12.4 Creating Escher-Style Tessellations

***MathLinks 8*, pages 461–465**

Suggested Timing
50–60 minutes

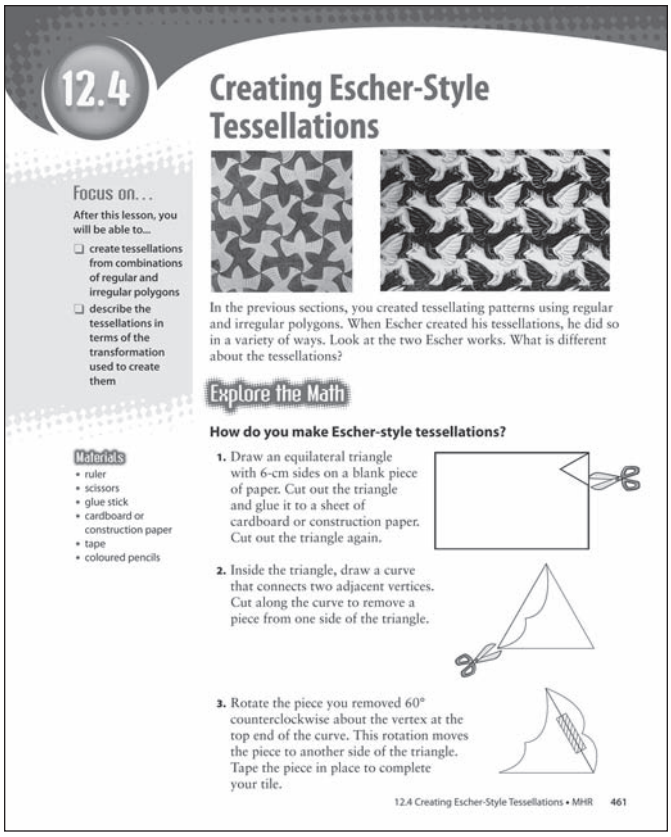
Materials
- ruler
- scissors
- glue stick
- cardboard or construction paper
- transparent tape
- coloured pencils
- pattern blocks (optional)
- protractor
- tracing paper (optional)

Blackline Masters
Master 7 Isometric Dot Paper
Master 9 0.5 Centimetre Grid Paper
BLM 12–3 Chapter 12 Warm-Up
BLM 12–16 Section 12.4 Extra Practice
BLM 12–17 Section 12.4 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☐ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
SS6 Demonstrate an understanding of tessellation by:
- explaining the properties of shapes that make tessellating possible
- creating tessellations
- identifying tessellations in the environment.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	1, 3–5, 8, Math Link
Typical	1, 3–5, 6 *or* 7, 8 *or* 9 *or* 10, Math Link
Extension/Enrichment	1, 3, 6 *or* 7, 8 *or* 9 *or* 10, 11


12.4 Creating Escher-Style Tessellations

Focus on...
After this lesson, you will be able to...
- create tessellations from combinations of regular and irregular polygons
- describe the tessellations in terms of the transformation used to create them

In the previous sections, you created tessellating patterns using regular and irregular polygons. When Escher created his tessellations, he did so in a variety of ways. Look at the two Escher works. What is different about the tessellations?

Explore the Math

How do you make Escher-style tessellations?

Materials
- ruler
- scissors
- glue stick
- cardboard or construction paper
- tape
- coloured pencils

1. Draw an equilateral triangle with 6-cm sides on a blank piece of paper. Cut out the triangle and glue it to a sheet of cardboard or construction paper. Cut out the triangle again.
2. Inside the triangle, draw a curve that connects two adjacent vertices. Cut along the curve to remove a piece from one side of the triangle.
3. Rotate the piece you removed 60° counterclockwise about the vertex at the top end of the curve. This rotation moves the piece to another side of the triangle. Tape the piece in place to complete your tile.

12.4 Creating Escher-Style Tessellations • MHR 461

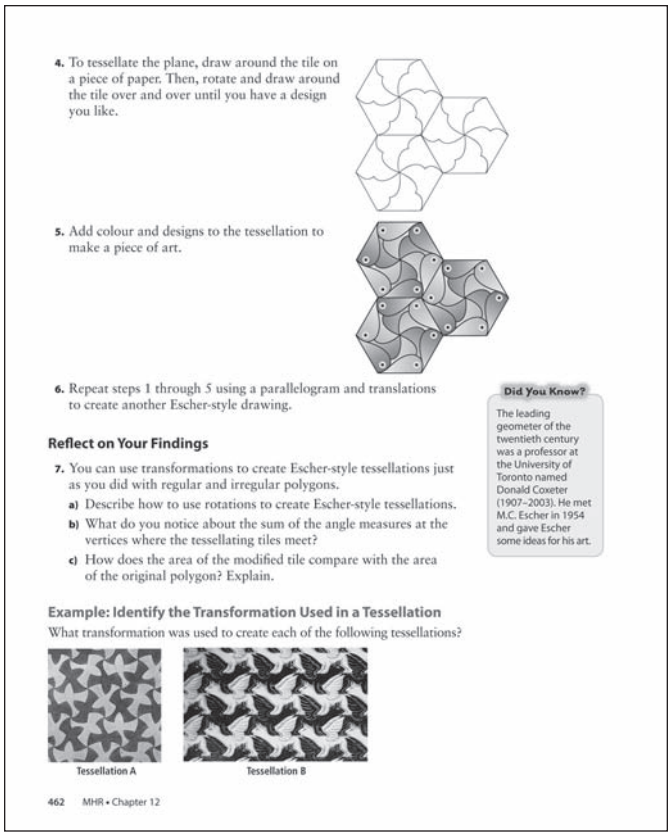
Planning Notes

Have students complete the warm-up questions on **BLM 12–3 Chapter 12 Warm-Up** to reinforce material learned in previous sections.

As a class, read and discuss the information about M.C. Escher's work presented in the student resource. Discuss with students what an Escher-style tessellation is. This should include a brief discussion of M.C. Escher and his contributions to art. Help students summarize what they learned in the previous sections and explain that they will be applying these skills to create their own Escher-style tessellation.

Explore the Math

In this exploration, students make Escher-style tessellations, and then confirm what they have already learned about using rotations to make tessellations, the sum of angle measures at the vertices where tessellating tiles meet, and conservation of area.



4. To tessellate the plane, draw around the tile on a piece of paper. Then, rotate and draw around the tile over and over until you have a design you like.

5. Add colour and designs to the tessellation to make a piece of art.

6. Repeat steps 1 through 5 using a parallelogram and translations to create another Escher-style drawing.

Did You Know?

The leading geometer of the twentieth century was a professor at the University of Toronto named Donald Coxeter (1907–2003). He met M.C. Escher in 1954 and gave Escher some ideas for his art.

Reflect on Your Findings

7. You can use transformations to create Escher-style tessellations just as you did with regular and irregular polygons.
   a) Describe how to use rotations to create Escher-style tessellations.
   b) What do you notice about the sum of the angle measures at the vertices where the tessellating tiles meet?
   c) How does the area of the modified tile compare with the area of the original polygon? Explain.

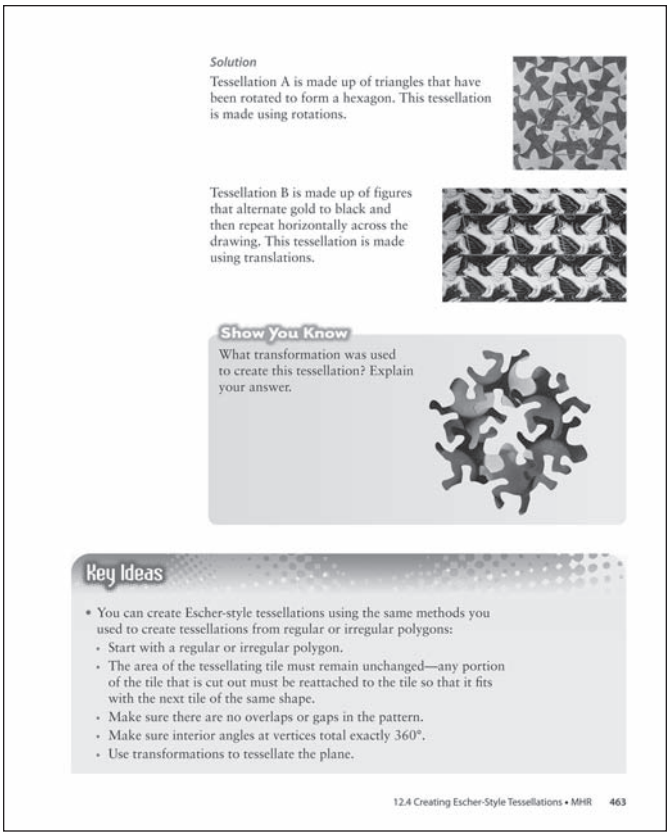
Example: Identify the Transformation Used in a Tessellation

What transformation was used to create each of the following tessellations?

Tessellation A

Tessellation B

462 MHR • Chapter 12



Solution

Tessellation A is made up of triangles that have been rotated to form a hexagon. This tessellation is made using rotations.

Tessellation B is made up of figures that alternate gold to black and then repeat horizontally across the drawing. This tessellation is made using translations.

Show You Know

What transformation was used to create this tessellation? Explain your answer.

Key Ideas

- You can create Escher-style tessellations using the same methods you used to create tessellations from regular or irregular polygons:
  - Start with a regular or irregular polygon.
  - The area of the tessellating tile must remain unchanged—any portion of the tile that is cut out must be reattached to the tile so that it fits with the next tile of the same shape.
  - Make sure there are no overlaps or gaps in the pattern.
  - Make sure interior angles at vertices total exactly 360°.
  - Use transformations to tessellate the plane.

12.4 Creating Escher-Style Tessellations • MHR 463

Method 1 Have students work in pairs to complete the exploration and discuss their answers. After students have completed #6, have them compare their tessellations to see how the patterns differ when using parallelograms or triangles.

Have students complete #7 independently and then discuss their answers with a partner.

Method 2 Demonstrate the method for making Escher-style tessellations using the overhead. Ensure students understand how to draw and cut the curve from the triangle and how to rotate the piece. When you have demonstrated the method, have students work independently or in pairs to complete the exploration.

Have students complete #7 in pairs, and then discuss the answers as a class.

The Did you Know? beside the Reflect on Your Findings question points out the link between geometry and M.C. Escher's work.

Example

Reinforce using concrete materials to help students identify the shapes that make up the tessellations.

After discussing the Example, have students complete the Show You Know question to make sure that they are ready to move on.

Meeting Student Needs

- Encourage students to work with concrete objects or computer programs to help them identify the shapes and transformations.

ELL

- Ensure students understand the following ter ms: *reflected*, *repeated*, *overlaps*, *gaps*, *regular* and *irregular polygons*, and *principles*.

Common Errors

- These shapes are more complex, so students may have difficulty identifying the original shape and determining how it was modified and translated.

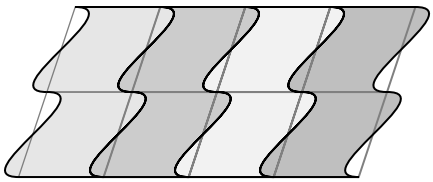
R_x Encourage students to use concrete materials to help them identify the shapes used in the tessellations.

Answers

Explore the Math

6. Answers may vary. Look for the following:
- The drawing should be made up of a parallelogram and translations.
- There should be no gaps between the tiles.

Example:



7. a) Answers may vary. Example:
- Cut out a regular polygon.
- Trace it on a piece of paper.
- Remove a piece from one side of the polygon.
- Rotate this piece about a vertex of the polygon.
- Tape it to the original piece to complete a tile.
- Rotate the tile and draw around it to create an Escher-style tessellation.

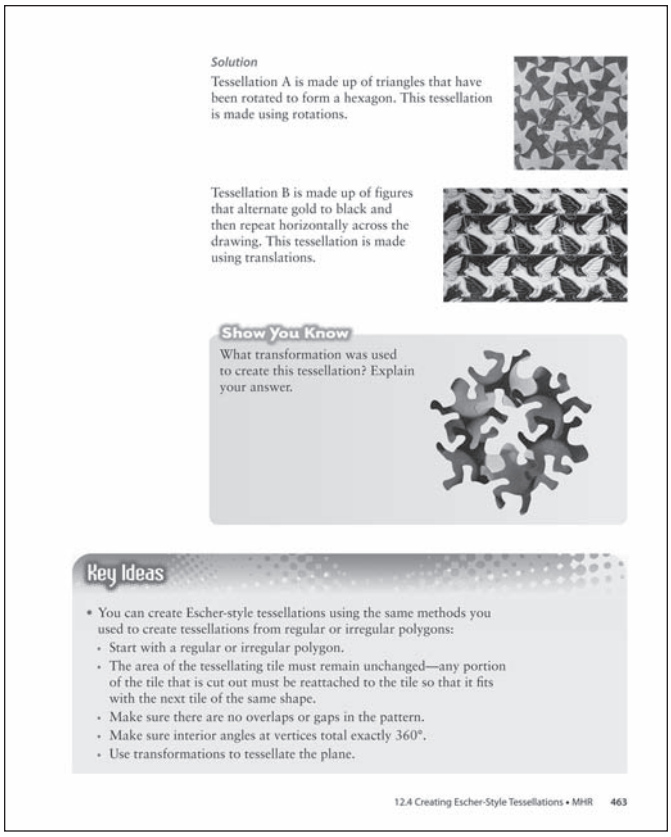
b) Answers may vary. Example: The sum of the angles at the vertices is 360°.

c) Answers may vary. Example: The area remains unchanged.

Show You Know: Example

Answers may vary. Example: A 60° rotation was used, since the same shape has been rotated five times to form the tessellation.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect on Your Findings Listen as students discuss how to use rotations to create an Escher-style tessellation. Try to have students generalize their conclusions. Students should understand that the shapes used must have interior angles that sum to 360° at the point of rotation.	• Encourage students to use manipulatives or computer programs to create Escher-style tessellations using different combinations of shapes, rotations, translations, and reflections. • For #7b), students may benefit from using a protractor to verify the measurements of the angles.
Assessment *for* Learning	
Example Have students do the Show You Know related to the Example.	• Have students talk through their thinking and work with a partner. • You may wish to provide additional examples of Escher-style tessellations that clearly show the shape and transformations used. • Use an overhead with different Escher-style tessellations on acetate to provide an effective visual for outlining the original shape, how it was altered, and the transformations used to create the tessellation. • Students may benefit from using manipulatives of basic shapes. • You may wish to modify the assignment to use basic designs of a square with a corner cut off, to help students get started.



Solution

Tessellation A is made up of triangles that have been rotated to form a hexagon. This tessellation is made using rotations.

Tessellation B is made up of figures that alternate gold to black and then repeat horizontally across the drawing. This tessellation is made using translations.

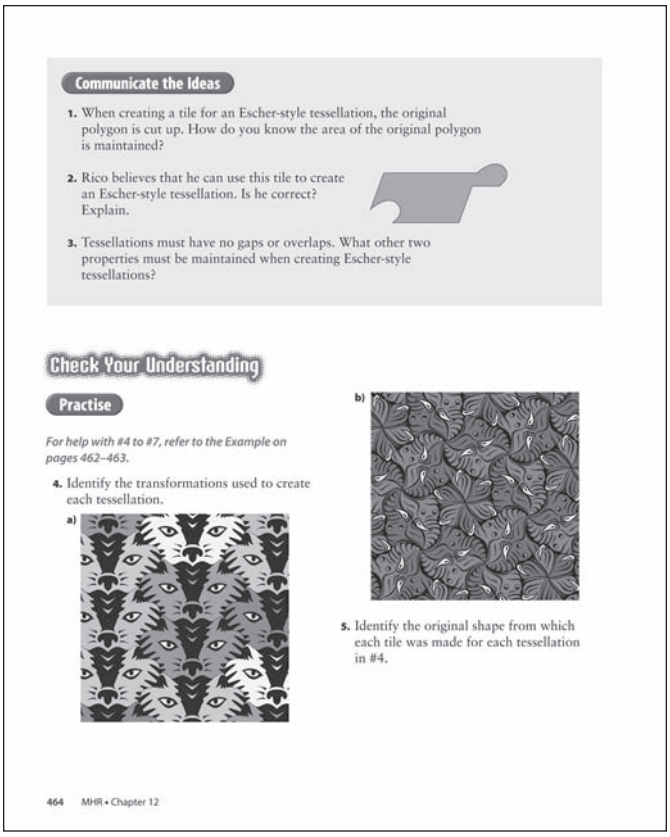
Show You Know

What transformation was used to create this tessellation? Explain your answer.

Key Ideas

- You can create Escher-style tessellations using the same methods you used to create tessellations from regular or irregular polygons:
  - Start with a regular or irregular polygon.
  - The area of the tessellating tile must remain unchanged—any portion of the tile that is cut out must be reattached to the tile so that it fits with the next tile of the same shape.
  - Make sure there are no overlaps or gaps in the pattern.
  - Make sure interior angles at vertices total exactly 360°.
  - Use transformations to tessellate the plane.

12.4 Creating Escher-Style Tessellations • MHR 463



Communicate the Ideas

1. When creating a tile for an Escher-style tessellation, the original polygon is cut up. How do you know the area of the original polygon is maintained?
2. Rico believes that he can use this tile to create an Escher-style tessellation. Is he correct? Explain.
3. Tessellations must have no gaps or overlaps. What other two properties must be maintained when creating Escher-style tessellations?

Check Your Understanding

Practise

For help with #4 to #7, refer to the Example on pages 462–463.

4. Identify the transformations used to create each tessellation.

a)

b)

5. Identify the original shape from which each tile was made for each tessellation in #4.

464 MHR • Chapter 12

Have students read and review the Key Ideas. Remind students that they can create Escher-style tessellations from regular or irregular polygons, as long as the interior angles where the vertices meet total exactly 360°. Review the process as outlined in the Key Ideas section. Provide examples of translations, reflections, and rotations, as necessary. Have students prepare their own summary of the Key Ideas and record them in their chapter Foldable.

Communicate the Ideas

You may wish to have students complete the questions in groups and discuss their answers. For #1, you may wish to have students actually cut up a polygon to explore how the area is maintained.

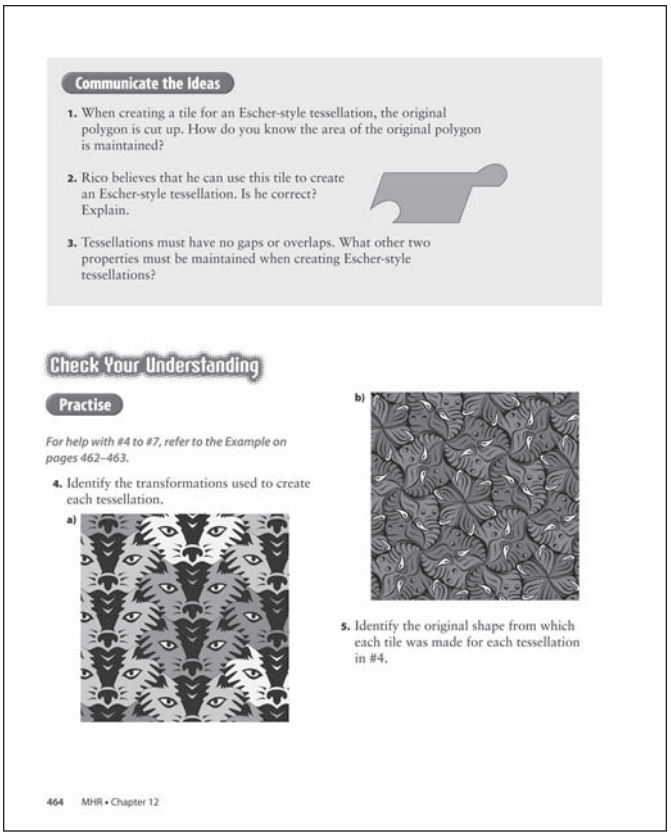
Encourage students to use a protractor to measure the angles in #2. Use students' responses to determine their understanding of how polygon shapes can be altered and rotated to form Escher-style tessellations. Briefly discuss the major points after students have attempted the questions.

Answers

Communicate the Ideas

1. Answers may vary. Example: The pieces that are cut out are attached back to the original shape to form the tile.
2. Yes. Answers may vary. Example: Think of the original piece as a parallelogram. The piece that was cut out is the same size and shape as the piece that sticks out of the parallelogram.
3. Answers may vary. Example: The interior angles at the vertices total exactly 360° and the area of the tessellating tile remains unchanged.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and #3.	• As a class, discuss possible answers to #1 and #3. Focus on the idea that the shapes must have interior angles that sum to 360° at the point where the tessellating tiles meet. • Students may benefit from observing while you cut off a piece of a polygon and reattach it, to confirm that area is maintained. • Have students write the properties of Escher-style tessellations in their Foldable.



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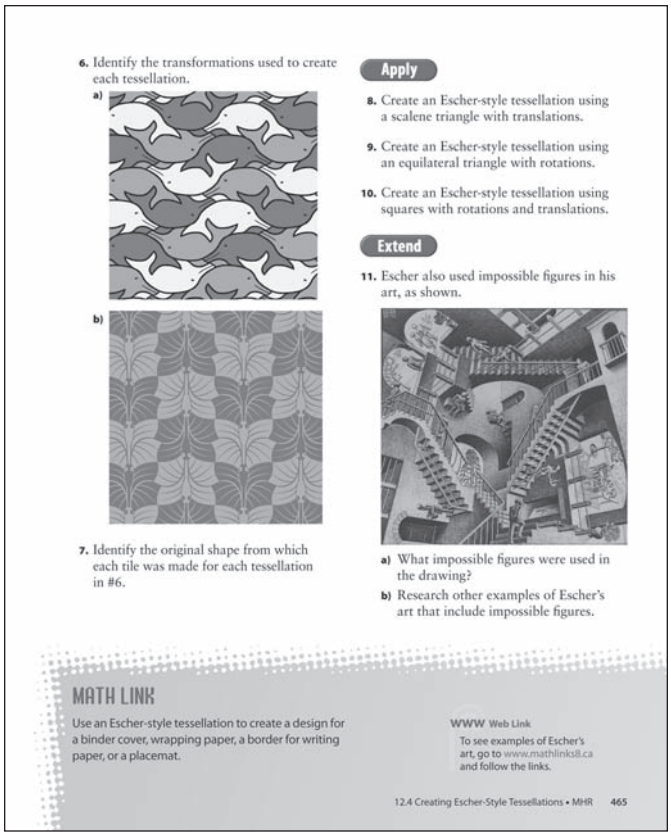
464 MHR • Chapter 12



6. Identify the transformations used to create each tessellation.

a)

b)

7. Identify the original shape from which each tile was made for each tessellation in #6.

Apply

8. Create an Escher-style tessellation using a scalene triangle with translations.

9. Create an Escher-style tessellation using an equilateral triangle with rotations.

10. Create an Escher-style tessellation using squares with rotations and translations.

Extend

11. Escher also used impossible figures in his art, as shown.

a) What impossible figures were used in the drawing?

b) Research other examples of Escher's art that include impossible figures.

MATH LINK

Use an Escher-style tessellation to create a design for a binder cover, wrapping paper, a border for writing paper, or a placemat.

WWW Web Link

To see examples of Escher's art, go to www.mathlinks8.ca and follow the links.

12.4 Creating Escher-Style Tessellations • MHR 465

Check Your Understanding

Practise

For #4 and #6, ensure that students are able to identify the correct transformations used to create a given Escher-like tessellation.

Apply

For the Apply questions, students need to be able to apply their knowledge from the entire chapter to answer problems. You may wish to provide students with pattern blocks, **Master 7 Isometric Dot Paper**, and **Master 9 0.5 Centimetre Grid Paper** to complete these questions. They will also need protractors.

Math Link

In this Math Link, students use an Escher-style tessellation to create a design. Make sure that students select shapes that can be altered and used in a rotating pattern that can then be translated or reflected. Ensure that they measure the interior angles to show that they are using a shape that can be rotated.

Extend

In #11, students are asked to identify the impossible shapes used in M.C. Escher's art. You may wish to have students work in pairs to complete this question.

Meeting Student Needs

- Provide **BLM 12–16 Section 12.4 Extra Practice** to students who would benefit from more practice.

Common Errors

- Students may change the shapes in ways that result in the interior angles not adding up to 360° at the point of rotation, thus creating tessellations with gaps or overlaps.

R_x Encourage students to measure the interior angles of the different shapes before using them to create a tiling pattern.

Web Link

For examples of tessellations by Escher and others, along with different ways of creating tessellations that students can explore, go to www.mathlinks8.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #4, #5, and # 8. Students who have no problems with these questions can go on to do the remaining Apply questions.	• Students who need assistance with #4 may need additional coaching with the Example to identify the type of transformations used. • For #5, students may benefit from using manipulatives or tracing paper. Encourage students to build a similar model with manipulatives in order to identify shapes or have them trace the shapes on tracing paper and identify the individual shapes that way. • Students who need assistance with #8 may need to review their work on the Explore the Math.
Math Link The Math Link on page 465 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 469.	• Have all students do this Math Link, since they will use these basic skills when they design and construct their own mosaic in the Wrap It Up! • Encourage students to design something that has meaning to them. • You may wish to ask additional questions to clarify the task. For example: – Which shapes and colours would you use in your tessellation? – Where could you use the tessellation? To fill in the lettering? As a border? As a central design? • Discuss students' designs and have them show you one of the patterns before getting started on their final product. • To help them get started, some students may benefit from using **BLM 12–17 Section 12.4 Math Link**, which provides scaffolding for this activity.
Assessment *as* Learning	
Math Learning Log Have students answer the following questions: • What is the result if the interior angles of the polygons where the tessellating tiles meet do not sum to 360°? • How do you know that the area of the original polygon used in an Escher-style tessellation is not changed?	• Encourage students to use diagrams or polygon tiles when rotating polygons (regular or irregular). • Students may benefit from using tracing paper to determine the shapes that make up a pattern. • Encourage students to use the What I Need to Work On section of their chapter Foldable to note what they continue to have difficulties with.