TEACHER'S RESOURCE

McGraw-Hill Ryerson

MathLinks 9

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McGraw-Hill Ryerson
MathLinks 9 Teacher's Resource

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Please quote the ISBN and title when placing your order.

ISBN-13: 978-0-07-097359-6
ISBN-10: 0-07-097359-8

http://www.mcgrawhill.ca

1 2 3 4 5 6 7 8 9 0 XBS 0 9

Printed and bound in Canada

MATH PUBLISHER: Linda Allison
PROJECT MANAGER: Helen Mason
CONTENT MANAGER: Susan Till
DEVELOPMENTAL EDITORS: Susan Till, Kelly Cochrane, Ann Firth, Adrienne Montgomerie, Rita Vanden Heuvel
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EDITORIAL COORDINATOR: Jennifer Keay
ASSISTANT PROJECT COORDINATOR: Janie Reeson
MANAGER, PRODUCTION SERVICES: Yolanda Pigden
SENIOR PRODUCTION COORDINATOR: Paula Brown
COVER DESIGN: Valid Design & Layout
INTERIOR DESIGN: Pronk & Associates
ELECTRONIC PAGE MAKE-UP: ArtPlus Ltd. & SR Nova Pvt. Ltd.
COVER IMAGE: Emanuele Taroni

To the Teacher

The McGraw-Hill Ryerson *MathLinks* program has several components.

Student Resource

The student resource introduces topics in real-world contexts. Each chapter starts with a two-page spread that introduces students to what they will learn in the chapter. This includes a **Literacy Link** that provides tips to help the adolescent learner read, interpret, and summarize the chapter content.

FOLDABLES™ Study Tool

Each chapter provides a **Foldable™** to help students keep track of what they are learning and need to work on. Each Foldable helps students to organize what they are learning and to self-assess their progress.

This is followed by an introduction to the **Math Link**. The Math Link introduction:

- engages students
- reactivates student learning and reinforces skills holistically
- encourages exploration
- generates discussions
- increases thoughtfulness

Work on the Math Link throughout the chapter promotes depth of understanding through listening, speaking, reading, and writing math, thus enabling student connections and increased conceptual understanding.

In each section, **Explore** activities encourage students to develop their own understanding of new concepts.

This is followed by **Link the Ideas**. Some of these start with a piece of text that will help students connect what they did in the Explore to the examples that follow. Worked **examples** present solutions in a clear, step-by-step manner. The **Link the Ideas** ends with **Key Ideas** that summarize the new principles.

The third part of each lesson is the **Check Your Understanding**. **Communicate the Ideas** questions at the beginning of this section help students communicate their understanding of what they are learning. The following questions allow students to **Practise**, **Apply**, and **Extend** the skills and concepts.

The student resource includes sections that can be used as assessment tools: **chapter review**, **practice test**, **Wrap It Up!**, **Challenge in Real Life**, and **cumulative review**. Technology is integrated throughout the program and includes the use of calculators and the Internet.

Teacher's Resource

The teaching and assessment suggestions that are provided in this Teacher's Resource include:

- sample responses for the **Explore the Math** questions
- sample responses for the **Communicate the Ideas** questions
- common student errors and suggested remedies
- suggestions for **Assessment *as* Learning**, **Assessment *for* Learning**, and **Assessment *of* Learning**

Practice and Homework Book

The *MathLinks Practice and Homework Books* provide additional opportunities for students to develop the skills they used in the student resource.

- Each chapter begins with a **Get Ready** that can be used to help reactivate the skills students need to be successful with that chapter.
- The chapter content is divided into sections. Each section starts with a **Key Ideas Review**. This is followed by a series of questions that allow students to **Practise and Apply** the skills and concepts from that section in the student resource.
- The end of each chapter includes a **Link It Together** activity that challenges students to combine the skills and concepts they learned during the chapter to solve problems.
- The final page of each chapter is a **Vocabulary Link** that reviews the **Key Words** and other important words from each chapter in the form of a word puzzle.
- Answers for all questions appear at the end of the Practice and Homework Book.

Solutions CD-ROM

The solutions CD-ROM provides fully worked solutions for all questions in the numbered sections of the student resource, as well as for questions in the **chapter review**, **practice test**, and **cumulative review** features.

Computer Assessment Bank

A computerized assessment bank CD-ROM contains more than 1100 questions and answers based on the material presented in the student resource, with each of the 11 chapter banks pertaining to the corresponding student chapter. The question types include: Multiple Choice, Completion, Matching, Short Answer, and Problem. Each question in the CAB is correlated with the appropriate chapter, section, topic, and Curriculum Outcome from the WNCP Mathematics Curriculum.

The design of the computerized assessment bank facilitates the process of building worksheets, tests, or exams that best suit individual classes. You can enter your own questions, edit the questions, and customize the appearance of the worksheet/test/exam you have created in the ExamView® software.

Online LearningCentre

Online Learning Centre

Additional activities, as well as games and puzzles, are available in McGraw-Hill Ryerson's **Online Learning Centre**. Go to www.mathlinks9.ca and follow the links to the online **Teacher Centre**. There is also an online **Student Centre** and an online **Parent Centre**.

The first time you enter this site, use the following username and password:

Username: mathlinks9
Password: 9_teacher

Once you log in, you will be prompted to create your own unique username and password.

We feel confident that these materials will assist you and your students in meeting the outcomes of the new WNCP Math curriculum.

—The Authors, Consultants, and Advisors for *MathLinks 9*

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PROGRAM INTRODUCTION

MATHLINKS 9 CHAPTERS

BLACKLINE MASTERS

(Available on *MathLinks 9 Teacher's Resource* CD-ROM)

- This package is available on the *MathLinks 9 Teacher's Resource* CD-ROM. It provides generic resource masters, which include a generic project rubric, and chapter-specific worksheets and tests.
- Blackline master worksheets are provided in Microsoft® Word and PDF format. You may use the worksheet as is, or access the Microsoft® Word document and revise the worksheet to meet the specific needs of a particular student or of your entire class.
- Note that the last blackline master in each chapter provides the answers to all the questions on the chapter blackline masters.
- There are additional worksheets on the McGraw-Hill Ryerson Online Learning Centre. You can access these through the *MathLinks 9* book site at http://www.mcgrawhill.ca/mathlinks9.

Generic Masters

Master 1 Project Rubric
Master 2 Communication Peer Evaluation
Master 3 Integer Number Lines
Master 4 Number Lines
Master 5 Tangram
Master 6 Square Dot Paper
Master 7 Isometric Dot Paper
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
Master 10 2 Centimetre Grid Paper
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Master 21 Mathematical Symbols
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Master 23 Famous Mathematicians

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BLM 2–13 Chapter 2 Test
BLM 2–14 Chapter 2 Math Link: Wrap It Up!
BLM 2–15 Chapter 2 BLM Answers

CURRICULUM CORRELATION

Strand/Outcome	Chapter/Section	Pages
Strand: Number		
General Outcome *Develop number sense.*	Chapters 2–3	pp. 42–125
Specific Outcomes		
1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by: • representing repeated multiplication using powers • using patterns to show that a power with an exponent of zero is equal to one • solving problems involving powers. [C, CN, PS, R]	3.1–3.4 Math Link: Wrap It Up! Challenge: Develop Your Own Online Tournament Challenge: Stopping the Spread of Harmful Bacteria Task: How Many Times Can You Fold a Piece of Paper?	pp. 92–119 p. 123 p. 124 p. 125 p. 169
2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents. [C, CN, PS, R, T)	3.2–3.4 Math Link: Wrap It Up! Challenge: Develop Your Own Online Tournament Challenge: Stopping the Spread of Harmful Bacteria	pp. 99–119 p. 123 p. 124 p. 125
3. Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers • solving problems that involve arithmetic operations on rational numbers. [C, CN, PS, R, T, V]	2.1–2.4 Math Link: Wrap It Up! Challenge: Reaction Time Challenge: Going Up? Task: How Many Times Can You Fold a Piece of Paper?	pp. 46–81 p. 85 p. 86 p. 87 p. 169
4. Explain and apply the order of operations, including exponents, with and without technology. [PS, T]	3.3–3.4 Math Link: Wrap It Up! Challenge: Develop Your Own Online Tournament Challenge: Stopping the Spread of Harmful Bacteria Task: Choosing a Television to Suit Your Room	pp. 108–119 p. 123 p. 124 p. 125 p. 287
5. Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T]	2.4 Math Link: Wrap It Up! Challenge: Reaction Time Challenge: Going Up?	pp. 72–81 p. 85 p. 86 p. 87
6. Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, PS, R, T]	2.4 Math Link: Wrap It Up! Challenge: Reaction Time Challenge: Going Up?	pp. 72–81 p. 85 p. 86 p. 87
Strand: Patterns and Relations (Patterns)		
General Outcome *Use patterns to describe the world and solve problems.*	Chapter 6	pp. 206–249
Specific Outcomes		
1. Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution. [C, CN, PS, R, V]	6.1 Math Link: Wrap It Up! Challenge: Hot-Air Ballooning Challenge: Opening a Fitness Club	pp. 210–219 p. 247 p. 248 p. 249
2. Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems. [C, CN, PS, R, T, V]	6.2–6.3 Math Link: Wrap It Up! Challenge: Hot-Air Ballooning Challenge: Opening a Fitness Club Task: Choosing a Television to Suit Your Room Challenge: Global Warming	pp. 231–243 p. 247 p. 248 p. 249 p. 287 p. 448

Strand/Outcome	Chapter/Section	Pages
Strand: Patterns and Relations (Variables and Equations)		
General Outcome *Represent algebraic expressions in multiple ways.*	Chapters 5, 7–9	pp. 170–205, 250–373
Specific Outcomes		
3. Model and solve problems using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b, a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c, a \neq 0$ • $ax = b + cx$ • $a(x + b) = c$ • $ax + b = cx + d$ • $a(bx + c) = d(ex + f)$ • $\frac{a}{x} = b, x \neq 0$ where a, b, c, d, e and f are rational numbers. [C, CN, PS, V]	Task: Choosing a Television to Suit Your Room 8.1–8.4 Math Link: Wrap It Up! Challenge: School Store Challenge: Pair Up, Create, and Solve	p. 287 pp. 292–329 p. 333 p. 334 p. 335
4. Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context. [C, CN, PS, R, V]	9.1–9.3 Math Link: Wrap It Up! Challenge: Not for Profit Challenge: The Inequalities Game	pp. 340–367 p. 371 p. 372 p. 373
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Strand: Shape and Space (Measurement)		
General Outcome *Use direct or indirect measurement to solve problems.*	Chapter 10	pp. 374–409
Specific Outcomes		
1. Solve problems and justify the solution strategy using circle properties including: • the perpendicular from the centre of a circle to a chord bisects the chord • the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc • the inscribed angles subtended by the same arc are congruent • a tangent to a circle is perpendicular to the radius at the point of tangency. [C, CN, PS, R, T, V]	10.1–10.3 Math Link: Wrap It Up! Challenge: Dream Catcher	pp. 378–403 p. 407 p. 408
Strand: Shape and Space (3-D Objects and 2-D Objects)		
General Outcome *Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.*	Chapters 1, 4	pp. 2–41, 126–169
Specific Outcomes		
2. Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]	1.3 Math Link: Wrap It Up! Challenge: Making a Paper Airplane Challenge: Musical Instruments	pp. 26–35 p. 39 p. 40 p. 41

Strand/Outcome	Chapter/Section	Pages
Strand: Shape and Space (3-D Objects and 2-D Objects) continued		
3. Demonstrate an understanding of similarity of polygons. [C, CN, PS, R, V]	4.4 Math Link: Wrap It Up! Challenge: Shadow, Shadow Challenge: Graphic Designer Task: How Many Times Can You Fold a Piece of Paper?	pp. 154–159 p. 163 p. 164 p. 165 p. 169
Strand: Shape and Space (Transformations)		
General Outcome *Describe and analyze position and motion of objects and shapes.*	Chapters 1, 4	pp. 2–41, 126–169
Specific Outcomes		
4. Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V]	4.1–4.3 Math Link: Wrap It Up! Challenge: Shadow, Shadow Challenge: Graphic Designer	pp. 130–159 p. 163 p. 164 p. 165
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Strand: Statistics and Probability (Data Analysis)		
General Outcome *Collect, display and analyze data to solve problems.*	Chapter 11	pp. 410–452
Specific Outcomes		
1. Describe the effect of: • bias • use of language • ethics • cost • time and timing • privacy • cultural sensitivity on the collection of data. [C, CN, R, T]	11.1 Challenge: Global Warming Challenge: Probability in Society	pp. 414–421 p. 448 p. 449
2. Select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]	11.2 Challenge: Global Warming Challenge: Probability in Society	pp. 422–429 p. 448 p. 449
3. Develop and implement a project plan for the collection, display and analysis of data by: • formulating a question for investigation • choosing a data collection method that includes social considerations • selecting a population or a sample • collecting the data • displaying the collected data in an appropriate manner • drawing conclusions to answer the question. [C, PS, R, T, V]	11.1–11.4 Math Link: Wrap It Up! Challenge: Global Warming Challenge: Probability in Society	pp. 414–443 p. 443 p. 448 p. 449
Strand: Statistics and Probability (Chance and Uncertainty)		
General Outcome *Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.*	Chapter 11	pp. 410–452
Specific Outcomes		
4. Demonstrate an understanding of the role of probability in society. [C, CN, R, T]	11.3–11.4 Challenge: Probability in Society	pp. 430–443 p. 449

TIME LINES FOR *MATHLINKS 9*

The chart below shows estimated times, in minutes, for covering the material in *MathLinks 9*. Please note that times will vary depending on your particular class and its individual students. Field-testing shows that many classes can do some of this material in much less time than is outlined here, while it takes others more time. The chart shows an average. In most cases, the full course can be handled in 160 classes.

Also note that there are alternative ways to cover and assess many outcomes. For example, student achievement of chapter outcomes can be checked by having students do the **chapter review**, **practice test**, and **chapter test**, *or*, more holistically, by having students complete one of the **Challenges**, *or* by doing a combination of these things.

In a similar manner, you may wish to have some advanced students do one of the **Challenges** or a particular chapter while other students work on the sections. In other chapters, the **Challenges** may provide additional motivation for all students. Questions from the **cumulative review** could be used for extra practice for students who need it.

Chapter	1	2	3	4	5	6	7	8	9	10	11
Chapter Opener	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50
Section 1	80–100	80–100	45–60	80–100	80–100	80–100	60–80	80–100	50–60	50–60	60–80
Section 2	90–100	80–100	50–60	80–100	100–120	80–100	60–80	80–100	50–60	50–60	80–100
Section 3	120–135	80–100	50–60	80–100	100–120	80–100	60–80	80–100	50–60	50–60	80–100
Section 4		80–100	50–60	80–100				80–100			80–100
Chapter Review	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50
Practice Test	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	40–50	30–40	40–50
Math Link: Wrap It Up!	40–50	40–50	80–100	80–100	60–80	80–100	60–75	40–50	30–40	40–50	40–50
Challenge #1	40–100	40–50	40–50	40–50	40–60	60–80	80–100	40–50	30–40	50–60	80–100
Challenge #2	40–50	40–50	40–50	40–50	40–50	40–50	50–60	30–40	30–40		60–80
Cumulative Review				60–75			60–75				60–75
Task				40–50			40–50				

AN INTRODUCTION TO *MATHLINKS 9* TEACHER'S RESOURCE

The teaching notes for each chapter have the following structure:

Opening Matter and Charts

- These are provided on a four-page foldout immediately after each chapter tab.
- These pages provide:
  - an overview of the chapter outcomes and the concepts, skills, and processes that will be assessed
  - assessment suggestions for the use of the **Literacy Link** in the chapter opener, the **Math Link** introduction, the **Foldable**, and the section **Warm-Ups**
  - an introduction to the **Problems of the Week**
- The **Chapter Planning Chart** provides
  - suggested timing for the numbered sections, chapter review, practice test, Math Links: Wrap It Up!, games, and Challenges
  - a list of prerequisite skills for each section
  - suggested assignments for most students
  - a list of related blackline masters available on the CD-ROM
  - a list of materials and technology tools needed for each lesson
  - the location of Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning opportunities in the chapter
  - suggested sources for extra support

Chapter Opener

The Chapter Opener includes:

- a description of the math that will be covered in the chapter
- suggestions for introducing students to the chapter's topics
- ideas about how to introduce the Math Link
- suggestions for how students could use the Foldable most effectively

FOLDABLES™ Study Tool

Numbered Sections

The opening page lists:

- **Specific Outcomes** and **Mathematical Processes** that the section covers in whole or in part
- **Materials** needed for the section
- **Suggested Timing** for the section
- **Blackline Masters** useful for extra practice, assessment, and adaptations. This includes a Warm-Up master that provides exercises for reinforcing material in previous sections of the student resource, as well as mental math skills.
- a **question planning chart** that specifies the questions to be assigned.
  - Essential: the minimum, usually knowledge and skill questions, that all students should be able to complete to address the outcomes
  - Typical: questions that most students should be fairly successful with
  - Extension/Enrichment: questions that extend the concepts horizontally and provide additional challenge

Teaching Notes

The key items include the following:

- Answers for the **Explore** questions let you know the expected outcome of these activities.
- **Planning Notes** give insights about and suggestions for the three parts of the lesson: **Explore**, **Link the Ideas**, and **Check Your Understanding**.
- Sample responses for the **Communicate the Ideas** questions provide the type of answers students are expected to give.
- **Assessment** boxes give a variety of short assessment strategies and related supported learning for Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning. These boxes are provided for each of the activities in the student resource numbered sections.
- A **Math Link** box describes what students will achieve with the Math Link activity and provides strategies for students to complete it successfully.

End of Chapter Items

- The chapter sections are followed by a **chapter review** and a **practice test**.
- The chapter problem is finalized in a **Math Link: Wrap It Up!** Related notes provide ideas for handling this assessment opportunity. A rubric and suggested assessment notes are provided.
- The final two pages in each chapter provide one or two **Challenges**. These activities can be used as holistic assessment tools, as extra activities for gifted and enriched students, and/or by all students as motivating activities related to real life. A rubric and suggested assessment notes are provided for Challenges that do not consist solely of students playing a game to reinforce skills.
- **Cumulative reviews** reinforce the previous four chapters.
- In chapters 4 and 7, the cumulative reviews are followed by a **Task**. This activity can be used as a holistic assessment tool for cross-strand work or as an extra activity for gifted and enriched students. A rubric and suggested assessment notes are provided.

The Teacher's Resource CD-ROM also provides editable masters:

- **Generic Masters** such as grid paper
- **Blackline Masters** related to each chapter:
  - an open-ended diagnostic assessment opportunity
  - **Warm-Up** questions that provide exercises for reinforcing material in previous sections and mental math skills
  - **Problems of the Week**, including innovative problems that require students to think outside the box and experiment with a variety of approaches
  - **Extra Practice** questions for each section
  - scaffolding for each **Math Link** and **Math Link: Wrap It Up!** for students who need supported learning
  - a **chapter test**
  - answers for blackline master questions

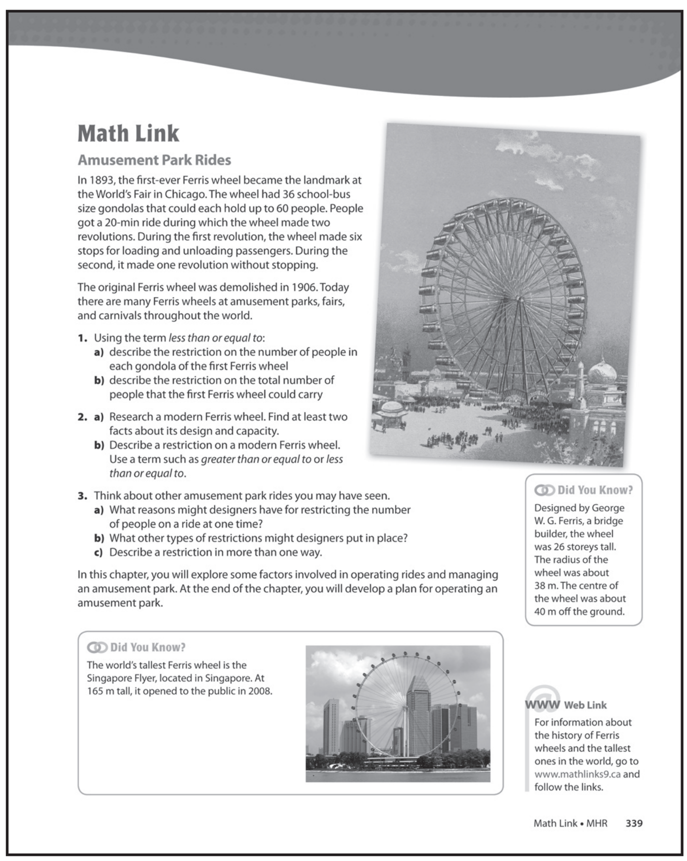
CHARACTERISTICS OF MCGRAW-HILL RYERSON'S *MATHLINKS* PROGRAM

McGraw-Hill Ryerson's *MathLinks* program is based on a view that all students can be successful in mathematics and should have the opportunity and support to learn mathematics for depth and understanding (NCTM, 2000). The goal is to assist students in becoming more responsible, thoughtful, and active learners. The program is built on principles of effective practice and on research about how early adolescents learn—prerequisites for achieving a balanced approach to instruction in mathematics.

Mathematics: Making Links

Throughout the *MathLinks* student resource, students are given the opportunity to see the links between real life and mathematics.

- Every chapter is introduced with a **Math Link** problem that models mathematics in the real world, engages students' interest, and gives students a meaningful purpose for learning the mathematics presented in the chapter. The Math Link provides an important foundation for the concepts and skills developed throughout the chapter. The problem is designed to engage students by making links between the mathematics in the chapter and students' personal experiences, as well as between mathematics and the real world.



Math Link

Amusement Park Rides

In 1893, the first-ever Ferris wheel became the landmark at the World's Fair in Chicago. The wheel had 36 school-bus size gondolas that could each hold up to 60 people. People got a 20-min ride during which the wheel made two revolutions. During the first revolution, the wheel made six stops for loading and unloading passengers. During the second, it made one revolution without stopping.

The original Ferris wheel was demolished in 1906. Today there are many Ferris wheels at amusement parks, fairs, and carnivals throughout the world.

1. Using the term *less than or equal to*:
   a) describe the restriction on the number of people in each gondola of the first Ferris wheel
   b) describe the restriction on the total number of people that the first Ferris wheel could carry
2. a) Research a modern Ferris wheel. Find at least two facts about its design and capacity.
   b) Describe a restriction on a modern Ferris wheel. Use a term such as *greater than or equal to* or *less than or equal to*.
3. Think about other amusement park rides you may have seen.
   a) What reasons might designers have for restricting the number of people on a ride at one time?
   b) What other types of restrictions might designers put in place?
   c) Describe a restriction in more than one way.

In this chapter, you will explore some factors involved in operating rides and managing an amusement park. At the end of the chapter, you will develop a plan for operating an amusement park.

Did You Know?
Designed by George W. G. Ferris, a bridge builder, the wheel was 26 storeys tall. The radius of the wheel was about 38 m. The centre of the wheel was about 40 m off the ground.

Did You Know?
The world's tallest Ferris wheel is the Singapore Flyer, located in Singapore. At 165 m tall, it opened to the public in 2008.

Web Link
For information about the history of Ferris wheels and the tallest ones in the world, go to www.mathlinks9.ca and follow the links.

Math Link • MHR 339

- The **Math Link** is revisited at the end of most lessons. This provides students with the opportunity to apply newly acquired concepts and skills in the context of the original problem.

Math Link

For safety reasons, some amusement park rides have age and height restrictions for riders.

a) Choose an amusement park ride that you have seen or design one of your own. Describe your ride.

b) For your ride, consider the safety restrictions or conditions that you might impose on riders. List at least three restrictions. Use terms of your choice.

c) Represent each restriction algebraically using a different variable for each.

d) Sketch a sign. Use words and graphics that clearly inform riders about each of your restrictions.

- At the end of the chapter, the **Math Link: Wrap It Up!** offers an open-ended assessment opportunity for students to demonstrate their understanding by solving the problem introduced at the beginning of the chapter.

Math Link: Wrap It Up!

You are an amusement park manager who has been offered a job planning a new park in a different location.

a) Give your park a name and choose a location. Explain how you made your choice. State the population of the area around the park that you chose.

b) Choose a reasonable number of rides for your park. Assume that the fixed costs include \$5000 in addition to maintenance and wages. Assume maintenance and repairs cost \$400 per ride and that it takes eight employees to operate and supervise each ride. Conduct research and then decide:
- the number of hours that rides will be open
- the average hourly wage for employees

What might be the problem if you choose too few or too many rides?

c) Organize your estimates about operating expenses and revenues for the park. You can use the table in the Math Link on page 367 as a reference.

d) Write an expression to represent each of the following for the number of rides you chose:
- expenses per visitor
- revenue per visitor

e) For the number of rides you chose, how many visitors will be needed for the park to make a profit? Show all your work. Justify your solution mathematically.

f) Assume that you have now opened your park. You find that 0.1% of the people in the area come to the park per day, on average. Using this information, will your park earn a profit? If not, explain what changes you could make. Show all your work and justify your solution.

- Most concepts or procedures in the chapter are introduced in a real-life context.
- **What You Will Learn** at the beginning of each chapter lists in student-friendly language the outcomes of the mathematics curriculum that are covered in the chapter. These outcomes may be from different strands that naturally fit together and further illustrate how the program makes important links among concepts within the discipline and with the real world.
- Connections with other curriculum areas, such as science, geography, history, language, and art, are evident in a number of lessons in several of the chapters. Some of these are in subject link and Did You Know? boxes. For example, the Did You Know? shown here is part of a Chapter 6 Math Link that talks about the Intertropical Convergence Zone (ITCZ). The Teacher's Resource also identifies the various ways that concepts developed in the chapter are linked to concepts in the different strands.

Did You Know?

The ITCZ is located between 5° north and 5° south of the equator and is approximately 1100 km wide. Note how the position of the ITCZ moves during the year.

Procedural Fluency and Conceptual Understanding

The three-part lesson structure in McGraw-Hill Ryerson's *MathLinks* program is designed to engage students in learning that develops their conceptual understanding and procedural fluency. The three parts are described below.

1. Explore

- begins with a focus question that identifies the learning objective of the lesson

Explore Using a Graph to Solve Problems

Richard is paid $64/day plus a commission of 10% of sales for selling riverboat cruises.

1. The table provides some information about his sales and daily pay.

Day	River Cruises Sold ($)	Pay for the Day ($)
Monday	1500	214
Tuesday	1250	
Wednesday	800	
Thursday	0	64
Friday	2010	

a) On the graph, locate the data points for each day.
b) How could you use the graph to estimate the missing values on the table? Try your method.

- provides an opportunity for students to generalize learning about the key concepts and to answer the original focus question in the **Reflect on Your Findings** in *MathLinks 7* and *MathLinks 8*, and **Reflect and Check** in *MathLinks 9*

2. Link the Ideas

- some start with a piece of text that helps students connect what they did in Explore to the Examples
- provides worked **examples** of the mathematics being modelled, often with multiple approaches to a solution
- makes use of commonly available concrete materials and mathematics manipulatives
- provides opportunities for students to check their understanding of concepts, through **Show You Know** questions, before proceeding to the next example
- **Key Ideas** summarize the key concepts or big ideas of the lesson

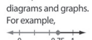
Link the Ideas

Reading an inequality depends on the inequality symbol used.

Inequality	Meaning
$a > b$	*a* is greater than *b*
$a < b$	*a* is less than *b*
$a \geq b$	*a* is greater than or equal to *b*
$a \leq b$	*a* is less than or equal to *b*
$a \neq b$	*a* is not equal to *b*

Literacy Link

Inequalities can be expressed three ways:
- *Verbally* using words. For example, "all numbers less than or equal to 0.75."
- *Graphically* using visuals, such as diagrams and graphs. For example,



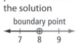
- *Algebraically* using mathematical symbols. For example, $x \leq 0.75$.

Example 1: Represent Inequalities

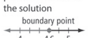
Many jobs pay people a higher rate for working overtime. Reema earns overtime pay when she works more than 40 h a week.

a) Give four possible values that would result in overtime pay.
b) Verbally express the amount of time that qualifies for overtime as an inequality.
c) Express the inequality graphically.
d) Express the inequality algebraically.
e) Represent the amount of time that does not qualify for overtime as an inequality. Express the inequality verbally, graphically, and algebraically.

boundary point
- separates the values less than from the values greater than a specified value
- may or may not be a possible value in a solution
- an open circle shows that the boundary point is not included in the solution



- a closed circle shows that the boundary point is included in the solution

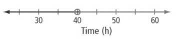


Solution

a) Reema does not qualify for overtime if she works exactly 40 h. She qualifies only if she works more than 40 h. Some examples include 40.5 h, 42 h, 46.25 h, and 50 h.

b) In order to qualify for overtime, Reema needs to work more than 40 h.

c) Draw a number line to represent the inequality graphically. Display the value 40 and values close to 40. The value 40 is a boundary point. This point separates the regular hours from the overtime hours on the number line. Draw an open circle at 40 to show the boundary point. Starting at 40, draw an arrow pointing to the right to show that the possible values of *t* are greater than but not equal to 40.



The open circle shows that the value 40 is not a possible value for the number of hours that qualify for overtime.

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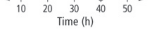
d) The inequality is $t > 40$, where *t* represents the amount of time, in hours, that Reema works in a week.

Which of the three representations of an inequality do you prefer?

e) Verbally: Reema does not qualify for overtime if the number of hours she works is less than or equal to 40 h.

Graphically: Draw a closed circle at 40. Draw an arrow pointing to the left of 40 to show the possible values of *t* less than or equal to 40.



The closed circle shows that 40 is a possible value for the number of hours that do not qualify for overtime.

Algebraically: Using *t* to represent the amount of time, in hours, that Reema works, $t \leq 40$.

Show You Know

In many provinces, you must be at least 16 years of age to get a driver's licence.
a) Sketch a number line to represent the situation.
b) Represent the situation algebraically.

Key Ideas

- A linear inequality compares linear expressions that may not be equal.
  $x \geq -3$ means that *x* is greater than or equal to -3.
- Situations involving inequalities can be represented verbally, graphically, and algebraically.
  A person must be under twelve years of age to qualify for a child's ticket at the movies. Let *a* represent the age of the person.
  - Verbally: Use words.
    The values of *a* are less than 12.
  - Graphically: Use visuals, such as diagrams and graphs.

  - Algebraically: Use mathematical symbols, such as numbers, variables, operation signs, and the symbols $<$, $>$, $\leq$, and $\geq$.
    The inequality is $a < 12$.
- An inequality with the variable on the right can be interpreted two ways.
  $8 < x$ can be read "8 is less than *x*." This is the same as saying "*x* is greater than 8."
- Double inequalities can be used to represent situations involving two conditions.
  A business used a minimum of 65 L and a maximum of 85 L of fuel each day. An inequality that represents the amount of fuel used is $65 \leq f \leq 85$.

3. Check Your Understanding

Check Your Understanding

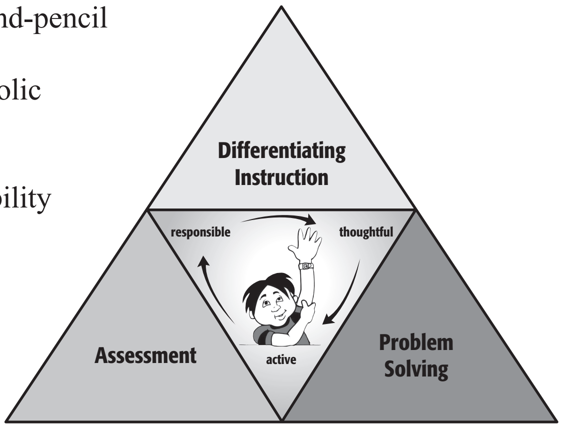
- **Communicate the Ideas** consolidates student learning through questions that include explaining or comparing concepts, identifying and correcting errors, and discussing as a group
- allows practice of new skills and application of learning to different situations
- provides opportunities for solving problems in a variety of contexts and using multiple approaches
- provides opportunities for students to extend their thinking (e.g., synthesizing, analysing, evaluating) by using what was discussed in the chapter in a different context or a different way

MathLinks balances:

- procedural fluency and conceptual understanding
- mental mathematics, paper-and-pencil arithmetic, and technology
- concrete, pictorial, and symbolic representations
- practice and application
- student and teacher responsibility

This approach is embedded in three cornerstones of the *MathLinks* program:

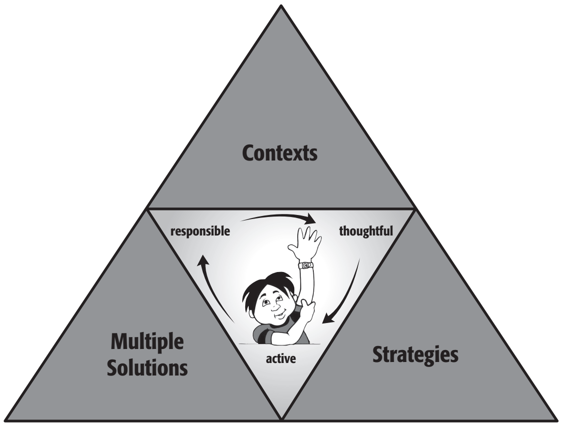
- problem solving
- differentiating instruction
- assessment



Problem Solving

Problem solving is central to the McGraw-Hill Ryerson *MathLinks* program. Significant emphasis has been placed on incorporating problems that:

- have a range of contexts
- can be solved using different problem solving strategies
- may have multiple solutions



A variety of problem solving experiences are provided throughout the lessons:

- A four-step **problem solving model** is outlined at the beginning of the student resource: Understand, Plan, Do It!, Look Back.
- **Problem solving strategies** are reinforced at the beginning of the student resource. These pages serve as a reference for students as they solve problems within the chapters.

Problem Solving

People solve mathematical problems at home, at work, at school, and at play. There are many different ways to solve problems. In *MathLinks 9*, you are encouraged to try different methods, look for alternative strategies, and use your own ideas. Your method may be different but it may also work.

A Problem Solving Model

Where do you begin with problem solving? It may help to use the following four-step process.

Understand Read the problem carefully.
- Note the key words, phrases, and important facts.
- Restate the problem in your own words.
- What information is given? What further information do you need?
- What is the problem asking you to do?

Plan Select a strategy for solving the problem. Carefully consider your reason for choosing that plan. Sometimes you need more than one strategy.
- Consider other problems you have solved successfully. Is this problem like one of them? Can you use a similar strategy? Strategies that you might use include:
  - Model It
  - Draw a Diagram
  - Make an Organized List or Table
  - Work Backward
  - Guess and Check
  - Look for a Pattern
  - Organize, Analyse, and Solve
  - Estimate and Check
  - Solve a Simpler Problem
  - Identify all Possibilities
  - Use a Variable
  - Solve an Equation
  - Make an Assumption
- Decide whether any of the following might help. Plan how to use them.
  - tools such as a ruler or a calculator
  - materials such as grid paper or a number line

Do It! Solve the problem by carrying out your plan.
- Use mental math to estimate a possible answer.
- Do the calculations.
- Consider an alternative plan if your plan does not help you find a solution.
- Record each of your steps.
- State your answer. Explain and justify your thinking.

Look Back Examine your answer. Does it make sense?
- Is your answer close to your estimate?
- Does your answer fit the facts given in the problem?
- Is the answer reasonable? If not, make a new plan. Try an alternative strategy.
- Consider solving the problem a different way. Do you get the same answer?
- Compare your methods with those of your classmates.

Problem Solving • MHR xv

Here are several strategies to help you solve problems. Your ideas on how to solve the problems might be different from any of these.

Problem 1 Leisa purchased 70 glass beads to make jewellery for her friends and family. The small beads cost \$1 each, and the large ones cost \$2 each. In total, Leisa spent \$99 on the beads. How many \$1 beads did she buy?

Strategy	Example
Estimate and Check	Estimate that there is an equal number of each bead. Therefore, $70 \div 2 = 35$ of each. Multiply the number of small beads by \$1 and the number of large beads by \$2 to determine the total cost. $1(35) + 2(35) = 35 + 70$ $= 105$ Too high You can organize further estimates in a chart.

Small	Large	Total Cost (\$)
45	25	$45 + 2(25) = 95$ Too low
40	30	$40 + 2(30) = 100$ Too high
43	27	$43 + 2(27) = 97$ Too low
41	29	$41 + 2(29) = 99$ Correct!

The number of \$1 beads Leisa purchased was 41.

Use an Equation

Let n represent the number of small beads she ordered.
The number of large beads can be represented by $70 - n$.
The cost of the small beads can be represented by $1n$, or n.
The cost of the large beads can be represented by $2(70 - n)$.
The total cost can be represented by $n + 2(70 - n)$.
The total cost is \$99.

$$n + 2(70 - n) = 99$$
$$n + 140 - 2n = 99$$
$$140 - n = 99$$
$$-n = -41$$
$$\left(\frac{-n}{-1}\right) = \left(\frac{-41}{-1}\right)$$
$$n = 41$$

The number of \$1 beads Leisa purchased was 41.

xvi MHR • Problem Solving

- Examples throughout the *MathLinks 7* student resource show the problem solving model and strategies being used in context. In *MathLinks 8* and *MathLinks 9*, the examples continue to show strategies in context.

Example 1: Use Diameter to Find Circumference

Traffic circles, or roundabouts, are used in some neighbourhoods to slow down traffic. Vehicles enter the circle and drive around in a counterclockwise direction.

a) Estimate the circumference of this traffic circle.
b) What is the circumference of the traffic circle, to the nearest tenth of a metre?
c) Is your estimate reasonable?

Solution

Understand You are given the diameter of the traffic circle. You need to find the circumference.
$C = ■$, $d = 5.2$ m

Plan Use the formula $C = \pi \times d$. Use an approximate value for π to estimate and calculate the circumference. Substitute the diameter into the formula.

Do It!

a) When estimating, use 3 as an approximate value for π. ME
The diameter of the traffic circle is about 5 m.
$C = \pi \times d$
$C \approx 3 \times 5$
$C \approx 15$
The circumference of the traffic circle is approximately 15 m.
The actual value should be higher because you estimated using numbers smaller than the actual numbers.

b) When calculating, use 3.14 as an approximate value for π.
$C = \pi \times d$
$C \approx 3.14 \times 5.2$ C 3.14 × 5.2 = 16.328
$C \approx 16.3$
The circumference of the traffic circle is approximately 16.3 m.

Check that you rounded your answer to the correct number of decimal places. Remember to use the proper units in your final answer.

Tech Link
If your calculator has a π key, you can use the π key instead of the value 3.14.

Look Back

c) The answer of 16.3 m is close to but a bit higher than the estimate of 15 m. The estimate of 15 m is reasonable.

Show You Know

Estimate and calculate the circumference of each circle, to the nearest tenth of a unit.

a)


b)

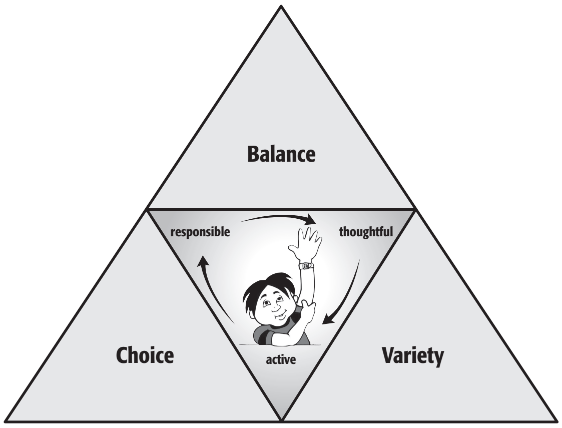

8.2 Circumference of a Circle • MHR 291

- Students are frequently asked to discuss their methods for solving problems. Doing so reinforces thinking and helps students realize that there may be multiple methods for solving a problem.
- The **Math Link** at the beginning of each chapter activates student knowledge of skills and concepts related to the topic in the chapter. The Math Link models a mathematics problem from the real world. This problem is wrapped up at the end of the chapter in the form of a performance task.
- A problem provides the focus for learning in **Explore the Math** in *MathLinks 7* and *MathLinks 8*, and Explore in *MathLinks 9*, often making use of concrete materials.
- Students are challenged to higher levels of thinking and to extend their thinking in the **Extend** section of the exercises, the **Problems of the Week** Blackline Master for each chapter, the **Extended Response** section in the practice test, the **Math Games**, and the **Challenges**.

Differentiating Instruction

Differentiating instruction provides educators with the tools needed to create a learning environment where students are actively involved and working together. Hands-on activities engage students and help to meet their diverse needs. Significant emphasis has been placed on:

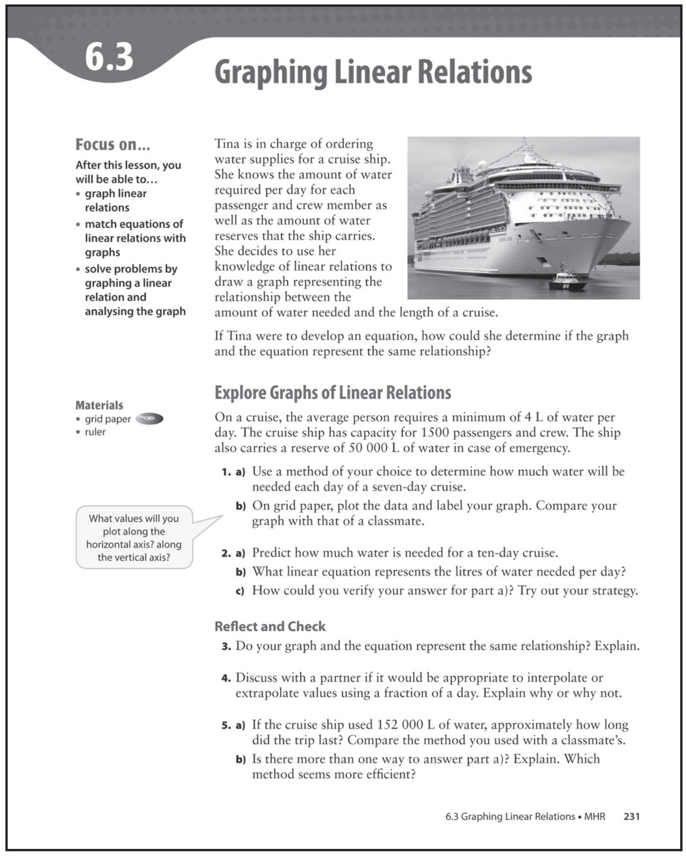
- variety — provides opportunities for students to be thoughtful about what and how they learn
- choice — encourages students to develop responsibility by making good personal decisions
- balance — is essential in having students actively involved in their learning. Students' needs are best met when they experience a variety of ways to develop and understand concepts.



Care has been taken in the McGraw-Hill Ryerson *MathLinks* program to ensure that all students—including special needs students (with learning disabilities or gifted), students at risk, English language learners, or students from different cultures—can access the mathematics and experience success.

- Visuals that illustrate how to carry out explorations accompany the instructions. These visuals help the student to "see" the process. They also aid in the acquisition of mathematical language.

- Visuals and graphics are paired with questions and content in other strategic locations in the student resource.


6.3

Graphing Linear Relations

Focus on...

After this lesson, you will be able to...
- graph linear relations
- match equations of linear relations with graphs
- solve problems by graphing a linear relation and analysing the graph

Tina is in charge of ordering water supplies for a cruise ship. She knows the amount of water required per day for each passenger and crew member as well as the amount of water reserves that the ship carries. She decides to use her knowledge of linear relations to draw a graph representing the relationship between the amount of water needed and the length of a cruise.

If Tina were to develop an equation, how could she determine if the graph and the equation represent the same relationship?

Explore Graphs of Linear Relations

Materials
- grid paper
- ruler

On a cruise, the average person requires a minimum of 4 L of water per day. The cruise ship has capacity for 1500 passengers and crew. The ship also carries a reserve of 50 000 L of water in case of emergency.

1. a) Use a method of your choice to determine how much water will be needed each day of a seven-day cruise.
   b) On grid paper, plot the data and label your graph. Compare your graph with that of a classmate.

What values will you plot along the horizontal axis? along the vertical axis?

2. a) Predict how much water is needed for a ten-day cruise.
   b) What linear equation represents the litres of water needed per day?
   c) How could you verify your answer for part a)? Try out your strategy.

Reflect and Check

3. Do your graph and the equation represent the same relationship? Explain.
4. Discuss with a partner if it would be appropriate to interpolate or extrapolate values using a fraction of a day. Explain why or why not.
5. a) If the cruise ship used 152 000 L of water, approximately how long did the trip last? Compare the method you used with a classmate's.
   b) Is there more than one way to answer part a)? Explain. Which method seems more efficient?

6.3 Graphing Linear Relations • MHR 231

- A **Literacy Link** at the beginning of each chapter provides tips to help students read and interpret the chapter content.

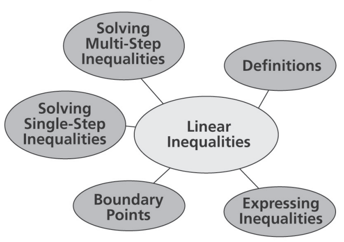
Literacy Link

A concept map can help you visually organize your understanding of math concepts.

Create a concept map in your math journal or notebook. Make each oval large enough to write in. Leave enough space to draw additional ovals. As you work through the chapter, complete the concept map.

- **Definitions**: Attach an oval for each term you learn. Write a definition for each one.
- **Expressing Inequalities**: Attach three ovals. For each oval, use an example to show a different way to express an inequality.
- **Boundary Points**: Attach two ovals. Identify and draw an example of a boundary point showing an open circle and a closed circle.
- **Solving Single-Step Inequalities**: Attach two ovals. For each oval, show how to solve a single-step inequality. Attach an oval to each example and show how to verify your solution.
- **Solving Multi-Step Inequalities**: Attach two ovals. For each oval, show how to solve a multi-step inequality. Include an example that illustrates reversing the inequality sign. Attach an oval to each example and show how to verify your solution.

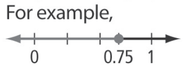
Discuss your ideas with a classmate. You may wish to add to or correct what you have written.



- Other Literacy Links throughout the chapters provide students with strategies for how to read and understand mathematical language.

Literacy Link

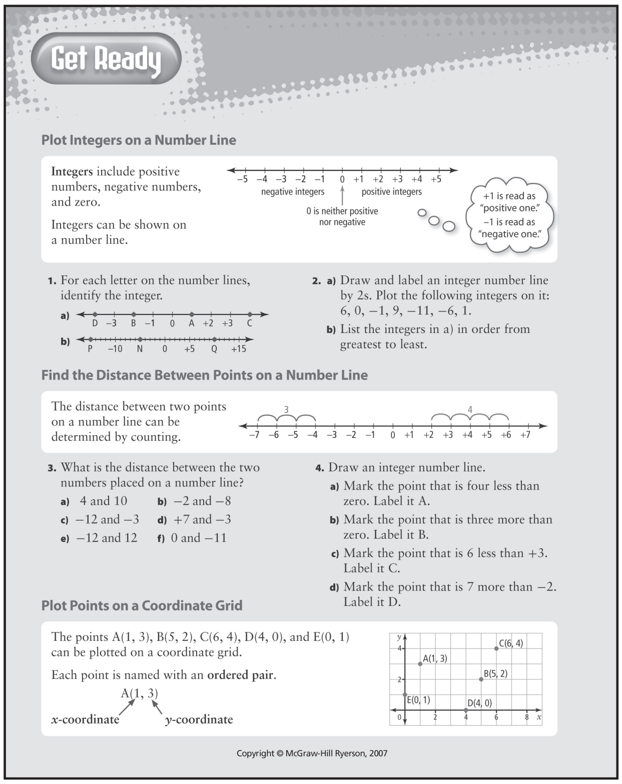
Inequalities can be expressed three ways:
- *Verbally* using words. For example, "all numbers less than or equal to 0.75."
- *Graphically* using visuals, such as diagrams and graphs. For example,



- *Algebraically* using mathematical symbols. For example, $x \leq 0.75$.

- The Teacher's Resource provides strategies and blackline master support for accommodating different learning styles, special needs, English language learners, First Nations, Métis, Inuit, and at-risk students.

- The **Get Ready** materials in the *MathLinks Practice and Homework Books* and on the *MathLinks* book site activate student knowledge and concepts related to the topic in the upcoming chapter.



Get Ready

Plot Integers on a Number Line

Integers include positive numbers, negative numbers, and zero.

Integers can be shown on a number line.

1. For each letter on the number lines, identify the integer.
   a) D −3 B −1 0 A +2 +3 C
   b) P −10 N 0 +5 Q +15
2. a) Draw and label an integer number line by 2s. Plot the following integers on it: 6, 0, −1, 9, −11, −6, 1.
   b) List the integers in a) in order from greatest to least.

Find the Distance Between Points on a Number Line

The distance between two points on a number line can be determined by counting.

3. What is the distance between the two numbers placed on a number line?
   a) 4 and 10
   b) −2 and −8
   c) −12 and −3
   d) +7 and −3
   e) −12 and 12
   f) 0 and −11
4. Draw an integer number line.
   a) Mark the point that is four less than zero. Label it A.
   b) Mark the point that is three more than zero. Label it B.
   c) Mark the point that is 6 less than +3. Label it C.
   d) Mark the point that is 7 more than −2. Label it D.

Plot Points on a Coordinate Grid

The points A(1, 3), B(5, 2), C(6, 4), D(4, 0), and E(0, 1) can be plotted on a coordinate grid.

Each point is named with an **ordered pair**.

A(1, 3)

x-coordinate *y*-coordinate

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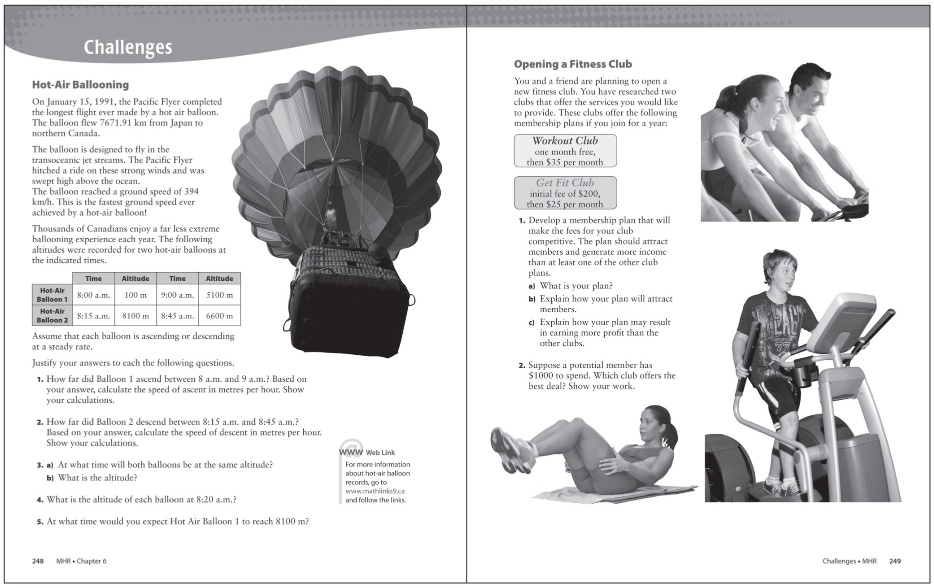
- Support for combined grades situations appears on the *MathLinks* book site at www.mathlinks.ca.
- The Teacher's Resource, *MathLinks Practice and Homework Books*, and *MathLinks* Online Learning Centre offer further support in the form of concrete activities, additional practice, and diagnostic strategies to support students who may have gaps in their learning.
- The open-ended nature of many of the problems and tasks accommodate the needs of all students by allowing for multiple entry points.



Challenges

Hot-Air Ballooning

On January 15, 1991, the Pacific Flyer completed the longest flight ever made by a hot air balloon. The balloon flew 7671.91 km from Japan to northern Canada.

The balloon is designed to fly in the transoceanic jet streams. The Pacific Flyer hitched a ride on these strong winds and was swept high above the ocean. The balloon reached a ground speed of 394 km/h. This is the fastest ground speed ever achieved by a hot-air balloon!

Thousands of Canadians enjoy a far less extreme ballooning experience each year. The following altitudes were recorded for two hot-air balloons at the indicated times.

	Time	Altitude	Time	Altitude
Hot-Air Balloon 1	8:00 a.m.	100 m	9:00 a.m.	5100 m
Hot-Air Balloon 2	8:15 a.m.	8100 m	8:45 a.m.	6600 m

Assume that each balloon is ascending or descending at a steady rate.

Justify your answers to each the following questions.

1. How far did Balloon 1 ascend between 8 a.m. and 9 a.m.? Based on your answer, calculate the speed of ascent in metres per hour. Show your calculations.
2. How far did Balloon 2 descend between 8:15 a.m. and 8:45 a.m.? Based on your answer, calculate the speed of descent in metres per hour. Show your calculations.
3. a) At what time will both balloons be at the same altitude?
   b) What is the altitude?
4. What is the altitude of each balloon at 8:20 a.m.?
5. At what time would you expect Hot Air Balloon 1 to reach 8100 m?

WWW Web Link
For more information about hot-air balloon records, go to www.mathlinks9.ca and follow the links.

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Opening a Fitness Club

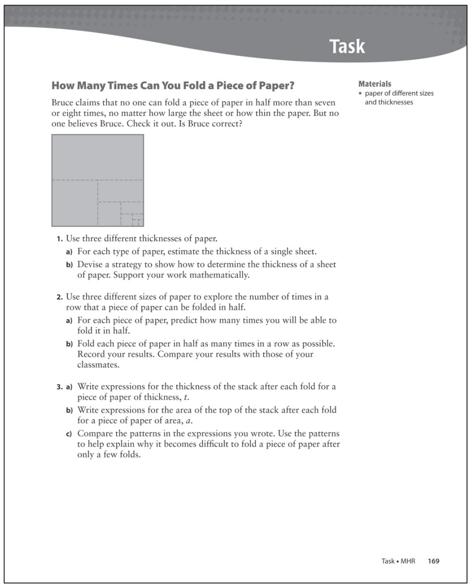
You and a friend are planning to open a new fitness club. You have researched two clubs that offer the services you would like to provide. These clubs offer the following membership plans if you join for a year:

Workout Club
one month free,
then $35 per month

Get Fit Club
initial fee of $200,
then $25 per month

1. Develop a membership plan that will make the fees for your club competitive. The plan should attract members and generate more income than at least one of the other club plans.
   a) What is your plan?
   b) Explain how your plan will attract members.
   c) Explain how your plan may result in earning more profit than the other clubs.
2. Suppose a potential member has $1000 to spend. Which club offers the best deal? Show your work.

Challenges • MHR 249



Task

How Many Times Can You Fold a Piece of Paper?

Materials
- paper of different sizes and thicknesses

Bruce claims that no one can fold a piece of paper in half more than seven or eight times, no matter how large the sheet or how thin the paper. But no one believes Bruce. Check it out. Is Bruce correct?

1. Use three different thicknesses of paper.
   a) For each type of paper, estimate the thickness of a single sheet.
   b) Devise a strategy to show how to determine the thickness of a sheet of paper. Support your work mathematically.
2. Use three different sizes of paper to explore the number of times in a row that a piece of paper can be folded in half.
   a) For each piece of paper, predict how many times you will be able to fold it in half.
   b) Fold each piece of paper in half as many times in a row as possible. Record your results. Compare your results with those of your classmates.
3. a) Write expressions for the thickness of the stack after each fold for a piece of paper of thickness, *t*.
   b) Write expressions for the area of the top of the stack after each fold for a piece of paper of area, *a*.
   c) Compare the patterns in the expressions you wrote. Use the patterns to help explain why it becomes difficult to fold a piece of paper after only a few folds.

Task • MHR 169

Did You Know?

After deep or long dives, scuba divers need to undergo decompression. They do this by ascending to the surface slowly in order to avoid decompression sickness, also known as the bends.

- **Did You Know?** boxes present interesting information related to the math or context of the lesson. In *MathLinks 9*, some provide connections to other subject areas.

Ten Needs of the Learner

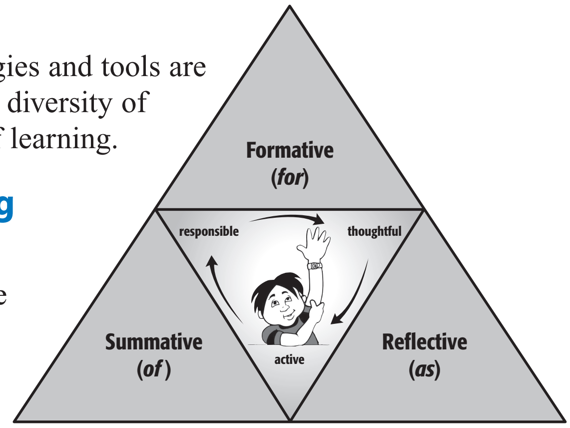
Anna Sfard (2003) has identified ten needs of the intermediate learner. She claims these needs are the driving force behind learning and must be fulfilled if the learning is to be successful. The needs are universal, but may be expressed differently in different individuals and at different ages.

1. **The Need for Meaning**
   Learners look for order, logic, and causal dependencies behind things, events, and experiences. This approach requires students to actively engage in generating the meaning for themselves. It also directs students' thinking so no time is lost investigating incorrect paths. While students are developing meaning for new concepts, they are guided to develop patience, persistence, and tolerance in the face of insufficient clarity.
2. **The Need for Structure**
   This need follows from the need for meaning. Meaning involves relationships among concepts. The connections among concepts already learned and new concepts being introduced should be an integral part of instruction. Such connections must include not only real-world applications and relevance, but also assistance in building mathematical abstractions, so students can see how the results can be transferred from one context to another (Wu, 1997). The more connections that exist among facts, ideas, and procedures, the better students' conceptual understanding.
3. **The Need for Repetitive Action**
   A person who has created meaning and structure for a mathematical concept is aware of a repetitive, constant structure in certain actions. A reasonable level of mastery of basic skills is an important element in constructing mathematical knowledge (Fuson and Briars, 1990; Fuson and Kwon, 1992; Hiebert and Wearne, 1996; Siegler, 2003; Stevenson and Stigler, 1992). If students are to reflect on the processes of mathematics, they must first master those processes to a sufficient degree. This does not mean a focus on rote repetition. Rather, it should be a process of reflective practice, where mastery of the action leads to reflection on the meaning of that action, which leads to further understanding and learning.
4. **The Need for Difficulty**
   True learning implies coping with difficulties. The goal of learning is to advance students from abilities they now possess to those they have not yet developed. The best way to accomplish this goal is to present students with tasks that are demanding but still within their reach.
5. **The Need for Significance and Relevance**
   Significance means linking new knowledge to existing knowledge, so this need also stresses the importance of helping students build connections. Significance and relevance do not come from only the concrete and the real; they also come from problems that are more abstract. Focusing only on real-life applications would lead to a fragmented, incomplete picture of mathematics.

6. **The Need for Social Interaction**
There is an inherent social nature to learning and making meaning. Jerome Bruner states that the fundamental vehicle of education is social interaction not "solo performance" (Bruner, 1985). The most obvious forms are student–teacher or student–student exchanges, but even interaction with a textbook is a form of social interaction (Sfard, 2003). Cooperative learning is another form of learning interaction in which the teacher does not have the central role.
7. **The Need for Verbal–Symbolic Interaction**
Interaction in learning means communication, and communication means using language (speech) and symbols (written language as well as mathematical symbols) to convey thoughts. If mathematical learning is to take place in an interactive setting, students must be encouraged to "talk" mathematics.
8. **The Need for a Well-Defined Discourse**
Discourse goes beyond the idea of a conversation. It refers to all communication practices of the classroom, both written and verbal. Discourse implies that the resulting communication follows specific rules. Researchers now recommend that rules be adjusted to the needs and potential of the learning child (Siegler, 2003). This does not mean giving up the need for rigour, but it does mean carefully choosing which rules we use and which rules we modify, and making these rules clear to students.
9. **The Need for Belonging**
The desire to belong and be counted as a member of a particular social group is a powerful force behind our actions. Learning by participation requires us to be a part of a learning community. Students need to feel respected and free to speak their mind in the classroom. However, the extent to which students value mathematics is influenced by the value given to mathematics by the wider community (Comiti and Ball, 1996). Thus, it may be difficult to instill a desire to embrace mathematics in an environment where mathematics is not valued. The most promising directions for improvement seem to be those that incorporate historical context in the mathematics content, portray mathematics as something unique in our world, and present it as something to be valued for its own sake (Sfard, 2003).
10. **The Need for Balance**
To meet learners' wide range of needs, the pedagogy must be variegated and rich in possibilities. The need for balance suggests an advantage in searching for the good in all theories. It does not imply that old and new are mutually exclusive. The reality is that there must be a bit of everything in the classroom: problem solving as well as skills practice, teamwork as well as individual learning and teacher exposition, real-life problems as well as abstract problems, and learning by talking as well as silent learning.

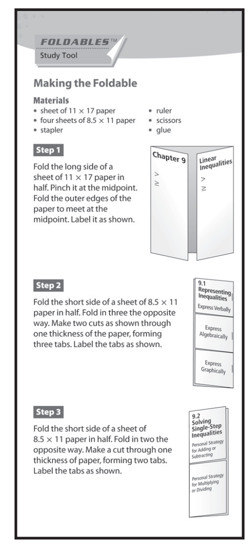
ASSESSMENT

A variety of assessment strategies and tools are employed to accommodate the diversity of students' abilities and styles of learning.



Assessment *as* Learning (Diagnostic)

These assessment tools include student reflection. They are provided throughout the *MathLinks* student resource and Teacher's Resource to assist the teacher in programming by identifying student weaknesses and gaps.



- The **Foldables™** activity in each chapter gives students a way to organize their learning and provides them with opportunities to express their understanding in their own words. A unique part of each chapter Foldable asks students to keep track of what they need to work on, allowing them to be self-directed learners.
- In *MathLinks 7* and *MathLinks 8*, the **Reflect on Your Findings** questions in each Explore the Math provide early opportunities for students to construct knowledge about the section content. In *MathLinks 9*, the Reflect and Check questions in the **Explore** serve the same purpose.
- The **Communicate the Ideas** questions allow students to explore their initial understandings of a concept.
- The **Warm-Up** exercises, journaling questions, and **Math Learning Log** suggestions in the Teacher's Resource provide additional support in identifying and facilitating student learning.
- The suggested assignments, questions, **Problems of the Week**, and activities in the **Meeting Student Needs** boxes in the Teacher's Resource address a variety of learner needs, including those of English language learners and gifted and enrichment students.
- Diagnostic support in the form of introductory questions designed to open discussion in the classroom and in the form of exploration activities are provided in the Teacher's Resource, where appropriate.

Assessment *for* Learning (Formative)

Formative assessment tools are provided throughout the *MathLinks* student resource and the Teacher's Resource.

- The **Math Link** and the **Literacy Link** at the beginning of each chapter activate learning necessary for students' success in the upcoming chapter.
- The *MathLinks Practice and Homework Books* and the *MathLinks* book site include a **Get Ready** section designed to provide teachers with an opportunity to activate student knowledge and assess the understanding that students should have to begin the chapter. In *MathLinks 7*, alternative, open-ended assessments for the Get Ready are provided as blackline masters.

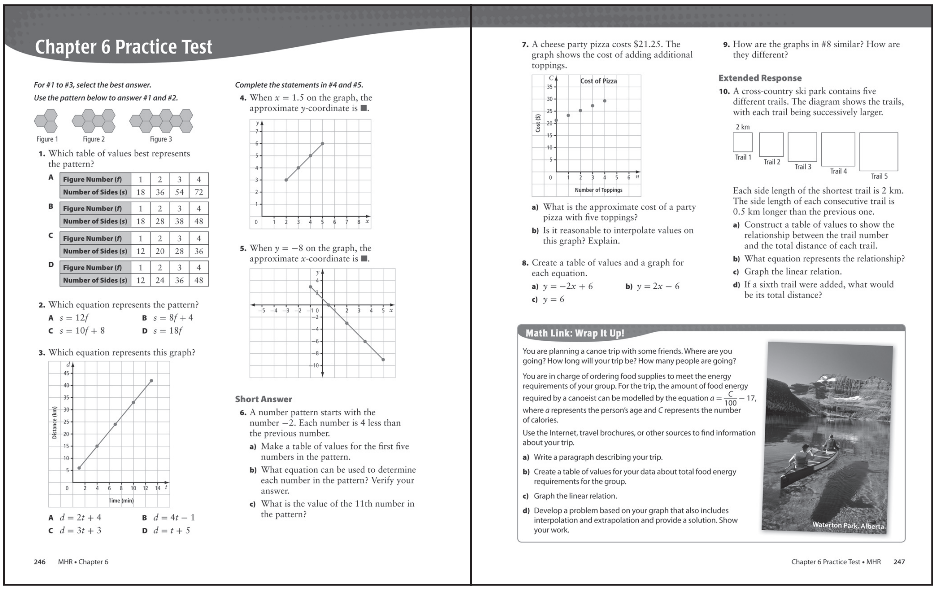
These assessments focus on determining if students possess both procedural knowledge and conceptual understanding of prerequisite concepts. In *MathLinks 8* and *MathLinks 9*, the alternative open-ended assessment is in the Assessment for Learning box on the back of the chapter fold-out.

- Additional support in the Teacher's Resource and on the *MathLinks* Online Learning Centre provides assistance for identifying and supporting weaknesses in students' learning.
- The *MathLinks* Teacher's Resource provides **Blackline Masters** that complement the student resource in areas where formative assessment indicates students may need further support.
- The **Reflect on Your Findings** in *MathLinks 7* and *MathLinks 8*, the **Reflect and Check** in *MathLinks 9*, and the **Communicate the Ideas** questions provide an opportunity to determine students' understanding of concepts through conversations and/or written work.
- The **Show You Know** questions target key skills of a section.
- Students can use the **Practise** assignments in each section to check their understanding.
- The **Math Links** at the end of most sections allow students to apply their understanding of the lesson's concepts to a problem that is linked to the **Math Link: Wrap It Up!** at the end of each chapter.
- The **chapter reviews** and **cumulative chapter reviews** provide opportunities to assess knowledge/understanding, applications, communications, mental math, and problem solving.

Assessment *of* Learning (Summative)

Summative assessment is provided in the following ways:

- **Practice tests** are provided at the end of the chapters in the student resource, and **chapter tests** are provided as blackline masters in the Teacher's Resource.



Chapter 6 Practice Test

For #1 to #3, select the best answer.

Use the pattern below to answer #1 and #2.

Figure 1 Figure 2 Figure 3

1. Which table of values best represents the pattern?

A
Figure Number (f)	1	2	3	4
Number of Sides (s)	18	36	54	72

B
Figure Number (f)	1	2	3	4
Number of Sides (s)	18	28	38	48

C
Figure Number (f)	1	2	3	4
Number of Sides (s)	12	20	28	36

D
Figure Number (f)	1	2	3	4
Number of Sides (s)	12	24	36	48

2. Which equation represents the pattern?
   A $s = 12f$ B $s = 8f + 4$
   C $s = 10f + 8$ D $s = 18f$

3. Which equation represents this graph?
   A $d = 2t + 4$ B $d = 4t - 1$
   C $d = 3t + 3$ D $d = t + 5$

Complete the statements in #4 and #5.

4. When $x = 1.5$ on the graph, the approximate y-coordinate is ■.

5. When $y = -8$ on the graph, the approximate x-coordinate is ■.

Short Answer

6. A number pattern starts with the number −2. Each number is 4 less than the previous number.
   a) Make a table of values for the first five numbers in the pattern.
   b) What equation can be used to determine each number in the pattern? Verify your answer.
   c) What is the value of the 11th number in the pattern?

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7. A cheese party pizza costs $21.25. The graph shows the cost of adding additional toppings.
   a) What is the approximate cost of a party pizza with five toppings?
   b) Is it reasonable to interpolate values on this graph? Explain.

8. Create a table of values and a graph for each equation.
   a) $y = -2x + 6$ b) $y = 2x - 6$
   c) $y = 6$

9. How are the graphs in #8 similar? How are they different?

Extended Response

10. A cross-country ski park contains five different trails. The diagram shows the trails, with each trail being successively larger.
    Each side length of the shortest trail is 2 km. The side length of each consecutive trail is 0.5 km longer than the previous one.
    a) Construct a table of values to show the relationship between the trail number and the total distance of each trail.
    b) What equation represents the relationship?
    c) Graph the linear relation.
    d) If a sixth trail were added, what would be its total distance?

Math Link: Wrap It Up!

You are planning a canoe trip with some friends. Where are you going? How long will your trip be? How many people are going?

You are in charge of ordering food supplies to meet the energy requirements of your group. For the trip, the amount of food energy required by a canoeist can be modelled by the equation $a = \frac{C}{100} - 17$, where a represents the person's age and C represents the number of calories.

Use the Internet, travel brochures, or other sources to find information about your trip.

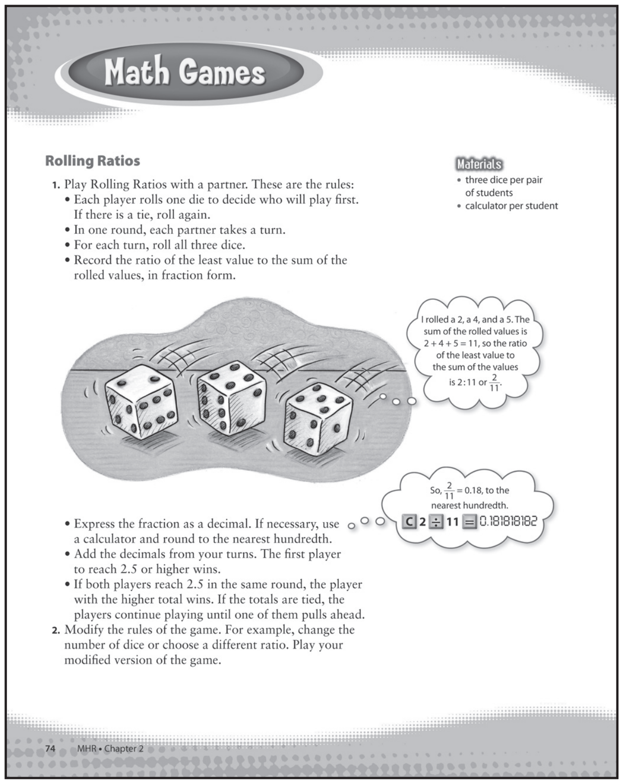
a) Write a paragraph describing your trip.
b) Create a table of values for your data about total food energy requirements for the group.
c) Graph the linear relation.
d) Develop a problem based on your graph that also includes interpolation and extrapolation and provide a solution. Show your work.

Waterton Park, Alberta

Chapter 6 Practice Test • MHR 247

- The **Math Link: Wrap It Up!** at the end of each chapter provides teachers with an opportunity to check whether students have synthesized the concepts and procedures. A rubric for each Wrap It Up! is included in the Teacher's Resource. Student exemplars are on the *MathLinks* Online Learning Centre.

- In *MathLinks 7* and *MathLinks 8*, **Math Games** at the end of each chapter give students and teachers another opportunity for assessment. These games are linked to concepts studied in the chapter. Some games also review outcomes in previous chapters. In *MathLinks 9*, some Challenges take the form of a game.



Math Games

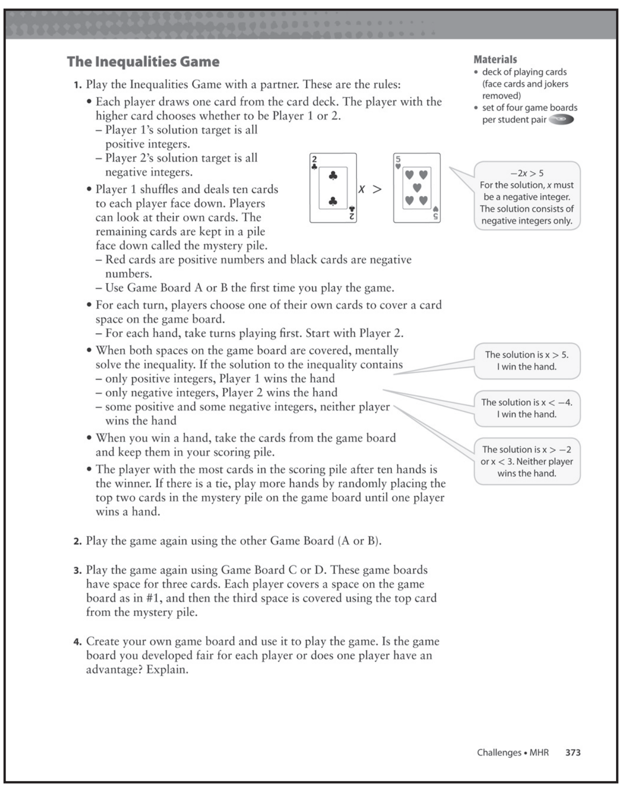
Rolling Ratios

Materials
- three dice per pair of students
- calculator per student

1. Play Rolling Ratios with a partner. These are the rules:
   - Each player rolls one die to decide who will play first. If there is a tie, roll again.
   - In one round, each partner takes a turn.
   - For each turn, roll all three dice.
   - Record the ratio of the least value to the sum of the rolled values, in fraction form.

   - Express the fraction as a decimal. If necessary, use a calculator and round to the nearest hundredth.
   - Add the decimals from your turns. The first player to reach 2.5 or higher wins.
   - If both players reach 2.5 in the same round, the player with the higher total wins. If the totals are tied, the players continue playing until one of them pulls ahead.
2. Modify the rules of the game. For example, change the number of dice or choose a different ratio. Play your modified version of the game.

74 MHR • Chapter 2



The Inequalities Game

Materials
- deck of playing cards (face cards and jokers removed)
- set of four game boards per student pair

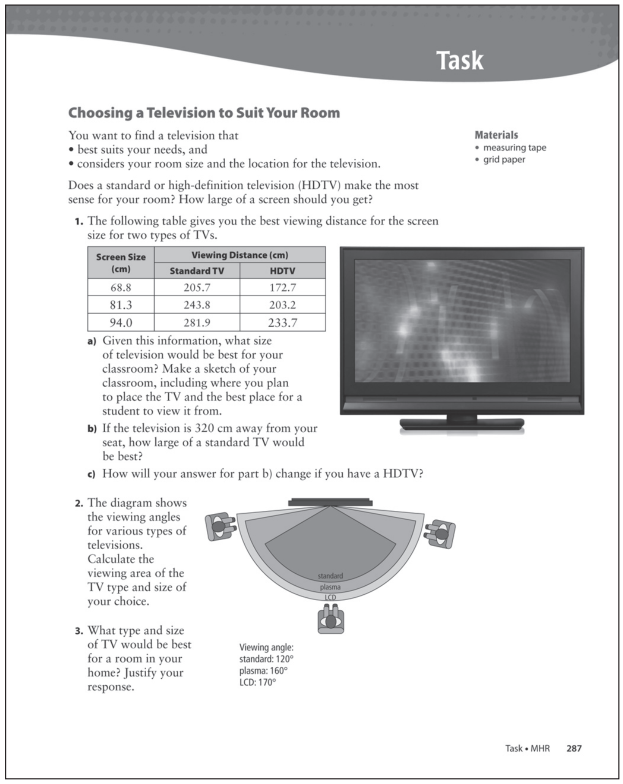
1. Play the Inequalities Game with a partner. These are the rules:
   - Each player draws one card from the card deck. The player with the higher card chooses whether to be Player 1 or 2.
     – Player 1's solution target is all positive integers.
     – Player 2's solution target is all negative integers.
   - Player 1 shuffles and deals ten cards to each player face down. Players can look at their own cards. The remaining cards are kept in a pile face down called the mystery pile.
     – Red cards are positive numbers and black cards are negative numbers.
     – Use Game Board A or B the first time you play the game.
   - For each turn, players choose one of their own cards to cover a card space on the game board.
     – For each hand, take turns playing first. Start with Player 2.
   - When both spaces on the game board are covered, mentally solve the inequality. If the solution to the inequality contains
     – only positive integers, Player 1 wins the hand
     – only negative integers, Player 2 wins the hand
     – some positive and some negative integers, neither player wins the hand
   - When you win a hand, take the cards from the game board and keep them in your scoring pile.
   - The player with the most cards in the scoring pile after ten hands is the winner. If there is a tie, play more hands by randomly placing the top two cards in the mystery pile on the game board until one player wins a hand.

2. Play the game again using the other Game Board (A or B).
3. Play the game again using Game Board C or D. These game boards have space for three cards. Each player covers a space on the game board as in #1, and then the third space is covered using the top card from the mystery pile.
4. Create your own game board and use it to play the game. Is the game board you developed fair for each player or does one player have an advantage? Explain.

Challenges • MHR 373

- A **Challenge in Real Life** is provided at the end of every chapter and it is accompanied by a rubric and suggested scoring in the Teacher's Resource. Student exemplars are on the *MathLinks* book site. There are one or two Challenges at the end of each chapter in *MathLinks 9*. Some challenges have students play or develop and play a game that will help them reinforce chapter outcomes.

- A **Task** is included after each cumulative review in *MathLinks 7* and *MathLinks 8*, and after the Chapter 4 and Chapter 7 cumulative reviews in *MathLinks 9*. An accompanying rubric and suggested scoring can be found in the Teacher's Resource. Student exemplars are provided on the *MathLinks* Online Learning Centre.


Task

Choosing a Television to Suit Your Room

You want to find a television that
• best suits your needs, and
• considers your room size and the location for the television.

Materials
• measuring tape
• grid paper

Does a standard or high-definition television (HDTV) make the most sense for your room? How large of a screen should you get?

1. The following table gives you the best viewing distance for the screen size for two types of TVs.

Screen Size (cm)	Viewing Distance (cm)	
	Standard TV	HDTV
68.8	205.7	172.7
81.3	243.8	203.2
94.0	281.9	233.7

a) Given this information, what size of television would be best for your classroom? Make a sketch of your classroom, including where you plan to place the TV and the best place for a student to view it from.
b) If the television is 320 cm away from your seat, how large of a standard TV would be best?
c) How will your answer for part b) change if you have a HDTV?

2. The diagram shows the viewing angles for various types of televisions. Calculate the viewing area of the TV type and size of your choice.

3. What type and size of TV would be best for a room in your home? Justify your response.

standard
plasma
LCD

Viewing angle:
standard: 120°
plasma: 160°
LCD: 170°

Task • MHR 287

- A **Computerized Assessment Bank (CAB)** offers a database of additional questions. The database includes a variety of question types (True/False, Multiple Choice, Completion, Matching, Short Answer, and Problems) and levels of difficulty (easy, average, and difficult). True/False questions are present in the *MathLinks 7 CAB*, but not in either the *MathLinks 8 CAB* or *MathLinks 9 CAB*.

Portfolio Assessment

Student-selected portfolios provide a powerful platform for assessing students' mathematical thinking. Portfolios provide the following benefits:
- help teachers assess students' growth and mathematical understanding
- give insight into students' self-awareness about their own progress
- help parents/guardians understand their child's growth

MathLinks 9 has many components that provide ideal portfolio items. Including any or all of the following chapter items is a non-threatening, formative way to gain insight into students' progress:
- student responses to the chapter opener
- answers to the **Reflect and Check** questions, which give students early opportunities to construct knowledge about the section content
- answers to the **Communicate the Ideas** questions, which allow students to explore their initial understanding of concepts

- journal and **Math Learning Log** responses, which show student understanding of the chapter skills and processes
- student responses to the **Math Link: Wrap It Up!** assignments
- **Task** and **Challenge** assignments, which show student understanding, usually across several chapters and strands

Master 1 Project Rubric

The **Master 1 Project Rubric** may be used for all assessments of **Math Links: Wrap It Up!** assignments, **Challenges**, and **Tasks**. This unique rubric includes

- a Score/Level grade ranging from 1 to 5 (Beginning to Standard of Excellence) **Note:** The Teacher Centre on the Online Learning Centre provides a four-level rubric and related exemplars.
- a Holistic Descriptor for each grade range, describing the level of understanding and communication skills
- Specific Question Notes, which provide suggested solutions typical of each grade range. These notes are meant to represent what the majority of students display. They are by no means exhaustive of all possible solutions. Teachers are encouraged to continually refer to both the specific and holistic pieces of the rubric.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	Demonstrates one of the following: • provides a complete response with weak communication or missing justification in one part of the question • provides a complete and correct response to all parts of the question except f); incorrectly uses 0.1% or is unable to determine changes needed to make a profit
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	Demonstrates one of the following: • correctly completes parts a) to d), with an error in one of the expressions • provides a correct response to parts a), c), d), and e) based on an incorrect part b) • provides partially correct answers to all parts of the question
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	Demonstrates one of the following: • correctly completes parts a) and b) and a significant, if not complete, part c) • provides a correct response to parts a) and b), with correct starts to any two remaining parts
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	Demonstrates one of the following: • provides a correct initial start to two parts of the problem • correctly completes part a), identifies a population, and starts part b); communication may be weak

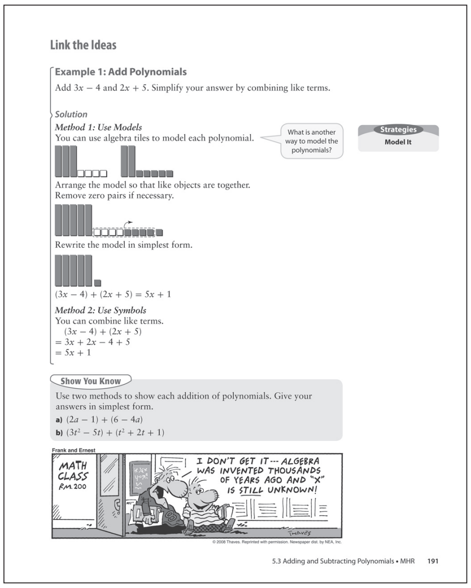
Teachers are encouraged to share the rubric with students early in the year. This will help them become active participants in their own assessment and program planning. Discussing and building the Specific Question Notes with students allows them to engage actively in their learning.

CONCRETE MATERIALS

The McGraw-Hill Ryerson *MathLinks* program engages students in a variety of worthwhile mathematical tasks that span the continuum from concrete to abstract.

Where appropriate, concept development in the program begins with students working with concrete materials. Most **Explore the Math** activities in *MathLinks 7* and *MathLinks 8*, and **Explore** activities in *MathLinks 9* have students using commonplace materials and conventional mathematical manipulatives in a hands-on approach. Pictorial images of the materials support the text and accommodate the stages of investigations in the absence of concrete materials. After an appropriate number of hands-on opportunities, students move from the pictorial to the symbolic in the **examples**, **Show You Know**, and **Check Your Understanding** exercises.

An example of how students move through the continuum of learning can be seen in the development of the concept of adding and subtracting polynomials. Students begin by using algebra tiles or other models to show addition and subtraction. Pictorial images of the models are paired in the text with sample solutions. Once students become confident with their understanding, they can use only the symbolic to add and subtract polynomials. Some students may need to stay at the concrete and/or pictorial stage longer than others.



Link the Ideas

Example 1: Add Polynomials

Add $3x - 4$ and $2x + 5$. Simplify your answer by combining like terms.

Solution

Method 1: Use Models

You can use algebra tiles to model each polynomial.

What is another way to model the polynomials?

Strategies: Model It

Arrange the model so that like objects are together. Remove zero pairs if necessary.

Rewrite the model in simplest form.

$(3x - 4) + (2x + 5) = 5x + 1$

Method 2: Use Symbols

You can combine like terms.

$(3x - 4) + (2x + 5)$
$= 3x + 2x - 4 + 5$
$= 5x + 1$

Show You Know

Use two methods to show each addition of polynomials. Give your answers in simplest form.

a) $(2a - 1) + (6 - 4a)$

b) $(3t^2 - 5t) + (t^2 + 2t + 1)$

Frank and Ernest

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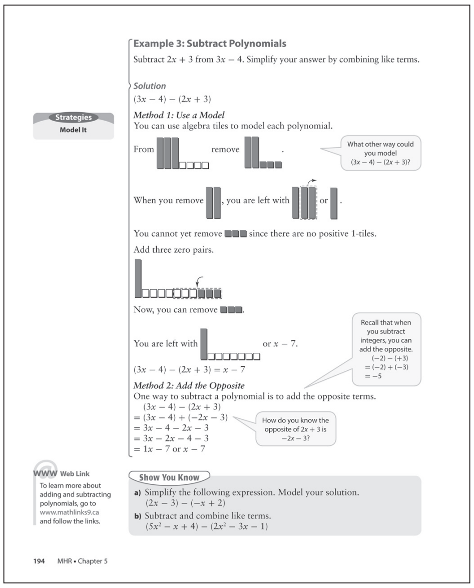
5.3 Adding and Subtracting Polynomials • MHR 191



Example 3: Subtract Polynomials

Subtract $2x + 3$ from $3x - 4$. Simplify your answer by combining like terms.

Solution

$(3x - 4) - (2x + 3)$

Strategies: Model It

Method 1: Use a Model

You can use algebra tiles to model each polynomial.

From [tiles] remove [tiles].

What other way could you model $(3x - 4) - (2x + 3)$?

When you remove [tiles], you are left with [tiles] or [tile].

You cannot yet remove [tiles] since there are no positive 1-tiles.

Add three zero pairs.

Now, you can remove [tiles].

You are left with [tiles] or $x - 7$.

$(3x - 4) - (2x + 3) = x - 7$

Recall that when you subtract integers, you can add the opposite. $(-2) - (+3) = (-2) + (-3) = -5$

Method 2: Add the Opposite

One way to subtract a polynomial is to add the opposite terms.

$(3x - 4) - (2x + 3)$
$= (3x - 4) + (-2x - 3)$
$= 3x - 4 - 2x - 3$
$= 3x - 2x - 4 - 3$
$= 1x - 7$ or $x - 7$

How do you know the opposite of $2x + 3$ is $-2x - 3$?

Web Link: To learn more about adding and subtracting polynomials, go to www.mathlinks9.ca and follow the links.

Show You Know

a) Simplify the following expression. Model your solution.
$(2x - 3) - (-x + 2)$

b) Subtract and combine like terms.
$(5x^2 - x + 4) - (2x^2 - 3x - 1)$

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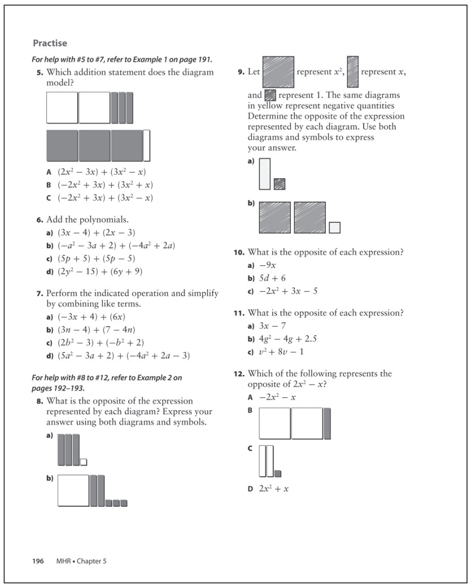

Practise

For help with #5 to #7, refer to Example 1 on page 191.

5. Which addition statement does the diagram model?

A $(2x^2 - 3x) + (3x^2 - x)$
B $(-2x^2 + 3x) + (3x^2 + x)$
C $(-2x^2 + 3x) + (3x^2 - x)$

6. Add the polynomials.

a) $(3x - 4) + (2x - 3)$
b) $(-a^2 - 3a + 2) + (-4a^2 + 2a)$
c) $(5p + 5) + (5p - 5)$
d) $(2y^2 - 15) + (6y + 9)$

7. Perform the indicated operation and simplify by combining like terms.

a) $(-3x + 4) + (6x)$
b) $(3n - 4) + (7 - 4n)$
c) $(2b^2 - 3) + (-b^2 + 2)$
d) $(5a^2 - 3a + 2) + (-4a^2 + 2a - 3)$

For help with #8 to #12, refer to Example 2 on pages 192–193.

8. What is the opposite of the expression represented by each diagram? Express your answer using both diagrams and symbols.

a)
b)

9. Let [tile] represent x^2, [tile] represent x, and [tile] represent 1. The same diagrams in yellow represent negative quantities Determine the opposite of the expression represented by each diagram. Use both diagrams and symbols to express your answer.

a)
b)

10. What is the opposite of each expression?

a) $-9x$
b) $5d + 6$
c) $-2x^2 + 3x - 5$

11. What is the opposite of each expression?

a) $3x - 7$
b) $4g^2 - 4g + 2.5$
c) $v^2 + 8v - 1$

12. Which of the following represents the opposite of $2x^2 - x$?

A $-2x^2 - x$
B
C
D $2x^2 + x$

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TECHNOLOGY

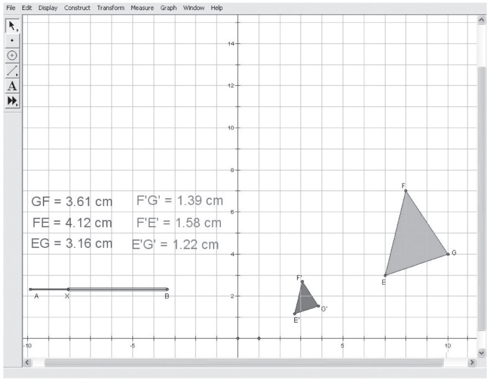
Where appropriate, lessons are designed to provide students with the opportunity to develop their skills in the use of calculators and spreadsheets, but not to rely on this technology to think mathematically. Students are also asked to use the Internet to research information related to problems they are required to solve.

The student resource provides technology learning that matches technology requirements for curriculum expectations. In *MathLinks 9*, some Tech Links refer students to the *MathLinks 9* Online Learning Centre where they can use software to extend their understanding of a concept.

Tech Link

Similarity and Scale Factors

In this activity, you can use dynamic geometry software to explore similarity and scale factors. To use this activity, go to www.mathlinks9.ca and follow the links.



Explore

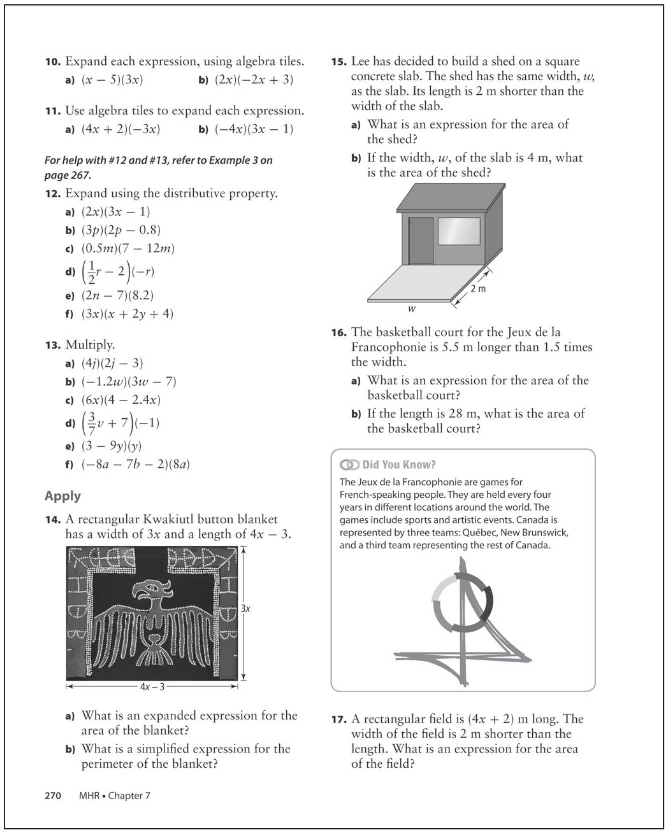
1. Slide point X along line segment AB and describe what happens to the image drawing.
2. How do the measures of the corresponding sides of the drawing change relative to each other? Explain.
3. Compare the scale factor to the lengths of the sides of the original drawing and the image drawing. Create and complete a table similar to the one below with measurements taken at different locations. Discuss your findings with a classmate. Hint: In the table, *m* means *the measure of*.

***m*FE**	***m*F′E′**	$\frac{m\text{AX}}{m\text{XB}}$

Blackline Masters of text-based technology activities that can easily be used in a computer laboratory are also included in the Teacher's Resource when grade-specific outcomes suggest these are needed. The masters include directions for using a number of different softwares common in many classrooms.

CAPITALIZING ON DIVERSITY AND REAL LIFE

Throughout the student resource, students are given opportunities to see how mathematics connects to real life by engaging in meaningful problem solving situations. Chapters are introduced with problems that model real life. Visual images used to introduce lessons, as well as those in the **Explore the Math** in *MathLinks 7* and *MathLinks 8*, in the **Explore** in *MathLinks 9*, and in the exercise sets, depict the cultural diversity within classrooms. Examples of mathematics from other cultures are evident throughout the text. Names used in the lessons and exercises also reflect the diversity of Canadian society.


10. Expand each expression, using algebra tiles.
a) $(x - 5)(3x)$ b) $(2x)(-2x + 3)$

11. Use algebra tiles to expand each expression.
a) $(4x + 2)(-3x)$ b) $(-4x)(3x - 1)$

For help with #12 and #13, refer to Example 3 on page 267.

12. Expand using the distributive property.
a) $(2x)(3x - 1)$
b) $(3p)(2p - 0.8)$
c) $(0.5m)(7 - 12m)$
d) $\left(\frac{1}{2}r - 2\right)(-r)$
e) $(2n - 7)(8.2)$
f) $(3x)(x + 2y + 4)$

13. Multiply.
a) $(4j)(2j - 3)$
b) $(-1.2w)(3w - 7)$
c) $(6x)(4 - 2.4x)$
d) $\left(\frac{3}{7}v + 7\right)(-1)$
e) $(3 - 9y)(y)$
f) $(-8a - 7b - 2)(8a)$

Apply

14. A rectangular Kwakiutl button blanket has a width of $3x$ and a length of $4x - 3$.

a) What is an expanded expression for the area of the blanket?
b) What is a simplified expression for the perimeter of the blanket?

15. Lee has decided to build a shed on a square concrete slab. The shed has the same width, w, as the slab. Its length is 2 m shorter than the width of the slab.
a) What is an expression for the area of the shed?
b) If the width, w, of the slab is 4 m, what is the area of the shed?

16. The basketball court for the Jeux de la Francophonie is 5.5 m longer than 1.5 times the width.
a) What is an expression for the area of the basketball court?
b) If the length is 28 m, what is the area of the basketball court?

Did You Know?
The Jeux de la Francophonie are games for French-speaking people. They are held every four years in different locations around the world. The games include sports and artistic events. Canada is represented by three teams: Québec, New Brunswick, and a third team representing the rest of Canada.

17. A rectangular field is $(4x + 2)$ m long. The width of the field is 2 m shorter than the length. What is an expression for the area of the field?

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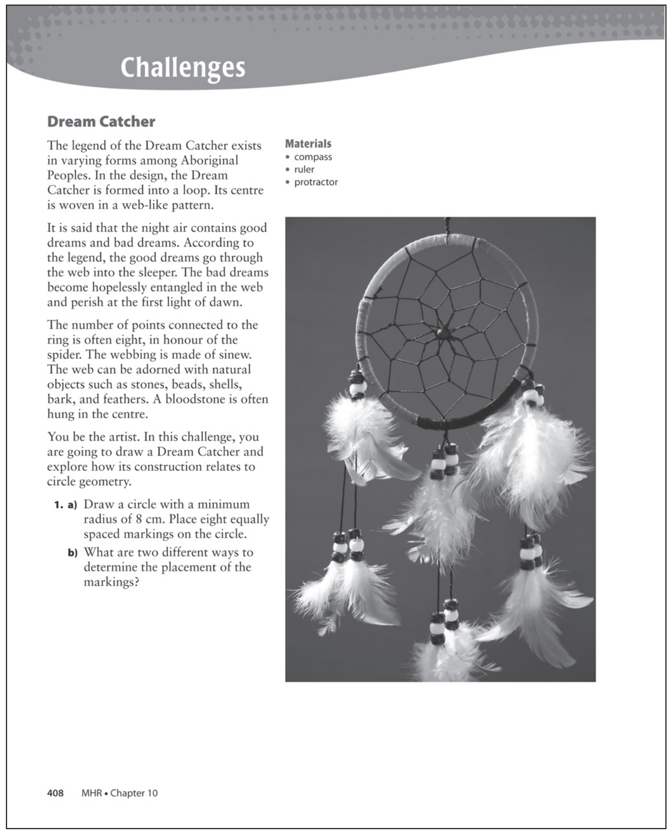
Challenges

Dream Catcher

The legend of the Dream Catcher exists in varying forms among Aboriginal Peoples. In the design, the Dream Catcher is formed into a loop. Its centre is woven in a web-like pattern.

It is said that the night air contains good dreams and bad dreams. According to the legend, the good dreams go through the web into the sleeper. The bad dreams become hopelessly entangled in the web and perish at the first light of dawn.

The number of points connected to the ring is often eight, in honour of the spider. The webbing is made of sinew. The web can be adorned with natural objects such as stones, beads, shells, bark, and feathers. A bloodstone is often hung in the centre.

You be the artist. In this challenge, you are going to draw a Dream Catcher and explore how its construction relates to circle geometry.

Materials
- compass
- ruler
- protractor

1. a) Draw a circle with a minimum radius of 8 cm. Place eight equally spaced markings on the circle.
   b) What are two different ways to determine the placement of the markings?

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Grouping

There are multiple opportunities throughout the program for teachers to use different types of student groupings. The **Explore the Math** sections in *MathLinks 7* and *MathLinks 8* and the **Explore** sections in *MathLinks 9* lend themselves to group work, but teachers are free to choose student groupings that meet their needs. Additional suggestions are also provided in the Teacher's Resource.

Home Connections

The design of the McGraw-Hill Ryerson *MathLinks* program recognizes that students' learning in mathematics also takes place outside of the classroom as they complete their homework, work with parents/guardians, and employ their mathematical skills in everyday life. The following features support learning outside of the classroom:

- **Key Ideas** provide summaries and worked examples to serve as references for students and parents/guardians when doing homework.
- Visuals and **Key Ideas** allow investigations to be easily followed independently.
- Opportunities for bringing mathematics activities home are provided through **Practise/Apply/Extend**, **Math Links**, **Math Games**, **Challenges**, and **Tasks**.
- The ***MathLinks Practice and Homework Books*** provide additional opportunities for parents/guardians to assist students in developing needed skills.
- Additional activities, as well as games and puzzles, are available on the McGraw-Hill Ryerson **Online Learning Centre**, which includes a **Parent Centre**.

Online Learning Centre

COOPERATIVE LEARNING

There are multiple opportunities throughout the program for teachers to use different types of classroom groupings. The explorations at the beginning of each section lend themselves to being completed in groups, but teachers are free to choose class groupings that meet the needs of their students. Additional suggestions are also provided in this Teacher's Resource.

Students learn effectively when they are actively engaged in the process of learning. Most sections of *MathLinks 9* begin with a hands-on activity that fosters this approach. These activities are best done through cooperative learning during which students work together—either with a partner or in a small group of three or four—to complete the activity and develop generalizations about the topic or process.

Group learning such as this is an important aspect of a constructivist educational approach. It encourages interactions and increases chances for students to communicate and learn from each other.[1]

Teachers' Role—In classrooms where students are adept at cooperative learning, the teacher becomes the facilitator, guide, and progress monitor. Until students have reached that level of group cooperation, however, the teacher will need to coach them in how to learn cooperatively. This may include

- making sure that the materials are at hand and directions perfectly clear, so that students know what they are doing before starting group work
- carefully structuring activities so that students can work together
- coaching how to provide peer feedback in a way that allows the listener to hear and attend
- constantly monitoring student progress and providing assistance to groups having problems with either group cooperation or the math at hand

Group Composition—The size of group may vary from activity to activity. Small-group settings allow students to take risks that they might not take in a whole class.[2] Research suggests that small groups are fertile environments for developing mathematical reasoning.[3]

Results of international studies suggest that groups of mixed ability work well in mathematics classrooms.[4] If your class is new to cooperative learning, you may wish to assign students to groups according to the specific skills of each individual. For example, pair a student who is talkative but weak in number sense and numeration with a quiet student who is strong in those areas. Pair a student who is weak in many parts of mathematics but has excellent spatial sense with a stronger mathematics student who has poor spatial sense. In this way, student strengths and weaknesses complement each other, and peers have a better chance of recognizing the value of working together.

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[1] Sternberg, R.J., and W.M. Williams, *Educational Psychology* (Boston, MA: Allyn & Bacon, 2002).

[2] Van De Walle, J., *Elementary and Middle School Mathematics: Teaching Developmentally*, 4th ed. (Boston, MA: Addison Wesley Longman, 2000).

[3] Artzt, A.F., and S. Yaloz-Femia, "Mathematical Reasoning During Small-Group Problem Solving," in L. Stiff and F. Curcio (eds.), *Developing Mathematical Reasoning in Grades K–12* (Reston, VA: National Council of Teachers of Mathematics, 1999), 115–26.

[4] Kilpatrick, J., J. Swafford, and B. Findell, *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: National Academy Press, 2001).

Cooperative Learning Skills—When coaching students about cooperative learning, consider task skills and working relationship skills.

Task Skills	Working Relationship Skills
• following directions • communicating information and ideas • seeking clarification • ensuring that others understand • actively listening to others • staying on task	• encouraging others to contribute • acknowledging and responding to the contributions of others • checking for agreement • disagreeing in an agreeable way • mediating disagreements within the group • sharing • showing appreciation for the efforts of others

Use class discussions, modelling, role-plays, and drama to provide positive task skills. For example, role-play different ways to provide feedback and have a class discussion on which ones students like and why. Discuss common group roles and how group members can use them. Make sure students understand that the same person can play more than one role.

Role	Job	Sample Comment
Leader	• makes sure the group is on task and everyone is participating • pushes group to come to a decision	Let's do this. Can we decide ... ? This is what I think we should do ...
Recorder	• manages materials • writes down data collected or measurements made	This is what I wrote down. Is that what you mean?
Presenter	• presents the group's results and conclusions	This is what the group thinks ...
Organizer	• watches time • keeps on topic • encourages getting the job done	Let's get started. Where should we start? So far we've done the following ... Are we on topic? What else do we need to do?
Clarifier	• checks that members understand and agree	Does everyone understand? So, what I hear you saying is ... Do you mean that ... ?

Types of Groups

Three group types are commonly used in the mathematics classroom.

Think/Pair/Share—This consists of having students individually think about a concept and then pick a partner to share their ideas. For example, students might work on the **Communicate the Ideas** questions and then choose a partner to discuss the concepts with. Working together, the partners could expand on what they understood individually. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

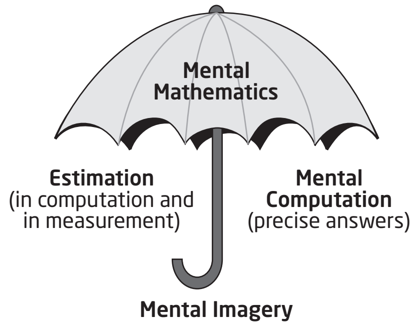
Cooperative Task Group—Task groups of two to four students can work on activities in the explorations at the beginning of each section. As a group, students can share their understanding of what is happening during the activity and how that relates to the mathematics topic, at the same time as they develop group cooperation skills.

Jigsaw—Another common cooperative learning group is called a jigsaw. In this technique, individual group members are responsible for researching and understanding a specific area of information for a project. Individual students then share what they have learned so that the entire group gets information about all areas being studied. For example, during data management, this type of group might have "experts" in the advantages of a particular type of graph. Group members could then coach each other on the best graph(s) to use a particular application.

Another way of using the jigsaw method is to assign "home" and "expert" groups during a large project. For example, students researching nutrition might have a home group in which each member is responsible for researching how carbohydrates, calcium, fibre, and protein contribute to good nutrition. Individual members could then move to expert groups. Expert groups would include all of the students responsible for researching each of the nutrients. Each of the expert groups would research foods that provide that nutrient and how much of the nutrient each food provides. They can use this information to develop math problems related to the outcomes being studied. Once the problems have been developed, individual members of the expert group could return to their home group and have other members do the problem(s) developed. They can coach students in how to solve particular problem(s).

MENTAL MATHEMATICS

A major goal of mathematics instruction for the twenty-first century is for students to make sense of the mathematics in their lives. The development of all areas of mental mathematics is a major contributor to this comfort and understanding. Mental mathematics is the mental manipulation of knowledge dealing with numbers, shapes, and patterns to solve problems.



The diagram above shows the various components under the umbrella of Mental Mathematics. All three are considered mental activities and interact with each other to make the connections required for mathematics understanding. Estimation and mental math are not topics that can be isolated as a unit of instruction; they must be integrated throughout the study of mathematics.

Estimation

Estimation refers to the approximate answers for calculations, a very practical skill in today's world. The development of estimation skills helps refine mental computation skills, enhances number sense, and fosters confidence in math abilities, all of which are key in problem solving. Over 80% of out-of-school problem solving situations involve mental computation and estimation.[5]

Estimation does not mean guessing at answers. Rather, it involves a host of computational strategies that are selected to suit the numbers involved. The goal is to refine these strategies over time with regular practice, so that estimates become more precise. The ultimate goal is for students to estimate automatically and quickly when faced with a calculation. These estimations allow for recognition of errors on calculator displays, provide learners with a strategy for checking the reasonableness of their calculations, and give students a strategy for finding an answer when only an approximation is necessary.

Mental Imagery

Mental imagery in mathematics refers to the images in the mind when one is doing mathematics. It is this mental representation, or conceptual knowledge, that needs to be developed in all areas of mathematics. Capable math students "see" the math and are able to perform mental manoeuvres in order to make connections and solve problems. These images are formed

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[5]Reys, B. J., and R.E. Reys, "One Point of View: Mental Computation and Computational Estimation—Their Time Has Come," *Arithmetic Teacher* (Vol. 33, No. 7, 1986), 4–5.

when students manipulate objects, explore numbers and their meanings, and talk about their learning. Students must be encouraged to look into their mind's eye and "think about their thinking."

Asking, "What do you see in your mind's eye?" when asked to visualize, as in the exercises below, forces students to think about the images they are using to help them solve problems. Students are often surprised when fellow students share their personal images; the discussion generated is very worthwhile.

Try these mental imaging exercises with students.

Example 1: Draw the mental image you have for each of the following: • $\frac{2}{3}$ • 75% of the questions on the page • a 175° angle	**Example 2:** Use mental imagery to answer the following: **1.** How many edges does a cube have? **2.** If I am facing east, what direction is to my left? **3.** What is the perimeter of a 90 cm × 30 cm shelf?

Mental Computation

Mental computation refers to an operation used to obtain the precise answer for a calculation. Unlike traditional algorithms, which involve one method of calculation for each operation, mental computations include a number of strategies—often in combination with each other—for finding the exact answer. As with estimation, strategies for mental computation develop in quantity and quality over time. A thorough understanding of, and facility with, mental computation allows students to solve complicated multi-step problems without spending needless time figuring out calculations and is a valuable prerequisite for proficiency with algebra. Students need regular practice in these strategies.

Some Points Regarding Mental Mathematics

- Students must have a knowledge of the basic facts (addition and multiplication) in order to estimate and calculate mentally. They learn the many strategies for fact learning in elementary school. With practice, they eventually commit these facts to memory. Without knowing the basic facts, it is unlikely that students will ever attempt to employ any estimation or mental math strategies, as these will be too tedious.
- The various estimation and mental calculation strategies must be taught and are best developed in context; opportunities must be provided for regular practice of these strategies. Having students share their various strategies is vital, as it provides possible options for classmates to add to their repertoire.
- Unlike the traditional paper-and-pencil algorithms, there are many mental algorithms to learn. With the learning, however, comes a greater facility with numbers. Key to the development of skills in mental math is the understanding of place value (number sense) and the number operations. This understanding is enhanced when students make mental math a focus as they calculate.

- Mental math strategies are flexible; the student needs to select one that is appropriate for the numbers in the computation. Practice should be in the form of practising the strategy itself, selecting appropriate strategies for a variety of computation examples, and using the strategies in problem solving situations.
- Although students should not be pressured with time constraints when first learning a mental math strategy, it is beneficial to provide timed tests once they have some facility with mental computation. If too much time is provided, many students will resort to the traditional algorithm and will not use mental strategies.
- Mental math algorithms are used with whole numbers, fractions, and decimal numbers.
- Sometimes mental math strategies are used in conjunction with paper-and-pencil tasks. The questions are rewritten to make the calculation easier.
- The ultimate goal of mental mathematics is for students to estimate for reasonableness and to look for opportunities to calculate mentally.
- Encourage students to refer to the strategies by their name (e.g., front-end strategy). Once the strategies have been taught, post them around the room. Have students write problems in which a mental strategy would be the appropriate computation. Share these problems with the class.
- Students need to identify why particular procedures work; they should not be taught computation "tricks" without understanding.
- Those who are skilled in using mental mathematics will be able to transfer, relate, and apply mental strategies to paper-and-pencil tasks.

Keep in Mind

Practice in classrooms has traditionally been in the form of asking students to write the answers to questions presented orally. This is particularly challenging for students who are primarily visual learners. Although we are sometimes faced with computations of numbers we cannot see, most often the numbers are written down. This makes it easier to select a strategy. In daily life, we see the numbers when solving written problems (e.g., when checking calculations on a bill or invoice, when determining what to leave for tips, when calculating discounted prices from a price tag). Provide students with mental math practice that is sometimes oral and sometimes visual.