12.3 Constructing Tessellations Using Rotations

***MathLinks 8*, pages 457–460**

Suggested Timing
50–60 minutes

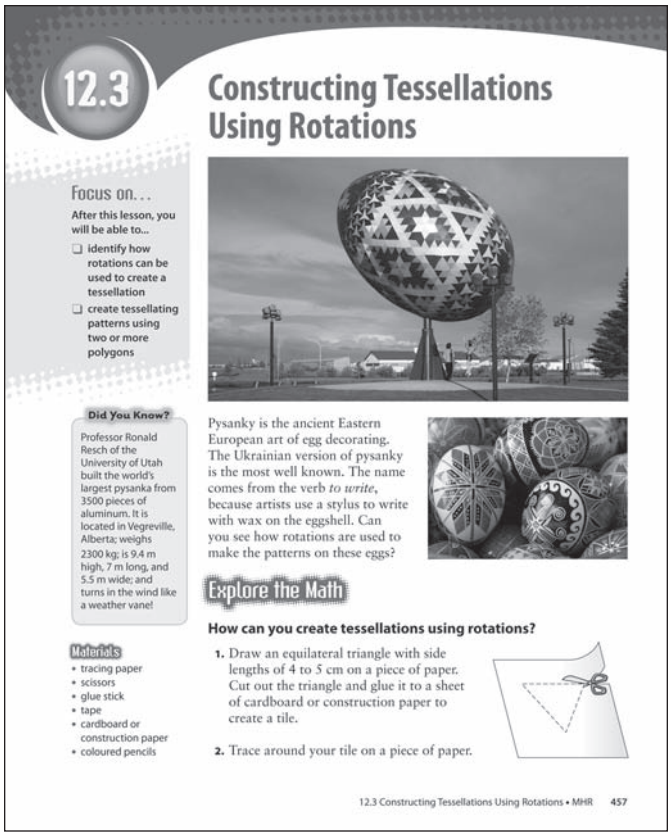
Materials
- tracing paper
- scissors
- glue stick
- transparent tape
- cardboard or construction paper
- coloured pencils
- ruler
- pattern blocks of equilateral triangles (optional)
- transparent shapes (optional)
- protractor

Blackline Masters
Master 7 Isometric Dot Paper
Master 9 0.5 Centimetre Grid Paper
BLM 12–3 Chapter 12 Warm-Up
BLM 12–10 Shapes
BLM 12–13 Extend Chart
BLM 12–14 Section 12.3 Extra Practice
BLM 12–15 Section 12.3 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☐ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
SS6 Demonstrate an understanding of tessellation by:
- explaining the properties of shapes that make tessellating possible
- creating tessellations
- identifying tessellations in the environment.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1–4, Math Link
Typical	1–4, 5 *or* 6, 7, Math Link
Extension/Enrichment	1, 2, 6–9


12.3 Constructing Tessellations Using Rotations

Focus on...
After this lesson, you will be able to...
- identify how rotations can be used to create a tessellation
- create tessellating patterns using two or more polygons

Did You Know?
Professor Ronald Resch of the University of Utah built the world's largest pysanka from 3500 pieces of aluminum. It is located in Vegreville, Alberta; weighs 2300 kg; is 9.4 m high, 7 m long, and 5.5 m wide; and turns in the wind like a weather vane!

Pysanky is the ancient Eastern European art of egg decorating. The Ukrainian version of pysanky is the most well known. The name comes from the verb *to write*, because artists use a stylus to write with wax on the eggshell. Can you see how rotations are used to make the patterns on these eggs?

Explore the Math

How can you create tessellations using rotations?

Materials
- tracing paper
- scissors
- glue stick
- tape
- cardboard or construction paper
- coloured pencils

1. Draw an equilateral triangle with side lengths of 4 to 5 cm on a piece of paper. Cut out the triangle and glue it to a sheet of cardboard or construction paper to create a tile.
2. Trace around your tile on a piece of paper.

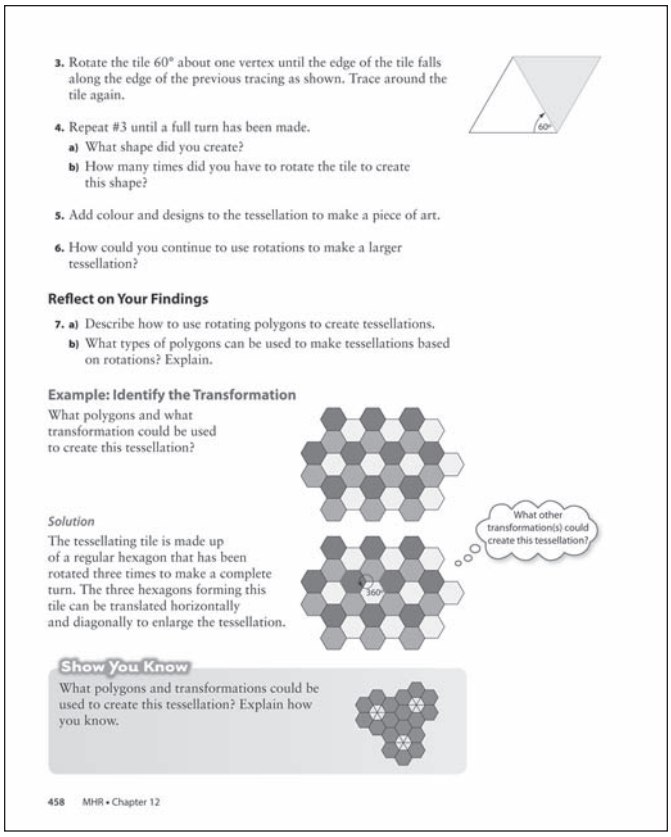
12.3 Constructing Tessellations Using Rotations • MHR 457

Planning Notes

Have students complete the warm-up questions on **BLM 12–3 Chapter 12 Warm-Up** to reinforce material learned in previous sections.

As a class, read and discuss the information about pysanky presented in the student resource. Have students discuss how rotations were used to make the patterns in the photos.

Reactivate students' knowledge about the characteristics of regular and irregular polygons (e.g., triangles, rectangles, pentagons, hexagons). Ask students to identify where they see these shapes in the real world.



3. Rotate the tile 60° about one vertex until the edge of the tile falls along the edge of the previous tracing as shown. Trace around the tile again.

4. Repeat #3 until a full turn has been made.
   a) What shape did you create?
   b) How many times did you have to rotate the tile to create this shape?

5. Add colour and designs to the tessellation to make a piece of art.

6. How could you continue to use rotations to make a larger tessellation?

Reflect on Your Findings

7. a) Describe how to use rotating polygons to create tessellations.
   b) What types of polygons can be used to make tessellations based on rotations? Explain.

Example: Identify the Transformation

What polygons and what transformation could be used to create this tessellation?

Solution

The tessellating tile is made up of a regular hexagon that has been rotated three times to make a complete turn. The three hexagons forming this tile can be translated horizontally and diagonally to enlarge the tessellation.

What other transformation(s) could create this tessellation?

Show You Know

What polygons and transformations could be used to create this tessellation? Explain how you know.

458 MHR • Chapter 12

Explore the Math

In this exploration, students use rotations to make a tessellation. They then identify the types of polygons that can result from rotations.

Method 1 Have students work in pairs to complete the exploration and discuss their answers. You may wish to give each student a pattern block of an equilateral triangle to use for reference. Ensure students draw the appropriate size of triangle. For #3, ensure students understand how the tile is to be rotated about one vertex.

Have students answer #6 and #7 independently and then compare their answers with a partner.

Method 2 Demonstrate the method for creating tessellations using rotations. Use the overhead or chart paper to show students how to create a tile and then rotate the tile 60° about one vertex. Continue demonstrating to students how to rotate the tile, and then have them complete the exploration on their own.

Have students answer #6 and #7 independently and then compare their answers with a partner or in a whole-class discussion.

Example

Reinforce the idea that in order to create a tessellation, the interior angles where the polygons meet must total exactly 360°. You may wish to demonstrate the rotation of hexagons using transparent shapes on the overhead.

After discussing the Example, have students complete the Show You Know question to make sure that they are ready to move on.

Meeting Student Needs

- Some students may benefit from using **Master 7 Isometric Dot Paper** to draw an equilateral triangle. They can draw the tessellation on the same paper.
- For #3 in the exploration, some students may benefit from a demonstration of the rotation using transparent shapes on the overhead.
- Encourage students to re-create a given tessellation using their descriptions of shapes and rotations using pattern blocks.
- You may wish to have students use a computer to create a design of a tessellation and rotate the design.

Common Errors

- Some students may attempt to rotate polygons in which the interior angles where the shapes meet do not add to 360°.

R_x Encourage students to use a protractor to measure the interior angles where the polygon they are rotating meets the other polygon(s). Manipulate various transparent shapes on the overhead to demonstrate that polygons whose interior angles do not sum to 360° where the vertices meet cannot tile the plane.

Answers

Explore the Math

4. a) hexagon

b) five times

6. Answers may vary. Example: Rotate the tile 60° about another vertex and repeat #3, or repeat #3 with an image polygon.

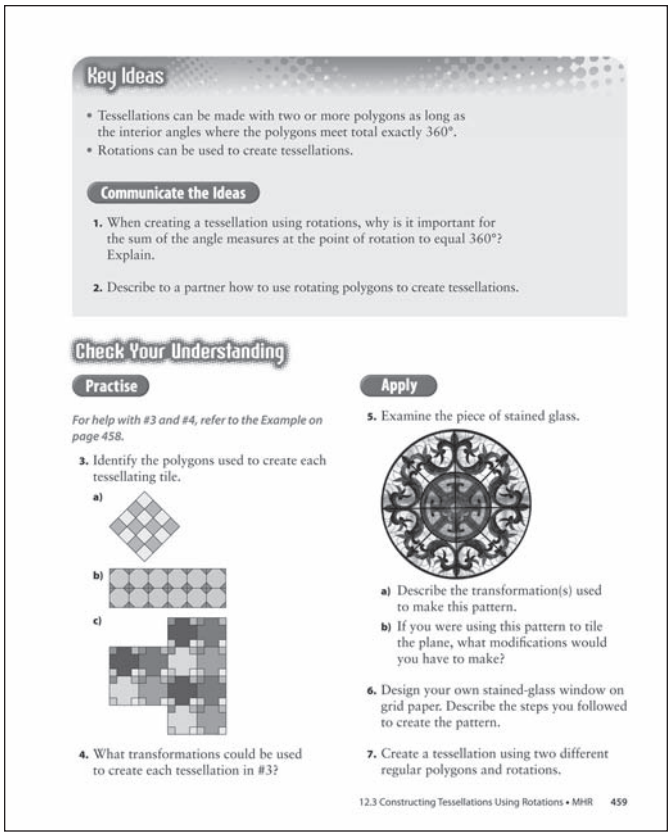
7. a) Answers may vary. Example: Use one vertex of the polygon as the turn centre and rotate the polygon until a full turn is made. Repeat with another vertex or rotate some image polygons about their vertices.

b) Answers may vary. Example: Regular polygons with the interior angle measure that is a factor of 360°; these polygons can be rotated about one vertex to make a full turn without overlapping or leaving gaps.

Show You Know: Example

Answers may vary. Example: A regular hexagon and an equilateral triangle could be used to create the tessellation by rotation. The yellow hexagon is formed by rotating the equilateral triangle about one of its vertices. The shape with a yellow hexagon in the centre of six blue hexagons can be rotated about a vertex of the blue hexagon to form the tessellation.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss how to use rotations to create a given tiling pattern. Try to have students generalize conclusions about their findings. Students should understand that the shapes used must have interior angles where the polygons meet that sum to 360°.	• Encourage students to use manipulatives or computer programs to re-create a given tessellation using different combinations of shapes and rotations. • Students may benefit from a discussion of the answer to #7b). It is important that students identify polygons that can be used to make a tessellation. • Have students verbalize their thinking and assist them with any misunderstandings.
Assessment *for* Learning	
Example Have students do the Show You Know related to the Example.	• Have students work with a partner and discuss their thinking. • You may wish to provide additional examples and non-examples of shape combinations and rotations that can be used to create the same tiling pattern. • Using an overhead with different shapes in different colours of acetate provides an effective visual for showing which shapes can be combined and which rotations result in the desired tiling pattern.



Key Ideas

- Tessellations can be made with two or more polygons as long as the interior angles where the polygons meet total exactly 360°.
- Rotations can be used to create tessellations.

Communicate the Ideas

1. When creating a tessellation using rotations, why is it important for the sum of the angle measures at the point of rotation to equal 360°? Explain.
2. Describe to a partner how to use rotating polygons to create tessellations.

Check Your Understanding

Practise

For help with #3 and #4, refer to the Example on page 458.

3. Identify the polygons used to create each tessellating tile.

a)

b)

c)

4. What transformations could be used to create each tessellation in #3?

Apply

5. Examine the piece of stained glass.

a) Describe the transformation(s) used to make this pattern.

b) If you were using this pattern to tile the plane, what modifications would you have to make?

6. Design your own stained-glass window on grid paper. Describe the steps you followed to create the pattern.

7. Create a tessellation using two different regular polygons and rotations.

12.3 Constructing Tessellations Using Rotations • MHR 459

Have students read and review the Key Ideas. Have students explain how rotations can be used to create tessellations. You may wish to provide additional examples of polygons that do and do not tile the plane.

Communicate the Ideas

You may wish to have students complete #1 in groups and discuss their answers. Encourage students to use manipulatives or diagrams to explain their answer. For #2, use students' responses to determine their understanding of how to use rotating polygons to create tessellations. Briefly discuss the major points after students have attempted the questions.

Meeting Student Needs

- Students may find it beneficial to develop visuals showing an example and a non-example of polygons with interior angles where the polygons meet that total exactly 360°. Have them label the tessellation and show how the interior angles add to 360°. Have them put a large X through the non-tessellation and show how the interior angles add to more or less than 360°.

Common Errors

- Students may use combinations of shapes that do not have interior angles that sum to 360°, thus creating tile patterns with gaps or overlaps.

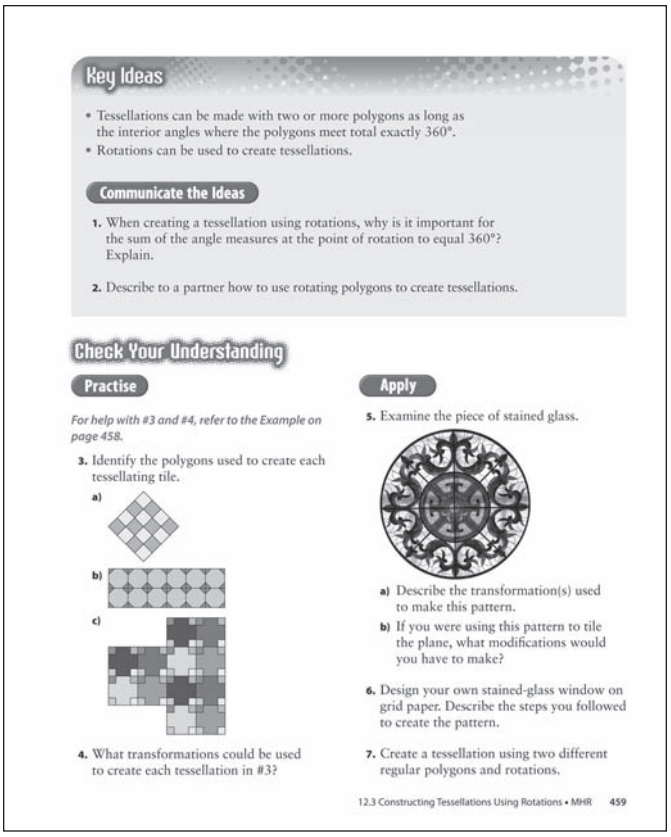
R_x Encourage students to measure the interior angles of the different shapes before using them to create a tiling pattern.

Answers

Communicate the Ideas

1. Answers may vary. Example: If the angle sum is less than 360°, there will be gaps. If the angle sum is more than 360°, the shapes will overlap.
2. Answers may vary. Example:
   - Cut out a regular polygon with the interior angle measure that is a factor of 360°.
   - Trace the polygon on a piece of paper.
   - Rotate the polygon using a vertex as the turn centre until the edge of the polygon falls along the edge of the previous tracing.
   - Continue rotating and tracing until the plane is covered.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and #2.	• Consider having students work in groups to answer the questions. • As a class, discuss possible answers to #1. Focus on the idea that the shapes must have interior angles that sum to 360° at the point where the vertices meet, which is the point of rotation. • Encourage students to try creating patterns with different shapes.



Key Ideas

- Tessellations can be made with two or more polygons as long as the interior angles where the polygons meet total exactly 360°.
- Rotations can be used to create tessellations.

Communicate the Ideas

1. When creating a tessellation using rotations, why is it important for the sum of the angle measures at the point of rotation to equal 360°? Explain.
2. Describe to a partner how to use rotating polygons to create tessellations.

Check Your Understanding

Practise

For help with #3 and #4, refer to the Example on page 458.

3. Identify the polygons used to create each tessellating tile.

a)

b)

c)

4. What transformations could be used to create each tessellation in #3?

Apply

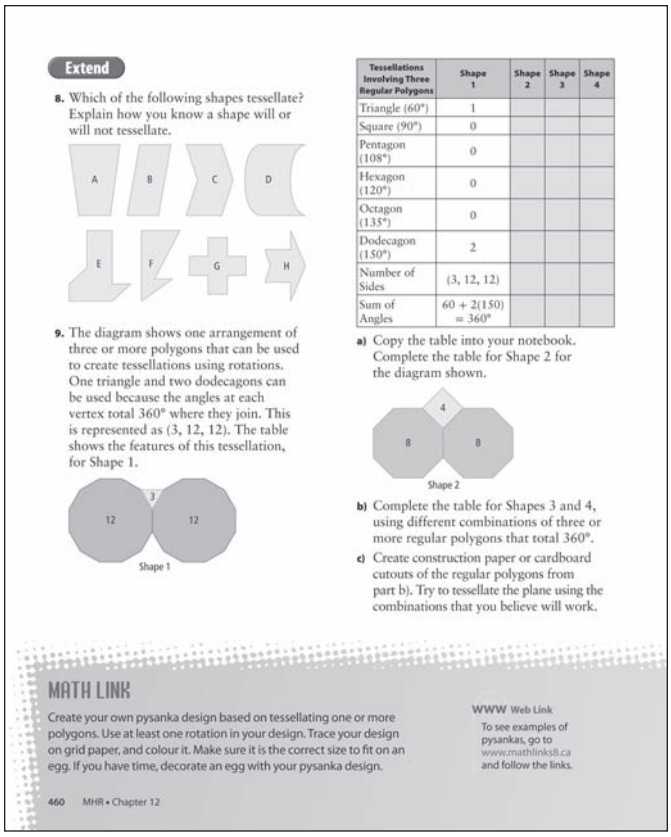
5. Examine the piece of stained glass.

a) Describe the transformation(s) used to make this pattern.

b) If you were using this pattern to tile the plane, what modifications would you have to make?

6. Design your own stained-glass window on grid paper. Describe the steps you followed to create the pattern.

7. Create a tessellation using two different regular polygons and rotations.



Extend

8. Which of the following shapes tessellate? Explain how you know a shape will or will not tessellate.

9. The diagram shows one arrangement of three or more polygons that can be used to create tessellations using rotations. One triangle and two dodecagons can be used because the angles at each vertex total 360° where they join. This is represented as (3, 12, 12). The table shows the features of this tessellation, for Shape 1.

Shape 1

Tessellations Involving Three Regular Polygons	Shape 1	Shape 2	Shape 3	Shape 4
Triangle (60°)	1			
Square (90°)	0			
Pentagon (108°)	0			
Hexagon (120°)	0			
Octagon (135°)	0			
Dodecagon (150°)	2			
Number of Sides	(3, 12, 12)			
Sum of Angles	60 + 2(150) = 360°			

a) Copy the table into your notebook. Complete the table for Shape 2 for the diagram shown.

Shape 2

b) Complete the table for Shapes 3 and 4, using different combinations of three or more regular polygons that total 360°.

c) Create construction paper or cardboard cutouts of the regular polygons from part b). Try to tessellate the plane using the combinations that you believe will work.

MATH LINK

Create your own pysanka design based on tessellating one or more polygons. Use at least one rotation in your design. Trace your design on grid paper, and colour it. Make sure it is the correct size to fit on an egg. If you have time, decorate an egg with your pysanka design.

WWW Web Link

To see examples of pysankas, go to www.mathlinks8.ca and follow the links.

Check Your Understanding

Practise

For #3 and #4, ensure that students are able to identify the correct shapes and transformations used to create a given tiling pattern.

Apply

For the Apply questions, students need to be able to apply their knowledge of regular and irregular polygons and understanding of rotations to answer problems.

For #6 and #7, provide students with **Master 7 Isometric Dot Paper** or **Master 9 0.5 Centimetre Grid Paper**.

Extend

The Extend problems require students to use greater creativity when visualizing, designing, and creating patterns. Provide students with protractors to complete #8 and #9. Students may benefit from using **BLM 12–13 Extend Chart** to complete #9.

Math Link

Make sure that students select a shape that can be used in a rotating pattern. Encourage students to measure the interior angles to show that they are using a shape that can be rotated.

Meeting Student Needs

- Students may benefit from using **BLM 12–10 Shapes** to complete #3.
- Provide **BLM 12–14 Section 12.3 Extra Practice** to students who would benefit from more practice.

ELL

- For #5 and #6, ensure students understand the meaning of *stained glass window*.

WWW Web Link

For a site that allows students to practise making tessellations using the computer, go to www.mathlinks8.ca and follow the links.

To view instructions on transferring your Math Link design to an egg, go to www.mathlinks8.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise Have students do #3 and #4. Students who have no problems with these questions can go on to the Apply questions.	• Provide additional coaching with the Example to students who need assistance with #3. • Students may benefit from using manipulative tiles to try out different combinations. • For #4, students may benefit from verbalizing the process as they identify the transformations used.
Math Link The Math Link on page 460 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 469.	• Have all students do this Math Link. They will use these basic skills when they design and construct their own mosaic in the Wrap It Up! • To help them get started, some students may benefit from using **BLM 12–15 Section 12.3 Math Link**, which provides scaffolding for this activity.
Assessment *as* Learning	
Math Learning Log Have students answer the following questions: • Why must the interior angles of polygons sum to 360° at the point of rotation? • Can irregular polygons be used when using rotation to create a tiling pattern? Explain.	• Encourage students to use diagrams or polygon tiles when rotating polygons (regular or irregular) to make a pattern. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.