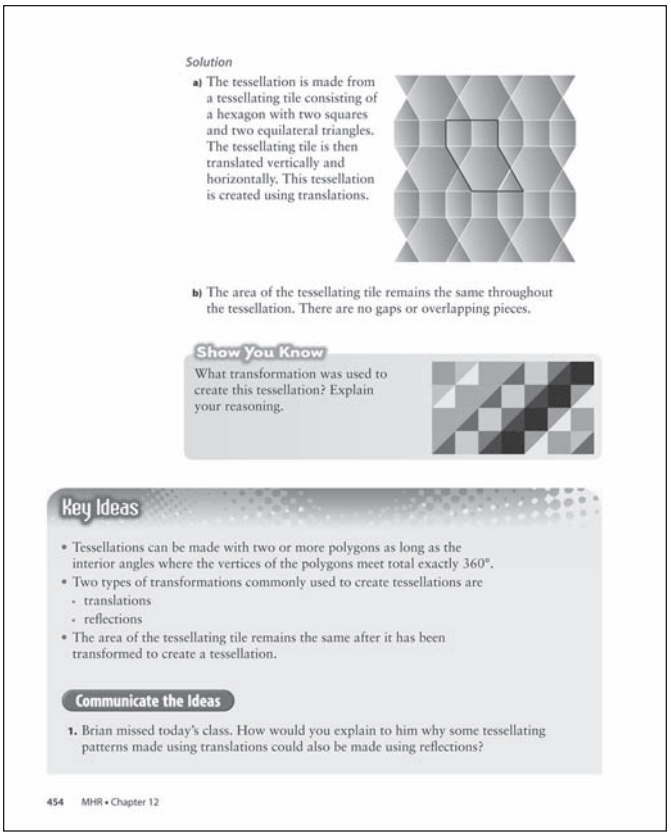

Solution

a) The tessellation is made from a tessellating tile consisting of a hexagon with two squares and two equilateral triangles. The tessellating tile is then translated vertically and horizontally. This tessellation is created using translations.

b) The area of the tessellating tile remains the same throughout the tessellation. There are no gaps or overlapping pieces.

Show You Know

What transformation was used to create this tessellation? Explain your reasoning.

Key Ideas

- Tessellations can be made with two or more polygons as long as the interior angles where the vertices of the polygons meet total exactly 360°.
- Two types of transformations commonly used to create tessellations are
  - translations
  - reflections
- The area of the tessellating tile remains the same after it has been transformed to create a tessellation.

Communicate the Ideas

1. Brian missed today's class. How would you explain to him why some tessellating patterns made using translations could also be made using reflections?

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Method 1 Have students work on the exploration in pairs or small groups and discuss their answers. Give each pair of students a supply of pattern blocks or cardboard cutouts of pattern block shapes. Have students compare their results and note any differences. Encourage students to answer #6 independently and then compare answers.

Method 2 Demonstrate how to create a tessellation using transformations on the overhead with transparent shapes. Begin by placing a hexagon on the overhead and asking students to trace the shape with their pattern blocks. Then, add the triangles for #2, and again ask students to trace the shape with pattern blocks. For #3, use several cutouts of the combined shape that you have prepared in advance, and demonstrate how the hexagon fits into the shape formed by the two triangles on the overhead. Have students complete the translations for their own pattern. Encourage students to answer #6 independently and then compare answers.

Example

In the Example, students identify which transformation was used to create the tessellation. Reinforce the use of concrete materials to identify the transformation and to determine if more than one transformation could be used to create the tessellation.

Meeting Student Needs

- Students may benefit from using **BLM 12–10 Shapes** to complete the exploration.
- Encourage students to re-create a given tessellation using their descriptions of shapes and transformation(s).
- Students may benefit from using a computer to find a visual that clearly shows translations and reflections. Encourage students to identify the shapes and transformations used to create the tessellations.

Common Errors

- Some students may describe transformations that will not result in the expected pattern.

R_x Demonstrate translations and reflections on the overhead using transparent shapes, to ensure students understand the differences between them.

Answers

Explore the Math

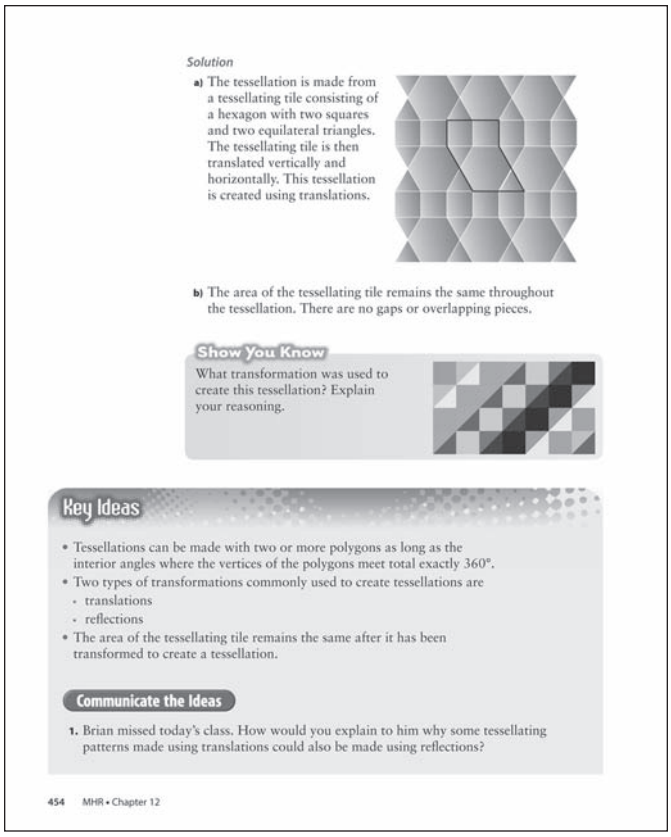
6. a) Answers may vary. Example: You can repeat translating shapes or a combination of shapes horizontally and vertically to fit spaces on the plane to form a tessellation.

b) Answers may vary. Example: You can reflect vertically the combined shape of a hexagon and two equilateral triangles and then translate to get the same pattern. The difference is that you use two different types of transformations instead of one.

Show You Know: Example

Answers may vary. Example: Translations of squares and isosceles right triangles of different colours were used to create the tessellation.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss which shapes and transformations have been used to create a given tiling pattern. Students should be able to identify multiple combinations of shapes and transformations that can be used to create the same tessellation. Have students try to generalize the conclusion about their findings.	• Reinforce the similarities and difference between tiling patterns created using translations versus reflections. • Encourage students to use manipulatives or computer programs to re-create a given tessellation using different combinations of shapes and transformations. • Encourage students to draw lines on a paper to use as a line of reflection. This may assist visual learners. • Students may find it easier to demonstrate to you the difference between a translation and a reflection than to write it. Have them verbalize the process as they demonstrate it in order to assist them in recognizing key words to write down.
Assessment *for* Learning	
Example Have students do the Show You Know related to the Example.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may wish to provide additional examples of shape combinations and transformations that can be used to create the same tiling pattern. Using an overhead with different shapes in different colours of acetate provides an effective visual for showing which shapes can be combined and which transformations result in the desired tiling pattern. • Provide students with manipulatives to re-create the given patterns so students can move them around and see what effects the changes have on the pattern or the reflection.



Solution

a) The tessellation is made from a tessellating tile consisting of a hexagon with two squares and two equilateral triangles. The tessellating tile is then translated vertically and horizontally. This tessellation is created using translations.

b) The area of the tessellating tile remains the same throughout the tessellation. There are no gaps or overlapping pieces.

Show You Know

What transformation was used to create this tessellation? Explain your reasoning.

Key Ideas

- Tessellations can be made with two or more polygons as long as the interior angles where the vertices of the polygons meet total exactly 360°.
- Two types of transformations commonly used to create tessellations are
  - translations
  - reflections
- The area of the tessellating tile remains the same after it has been transformed to create a tessellation.

Communicate the Ideas

1. Brian missed today's class. How would you explain to him why some tessellating patterns made using translations could also be made using reflections?

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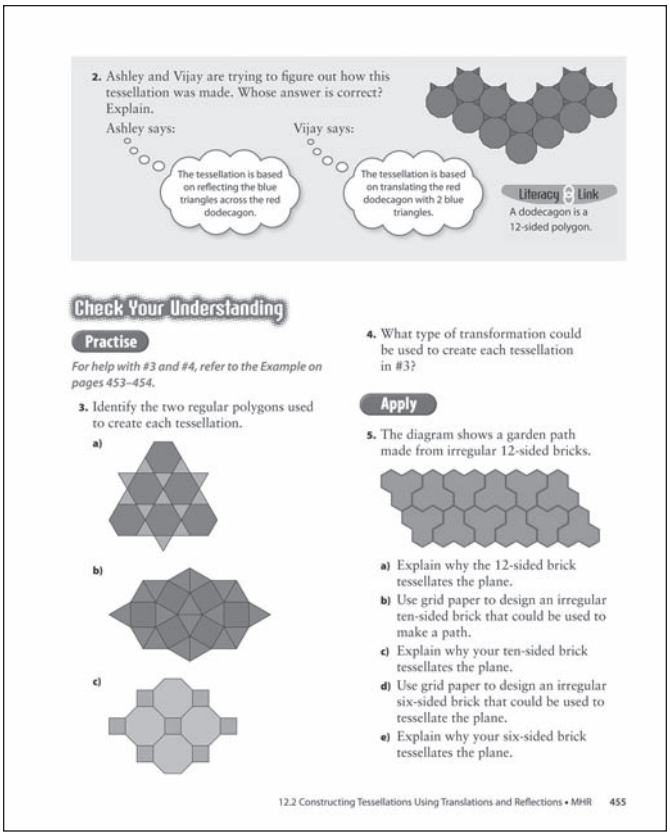

2. Ashley and Vijay are trying to figure out how this tessellation was made. Whose answer is correct? Explain.

Ashley says: The tessellation is based on reflecting the blue triangles across the red dodecagon.

Vijay says: The tessellation is based on translating the red dodecagon with 2 blue triangles.

Literacy Link: A dodecagon is a 12-sided polygon.

Check Your Understanding

Practise

For help with #3 and #4, refer to the Example on pages 453–454.

3. Identify the two regular polygons used to create each tessellation.

a)

b)

c)

4. What type of transformation could be used to create each tessellation in #3?

Apply

5. The diagram shows a garden path made from irregular 12-sided bricks.

a) Explain why the 12-sided brick tessellates the plane.
b) Use grid paper to design an irregular ten-sided brick that could be used to make a path.
c) Explain why your ten-sided brick tessellates the plane.
d) Use grid paper to design an irregular six-sided brick that could be used to tessellate the plane.
e) Explain why your six-sided brick tessellates the plane.

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Key Ideas

The Key Ideas develop the concept that different combinations of shapes and transformations can be used to create the same tiling pattern. Have students read and review the Key Ideas section. Student could prepare their own list of Key Ideas and put it in their chapter Foldable.

Communicate the Ideas

You may wish to have students complete these questions in groups and discuss their answers. For #1, students need to consider how to explain why some tessellations can be made using translations and can also be made using reflections. For #2, students need to figure out which description of a tessellation is correct. Use students' responses to determine their understanding of how different combinations of shapes and transformations can be used to tile the plane and why. Briefly discuss the major points after students have attempted the questions.

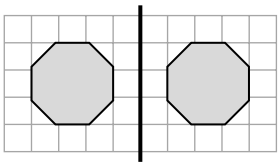
Meeting Student Needs

- Students may find it beneficial to develop visual examples for each point in the Key Ideas. Have them label the visuals with relevant points, such as the sum of interior angles where the vertices of polygons meet, the type of transformation used, and the area of the tessellating tile.

Answers

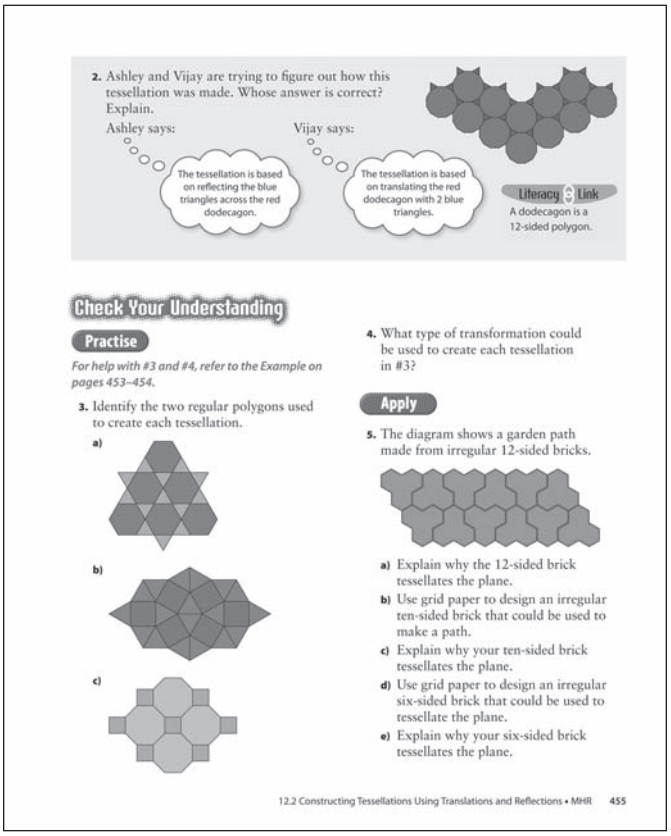
Communicate the Ideas

1. Answers may vary. Example: For many symmetrical shapes, translating the shape produces the same result as reflecting the shape in a line between the shape and its image.



2. Both answers are correct. Answers may vary. Example: Both answers result in the same pattern of equilateral triangles and dodecagons.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1.	• As a class, discuss possible answers to #1. Focus on the idea that combinations of different shapes and transformations can be used to create the same pattern. • Some students may benefit from using hands-on manipulatives to try out different transformations. • Ensure students understand why some designs or shapes tessellate the plane and others do not. • Use an overhead and tiling patterns or coloured transparency paper to model designs that will work and designs that will not. • Have students write a summary in their own words in their Foldable.



2. Ashley and Vijay are trying to figure out how this tessellation was made. Whose answer is correct? Explain.

Ashley says: The tessellation is based on reflecting the blue triangles across the red dodecagon.

Vijay says: The tessellation is based on translating the red dodecagon with 2 blue triangles.

Literacy Link: A dodecagon is a 12-sided polygon.

Check Your Understanding

Practise

For help with #3 and #4, refer to the Example on pages 453–454.

3. Identify the two regular polygons used to create each tessellation.

a) b) c)

4. What type of transformation could be used to create each tessellation in #3?

Apply

5. The diagram shows a garden path made from irregular 12-sided bricks.

a) Explain why the 12-sided brick tessellates the plane.
b) Use grid paper to design an irregular ten-sided brick that could be used to make a path.
c) Explain why your ten-sided brick tessellates the plane.
d) Use grid paper to design an irregular six-sided brick that could be used to tessellate the plane.
e) Explain why your six-sided brick tessellates the plane.

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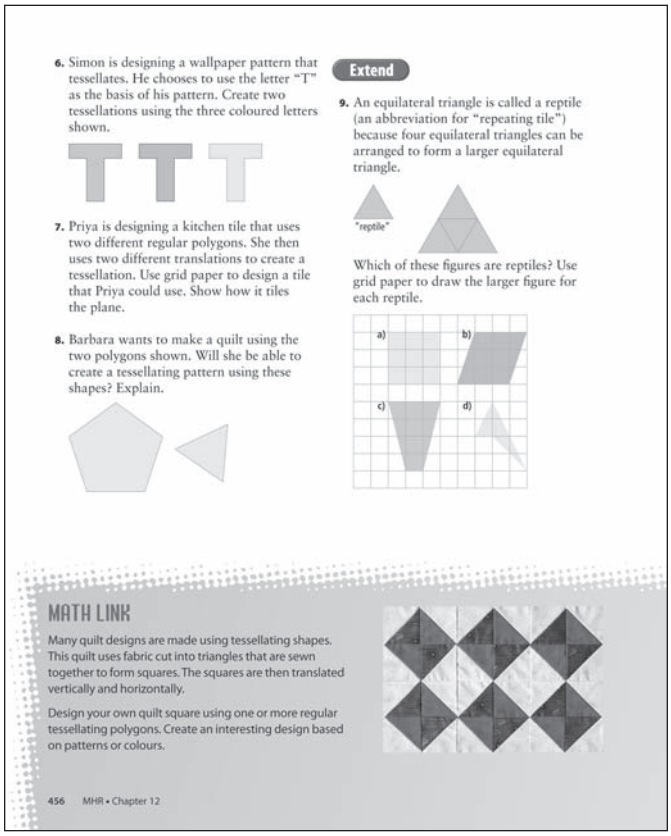

6. Simon is designing a wallpaper pattern that tessellates. He chooses to use the letter "T" as the basis of his pattern. Create two tessellations using the three coloured letters shown.

7. Priya is designing a kitchen tile that uses two different regular polygons. She then uses two different translations to create a tessellation. Use grid paper to design a tile that Priya could use. Show how it tiles the plane.

8. Barbara wants to make a quilt using the two polygons shown. Will she be able to create a tessellating pattern using these shapes? Explain.

Extend

9. An equilateral triangle is called a reptile (an abbreviation for "repeating tile") because four equilateral triangles can be arranged to form a larger equilateral triangle.

"reptile"

Which of these figures are reptiles? Use grid paper to draw the larger figure for each reptile.

a) b) c) d)

MATH LINK

Many quilt designs are made using tessellating shapes. This quilt uses fabric cut into triangles that are sewn together to form squares. The squares are then translated vertically and horizontally.

Design your own quilt square using one or more regular tessellating polygons. Create an interesting design based on patterns or colours.

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Check Your Understanding

Practise

Students will differ in the extent to which they continue to rely on concrete manipulatives to model tessellations. Assign Practise questions accordingly, and support students as necessary. Some students may benefit from working with a partner or in a group.

For #3 and #4, ensure that students are able to identify the correct shape combinations and transformations used to create a given tiling pattern.

Apply

For the Apply questions, students need to be able to apply their knowledge of regular and irregular polygons and understanding of translations and reflections to answer problems. Provide students with **Master 9 0.5 Centimetre Grid Paper** to complete #6 and #7 and protractors to complete #6 to #8.

Extend

The Extend problem requires students to use greater creativity when visualizing, designing, and creating patterns. Provide students with **Master 9 0.5 Centimetre Grid Paper** to complete #9.

Math Link

In this Math Link, students explore how quilt designs are made using tessellating shapes. Students are then asked to design their own quilt using tessellating polygons. Make sure that students select a regular polygon to create their pattern. The polygon must be one that can be used to tile the plane. Consider allowing students to practise tiling a plane by using the virtual manipulative described in the Web Link below. Ensure that students use the correct transformations (translations or reflections).

Meeting Student Needs

- Provide **BLM 12–11 Section 12.2 Extra Practice** to students who would benefit from more practice.

ELL

- Have students create a chart of the names of regular polygons, along with a picture of each one, to help them answer #3.

Web Link

For a virtual manipulative that students can use to practise tiling the plane, go to www.mathlinks8.ca and follow the links.

Common Errors

- Students may use combinations of shapes that do not have interior angles that sum to 360°, thus creating tile patterns with gaps or overlaps.

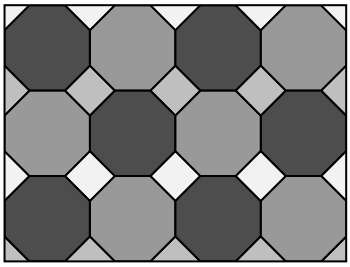
R_x Encourage students to measure the interior angles of the different shapes before using them to create a tiling pattern.

Answers

Math Link

Answers may vary. Look for the following:
- The quilt design should include one or more regular tessellating polygons.
- The design should be created using patterns or colours.

Example:



Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #3, #4, and #6. Students who have no problems with these questions can go on to do the rest of the Apply questions.	• Provide additional coaching with the Example to students who need assistance with #3 and #4. Have them verbalize their understanding. • Students may benefit from having manipulatives available that they can place over the given design. This will allow them to re-create the shape and see the effects of immediate changes. • Encourage students who need assistance with #6 to cut out models using stencils to translate the object. Ensure that they use a protractor to measure the angles of the vertices that meet.
Math Link The Math Link on page 456 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 469.	• Make sure that all students do this Math Link, since they will use these basic skills when they design and construct their own mosaic in the Wrap It Up! • Students may benefit from cutting out models from construction paper to start the problem. • You may wish to assign a single shape (such as a square or equilateral triangle) to students to simplify the question. • To help them get started, some students may benefit from using **BLM 12–12 Section 12.2 Math Link**, which provides scaffolding for this activity.
Assessment *as* Learning	
Math Learning Log Have students answer the following questions: • Why can different combinations of polygons result in the same shape used in a tiling pattern? • How can a translation of a specific shape result in the same pattern as a reflection?	• Encourage students to use diagrams or tiles when creating combinations of polygons to use to make a pattern. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.